International workshop on "Quantum Physics and Geometry"
Levico, July 4th-6th, 2017

Abstracts

ANGELO BASSI *(Università degli Studi di Trieste, Italy)*

Ways to modify the Schrödinger equation

The Schrödinger equation is one of the building blocks of quantum mechanics, and its structure is crucial for the consistency of the theory. An interesting question is to which extent the structure of the equation can be changed, without running into inconsistencies. I will show how the requirement of no-faster-then-light signaling and norm conservation (and little more) identifies a unique class of modified dynamics. I will describe physical applications to models of spontaneous wave function collapse.

DANIEL BRAUN *(Universität Tübingen, Germany)*

The Geometry of Quantumness and Entanglement

The quantum states of a spin-j can be considered "classical" if they can be represented as a convex sum of SU(2)-coherent states. These are pure (i.e.~rank-1) states with minimal uncertainty of the angular-momentum operators. At the same time, these "classical" states correspond to separable multi-qubit states, symmetric under particle exchange. Spin-j states can be represented by a "Bloch-tensor" that generalizes the well-know Bloch-vector of a spin-1/2.

The "entanglement problem" is to decide whether a given spin-j state is entangled under a certain bi-partition of the underlying qubits. It can be seen as a characterization of the convex hull of separable states. We show that the Bloch tensor when partitioned into a matrix, is similar to the partial transpose of the density matrix in the computational basis, enabling an immediate sufficient condition for entanglement on the basis of the positive partial transposition (PPT) criterion. More generally, we show that the entanglement problem can be mapped to the "truncated moment problem" studied in mathematics. It gives rise to a hierarchy of extensions of a semi-definite program with polynomial constraints and the PPT criterion as a first step. that generalizes and unifies on a more abstract level previous approaches. Flat extensions play a crucial role and are a new systematic ingredient that allows us to prove separability of a state and obtain its explicit deomposition into a convex sum of product states.

LUCA CHIANTINI *(Università di Siena, Italy)*

Minimal decompositions and the geometry of finite sets.

In the study of the rank decomposition of tensors, often one knows a particular decomposition, and the problem concerns its minimality or uniqueness. This happens, for instance, in the study of the complexity of block tensors (Strassen's conjecture) or in the study of the minimality of special decompositions (Comon's problem). We show that a geometric analysis of the finite projective set associated with a decomposition yields some conclusions on its minimality or uniqueness. The geometric methods are essentially based on the analysis of numerical invariants of the finite set (Hilbert function, Segre function). The results go beyond the numerical range of applicability of the Kruskal's criterion.

MATTHIAS CHRISTANDL *(Department of Mathematical Sciences, Copenhagen, Denmark)*

On the tensor rank of networks of entangled pairs: tensor surgery and the laser method

We prove upper bounds on the tensor rank of networks of entangled pairs. Any graph defines such a network by associating an entangled pair to each edge of the graph. We present two methods. First, we introduce a surgery-like procedure to transform a good decomposition of a well-chosen tensor into a good decomposition of a tensor of interest. We illustrate the method with surgery on the cycle graph, which corresponds to the iterated matrix multiplication tensor and obtain the first nontrivial rank results for large odd cycles and optimal asymptotic rank results for all cycles. Second, we generalize Strassen's laser method to higher- order tensors in order to show a nontrivial upper bound on the asymptotic rank for the complete graph. "Per edge" this improves on the best upper bound on the matrix multiplication exponent [LG14], for four or more vertices. In entanglement theory, our results amount to protocols for creating a network of entangled pairs from GHZ states by SLOCC. In communication complexity theory, our results imply new bounds on the nondeterministic quantum communication of equality games. Our work is inspired and tightly connected with the vast body of research on matrix multiplication.

ELISA ERCOLESSI *(Università di Bologna, Italy)*

Symmetric logarithmic derivative and the Fisher tensor for mixed states of an n-level quantum system.

We will discuss the notion of symmetric logarithmic derivative and define the Fisher tensor on the space of mixed states of an n-level quantum system. We will describe the geometric nature of such a tensor, by reviewing the algebraic and geometrical structures with which such space is naturally endowed.

FREDERIC HOLWECK *(University of Technology of Belfort-Montbéliard, France)*

Entanglement, auxiliary varieties and simple singularities

Algebraic geometry can be used to describe SLOCC entanglement classes of pure multipartite quantum systems by introducing auxiliary varieties (secants, duals) of the set of separable states. Many entanglement classifications can be recast in this language allowing us to establish a dictionnary between works of geometers and questions from quantum information theory. After looking, from this perspective, at various classifications proposed in the past 15 years in the physics literature, I'll focus on the concept of dual varieties. I will show how simple singularities of hypersurfaces can be attached to entanglement classes and discuss what it could be good for.

ROBERT KÖNIG *(Technische Universität München, Germany)*

Quantum computation with Turaev-Viro codes

The Turaev-Viro invariant for a closed 3-manifold is defined as the contraction of a certain tensor network. The tensors correspond to tetrahedra in a triangulation of the manifold, with values determined by a fixed spherical category. For a manifold with boundary, the tensor network has free indices that can be associated to qudits, and its contraction gives the coefficients of a quantum error-correcting code. The code has local stabilizers determined by Levin and Wen. By studying braid group representations acting on equivalence classes of colored ribbon graphs embedded in a punctured sphere, we identify the anyons, and give a simple recipe for mapping fusion basis states of the doubled category to ribbon graphs. Combined with known universality results for anyonic systems, this provides a large family of schemes for quantum computation based on local deformations of stabilizer codes. These schemes may serve as a starting point for developing fault-tolerance schemes using continuous stabilizer measurements and active error-correction.

This is joint work with Greg Kuperberg and Ben Reichardt.

JOSEPH LANDSBERG *(Texas A&M University, College Station, TX, USA)*

On the quantum max flow/min cut problem.

The quantum max flow fails to be equal to the quantum min cut, in contrast to the classical case where equality holds. I will discuss this failure and questions regarding tensor networks that the problem gives rise to.

This is joint work with Micheal Walter.

CHIARA MACCHIAVELLO *(Università di Pavia, Italy)*

Entanglement, complementarity and correlations

We provide an interpretation of entanglement based on classical correlations between measurement outcomes of complementary properties for composite quantum systems. We start with the bipartite case and discuss in particular what classical correlations in the measurements of these complementary properties tell us about the quantum correlations of the state of the system under consideration. We show that states that have correlations for complementary observables beyond a certain threshold value are entangled. The reverse is not true, however. We also show that, surprisingly, bipartite separable states with quantum correlations exhibit smaller correlations for complementary observables with respect to classical states.

We use mutual information as a measure of classical correlations, but we conjecture that the first result holds also for other measures (e.g. the Pearson correlation coefficient or the sum of conditional probabilities). We extend this approach to multipartite systems and introduce new measures of multipartite quantum correlations based on classical correlations of complementary outcomes. We show how these measures,based on the classical mutual information, can be used to detect high-dimensional tripartite entanglement by using only a few local measurements.

GIUSEPPE MARMO *(Università di Napoli "Federico II", Italy)*

Evolution on the stratified manifold of quantum states

We shall argue that the space of quantum states of a finite dimensional quantum systems is a stratified manifold.Stratified by the rank of thestates.Three classes of vector fields arise naturally, the first one generates isospectral transformations,the second one generates transformations which change the spectrum but preserve the rank ,and the third one does not preserve the rank. All three of them are necessary to describe a linear Markovian evolution.

DAVIDE PASTORELLO *(Università di Trento, Italy)*

Two-way quantum key distribution based on tripartite entanglement

Entanglement is a well-known resourse in quantum information and quantum cryptography. The celebrated EPR protocol exploits quantum correlations of entangled pairs to share a private key between two clients and uses the violation of Bell inequalities to detect eavesdropping attacks.

In this talk I propose a two-way QKD scheme based on preparation and processing of *tripartite entangled states which turns out to be an extension of standard EPR protocol with a significant increasing of the secret key rate. The security of the QKD scheme is basically guaranteed by the entanglement monogamy and any eavesdropping attack can be detected by the computation of a test statistic to check the violation of CHSH inequality.*

ERIC ROWELL *(Texas A&M University, College Station, TX, USA)*

Topological Quantum Computation

The topological model for quantum computation is an inherently fault-tolerant computation scheme, storing information in topological (rather than local) degrees of freedom with quantum gates typically realized by braiding quasi-particles in two dimensional media. Representation theory plays several key roles in this model, and I will describe a few of them along with some related open problems.

BASSANO VACCHINI (Università degli Studi di Milano, Italy)

Open quantum systems and non-Markovian evolutions

We will introduce the theory of open quantum systems, relevant for systems whose interaction with unobserved degrees of freedom cannot be neglected. The emphasis will be on the structure and characterization of quantum dynamical maps providing the reduced system dynamics, featuring decoherence and dissipative effects. In particular we will discuss recent developments in the characterization of the dynamics of such open quantum systems which allow for a possible definition of quantum non-Markovianity, considering the possible connection with Markovian and non-Markovian classical stochastic processes.

FRANK VERSTRAETE (Universitaet Wien, Austria)

Characterizing topological phases of matter using matrix product operator algebras

JON YARD *(University of Waterloo and Perimeter Institute, Canada)*

Lines, designs and quantum mechanics over class fields

Wigner showed that the automorphism group of complex projective HIlbert space, the natural arena for quantum mechanics, is generated by unitary transformations and complex conjugation. Projective spaces over number fields can have further symmetries via field automorphisms commuting with complex conjugation. In this talk, I will describe what is known about the existence of tight (and thus minimal) complex projective 2-

designs, which turn out to be naturally defined over number fields. These fields are certain abelian extensions of real quadratic number fields (arXiv:1604.06098) known to exist by general theorems of class field theory but had never been found 'explicitly' in Nature until now. The 2-designs are known as SIC-POVMs (Symmetric Informationally Complete Positive Operator-Valued Measures) in quantum information theory, and also as maximal sets of equiangular lines in algebraic combinatorics. They are obtained as orbits of finite Heisenberg groups, essentially known as generalized Pauli groups in quantum information. These 2-designs have unitary symmetries under the Weil representation, which is basically known as the Clifford group in quantum information theory, and are intimately related to the structure of the underlying number fields. A proper understanding of this relationship may ultimately lead to a general proof of their existence in all dimensions, rather than the finite number of examples currently proved to exist. Throughout this talk, I will introduce all of the relevant mathematics and in particular, assume no prior knowledge of number theory.

PAOLO ZANARDI (USC, CA, USA)

Algebraic Topology, Quantum Algorithms and Big Data

Extracting useful information from large data sets can be a daunting task. Algebraic-Topological methods for analyzing data sets provide a powerful technique for extracting such information. In this talk I'll illustrate the main ideas of the so-called persistent simplicial omology for topological data processing and sketch quantum algorithms for calculating persistent Betti numbers, and for finding eigenvectors and eigenvalues of the combinatorial Laplacian.