Asymptotic analysis of turbo-like codes: average spectra and minimum distances

Chiara Ravazzi
Dipartimento di Matematica, Politecnico di Torino
C.so Duca degli Abruzzi 24, 10129 Torino, Italy
chiara.ravazzi@polito.it

The topic of this talk falls within channel coding theory, and consists in the analysis of a particular class of turbo-like codes, defined by a multiple concatenation of an arbitrary outer encoder with m truncated convolutional encoders through uniform random permutations.

Fixed the number of inner encoders, structural properties of these coding schemes are studied when the truncation length goes to infinity.

As a first step in this study, we focus on truncated convolutional encoders, which are the constituent elements of turbo concatenations. We present a detailed analysis of the related weight distribution functions and of their exponential growth rate. In particular, the weight distribution functions are expressed as coefficients of the generating function of error events associated with a minimal realization of the encoder. Although these expressions can be computed for relatively small truncation lengths, they become prohibitively complex to compute as truncation lengths and weights increase. Fortunately, a very accurate approximation can be derived using the Multidimensional Saddle Point method. This approximation is substantially easier to evaluate and is used to obtain an expression for the asymptotic spectral function and to prove continuity and concavity. Finally, this approach is able to guarantee that the sequence of exponential growth rate converges uniformly to the asymptotic limit and to estimate the speed of this convergence.

Building upon these results, we show that for multiple concatenated coding schemes the average distance spectra can be obtained through the analysis of a dynamical system (dependent on the inner encoder) with initial condition equal to the asymptotic spectra of the outer encoder. Moreover, they are equal to 0 below a threshold distance δ_m and positive beyond it. Then, minimum distances are shown to scale linearly in the code-length with probability one, and the asymptotic normalized minimum distance to be exactly provided by δ_m. Under a very mild condition on the outer encoder, asymptotic spectral functions form a uniformly convergent sequence of functions. Their limit is the maximum between 0 and the average spectral shape of the random linear coding ensemble. As a consequence, the threshold sequence δ_m converges to the Gilbert-Varshamov distance, the best lower bound on the largest minimum distance achievable by a code.

More details can be found in the following papers.

- F. Fagnani, C. Ravazzi, "Spectra and minimum distances of Repeat Multiple Accumulate codes", in *Proc. of Information Theory and Applications Workshop*, pp. 77-86, La Jolla, CA, San Diego, Jan. 2008.
- C. Ravazzi, F. Fagnani, "Spectra and minimum distances of Repeat Multiple-Accumulate codes", *IEEE Transactions on Information Theory*, Vol. 55(11), pp. 4905-4924, Nov. 2009.
- C. Ravazzi, F. Fagnani, "Hayman-like techniques for computing input-output weight distribution of convolutional encoders", in *Proc. of IEEE International Symposium on Information Theory*, pp. 1110-114, Austin, Texas, June 2010.
- C. Ravazzi, F. Fagnani, "Minimum distance properties of multiple-serially concatenated codes", in *Proc. International symposium on turbo codes & iterative information processing*, pp. 88-92, Brest, France, Sep. 2010
- C. Ravazzi, F. Fagnani, "On the growth rate of input-output weight enumerators of convolutional encoders", submitted to *SIAM Journal on Discrete Mathematics*, 2011

All publications are available online at: http://calvino.polito.it/∼ravazzi