# BUNNYTN 72016 

## Monero vs Bitcoin

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Università degli Studi di Messina
16 November 2016, Trento

## Summary of Presentation

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- Reasons and Reviews


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- Monero


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- Monero vs Bitcoin


## Reasons and Reviews

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$$

Let $\mathfrak{m}$ be the message.

- SIGN: $\forall i=1 . . n \quad i \neq \pi$


## Monero

## MLSAG

Let $\left\{P_{i}^{j}\right\}_{i=1 \ldots n}^{j=1 \ldots m}$ the group public keys.

- KEYGEN: Let $\pi$ the secret index such that

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\forall j=1 \ldots m \quad x_{j} G=P_{\pi}^{j}(\bmod q)
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and compute the Keys Images

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## Monero

## MLSAG

Compute:

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Compute:

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L_{\pi}^{j}=a_{j} G \\
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c_{\pi+1}=h\left(\mathfrak{m}, L_{\pi}^{1}, R_{\pi}^{1}, \ldots, L_{\pi}^{m}, R_{\pi}^{m}\right)
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\end{gathered}
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$$
\begin{gathered}
L_{i}^{j}=s_{i}^{j} G+c_{i} P_{i}^{j} \\
R_{i}^{j}=s_{i}^{j} H\left(P_{i}^{j}\right)+c_{i} I_{j}
\end{gathered}
$$

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The complexity is $\mathcal{O}(m \cdot(n+1))$.

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## Monero

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## Theorem (MLSAG Unforgeability)

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Let $\mathcal{A}$ be a Probabilistic Polynomial Time (PPT) Adversary(Algorithm).

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Let $\mathcal{A}$ be a Probabilistic Polynomial Time (PPT) Adversary(Algorithm). Then the probability that $\mathcal{A}$ forges a verifying MLSAG Signature is Negligible under the (EC)DLOG Assumption.

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The probability that a PPT Algorithm $\mathcal{A}$ can create two verifying signatures $\sigma$ and $\sigma^{\prime}$ signed with the vectors $\bar{y}$ and $\bar{y}^{\prime}$ such that there exists the same public key y in both $\bar{y}$ and $\bar{y}^{\prime}$ is Negligible

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## Theorem (MLSAG Linkability)

The probability that a PPT Algorithm $\mathcal{A}$ can create two verifying signatures $\sigma$ and $\sigma^{\prime}$ signed with the vectors $\bar{y}$ and $\bar{y}^{\prime}$ such that there exists the same public key $y$ in both $\bar{y}$ and $\bar{y}^{\prime}$ is Negligible

## Theorem (MLSAG Anonimity)

The MLSAG protocol is Signer Ambiguous under the Decisional Diffie Hellman Assumption.

## Monero

## Monero

## Commitments

## Monero

## Commitments

Let $G$ be the Curve25519 Base Point

## Monero

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C(a, x)=x G+a H
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If i.e. there are 1 input and 2 outputs:

$$
\begin{gathered}
C_{\text {in }}=x_{C} G+a H \\
C_{\text {out }-1}=y_{1} G+b_{1} H \\
C_{\text {out }-2}=y_{2} G+b_{2} H
\end{gathered}
$$

## Monero

## Monero

## Commitments

 with:
## Monero

## Commitments

with:

- $x_{C}-y_{1}-y_{2}=z$
- $a=b_{1}+b_{2}$


## Monero

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so
$C_{\text {in }}-C_{\text {out }-1}-C_{\text {out }-2}=z G=C(0, z)$


## Monero

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with:

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## Ring Confidential Transactions

## Monero

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In practice $C_{i} i=1 \ldots n$ are the input commitments.

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In practice $C_{i} i=1 \ldots n$ are the input commitments. With the pairs $\left(P_{i}, C_{i}\right)$

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In practice $C_{i} i=1 \ldots n$ are the input commitments. With the pairs $\left(P_{i}, C_{i}\right)$ we create a Ring Signature of the form:

## Monero

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$$
\left\{P_{1}+C_{1}-\sum_{j} C_{j, \text { out }}, \ldots, P_{s}+C_{s}-\sum_{j} C_{j, \text { out }}, \ldots, P_{n}+C_{n}-\sum_{j} C_{j, \text { out }}\right\}
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with private key $z+x^{\prime}$

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with private key $z+x^{\prime}$ with $x^{\prime} G=P_{s}$

## Monero

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## Tag-Linkable Ring-CT with Multiple Inputs and OneTime Keys

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- Let $\left\{\left(P_{\pi}^{1}, C_{\pi}^{1}\right), \ldots,\left(P_{\pi}^{m}, C_{\pi}^{m}\right)\right\}$ be pairs of PubKeys/Commitments


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- Find $q+1$ collections $\left\{\left(P_{i}^{1}, C_{i}^{1}\right), \ldots,\left(P_{i}^{m}, C_{i}^{m}\right)\right\}, i=1 . . q+1$


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- Find $q+1$ collections $\left\{\left(P_{i}^{1}, C_{i}^{1}\right), \ldots,\left(P_{i}^{m}, C_{i}^{m}\right)\right\}, i=1 . . q+1$ not already Tag-Linked.


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- Find $q+1$ collections $\left\{\left(P_{i}^{1}, C_{i}^{1}\right), \ldots,\left(P_{i}^{m}, C_{i}^{m}\right)\right\}, i=1 . . q+1$ not already Tag-Linked.
- Choose a set of outputs $\left(Q_{i}, C_{i, \text { out }}\right)$ such that $\sum_{j=1}^{m} C_{\pi}^{j}-\sum_{i} C_{i, \text { out }}$


## Monero

## Tag-Linkable Ring-CT with Multiple Inputs and OneTime Keys

- Let $\left\{\left(P_{\pi}^{1}, C_{\pi}^{1}\right), \ldots,\left(P_{\pi}^{m}, C_{\pi}^{m}\right)\right\}$ be pairs of PubKeys/Commitments with private keys $x_{j} j=1 \ldots m$.
- Find $q+1$ collections $\left\{\left(P_{i}^{1}, C_{i}^{1}\right), \ldots,\left(P_{i}^{m}, C_{i}^{m}\right)\right\}, i=1 . . q+1$ not already Tag-Linked.
- Choose a set of outputs $\left(Q_{i}, C_{i, \text { out }}\right)$ such that $\sum_{j=1}^{m} C_{\pi}^{j}-\sum_{i} C_{i, \text { out }}$ is a commitment to 0 .


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\begin{gathered}
\Re:=\left\{\left\{\left(P_{1}^{1}, C_{1}^{1}\right), \ldots,\left(P_{1}^{m}, C_{1}^{m}\right),\left(\sum_{j} P_{1}^{j}+\sum_{j=1}^{m} C_{1}^{j}-\sum_{i} C_{i, \text { out }}\right)\right\}\right. \\
\ldots, \\
\left.\left\{\left(P_{q+1}^{1}, C_{q+1}^{1}\right), . .,\left(P_{q+1}^{m}, C_{q+1}^{m}\right),\left(\sum_{j} P_{q+1}^{j}+\sum_{j=1}^{m} C_{q+1}^{j}-\sum_{i} C_{i, \text { out }}\right)\right\}\right\}
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be the Generalized Ring which we wish to sign.

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- Compute MLSAG signature $\Sigma$ on $\Re$


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\mathfrak{R}:=\left\{\left\{\left(P_{1}^{1}, C_{1}^{1}\right), \ldots,\left(P_{1}^{m}, C_{1}^{m}\right),\left(\sum_{j} P_{1}^{j}+\sum_{j=1}^{m} C_{1}^{j}-\sum_{i} C_{i, \text { out }}\right)\right\},\right.
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## Conclusions on RCT

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RCTs ensure hiding of amount,origins and destination.

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be the Generalized Ring which we wish to sign.

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## Conclusions on RCT

RCTs ensure hiding of amount,origins and destination. In additon coin generation is trustless and verifyable secure.

## Monero vs Bitcoin

## Monero vs Bitcoin

## Monero and Bitcoin

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## Monero and Bitcoin

Bitcoin and Monero are just similar as they are different.

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-Blocksize Limit

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Bitcoin: has limit at 1 MB . Some people agree to remove the Limit, but it could overload the nodes.
Monero: has Scalability, it modify its size in scale with respect to memory requested.

## Monero vs Bitcoin

## Transaction Time

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| Crypto | Monero | Bitcoin |
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| Transaction <br> time | 1 minute | 10 minutes |

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Bitcoin: about 10 min.; Hash Algorithm is CPU-bound. Monero: about 1 min ; Hash Algorithm is Memory-bound.

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## Bitcoin:

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## Untraceability



Bitcoin: most trasparent currency,

## Monero vs Bitcoin

## Untraceability

| Crypto | Monero | Bitcoin |
| :--- | :--- | :--- |
| Untraceable | Yes | No |

Bitcoin: most trasparent currency, all transactions are public

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| Crypto | Monero | Bitcoin |
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Bitcoin: most trasparent currency, all transactions are public Monero: Untraceable thanks to Ring Confidential Transactions.

## Monero vs Bitcoin

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Bitcoin: most trasparent currency, all transactions are public Monero: Untraceable thanks to Ring Confidential Transactions. It is optionally transparent.

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## Safe Elliptic Curves

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## Safe Elliptic Curves

| Crypto | Monero | Bitcoin |
| :--- | :--- | :--- |
| Safe elliptic <br> curve | Yes | No |
| (Curve25519) | (Secp256k1) |  |

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Bitcoin Curve: Secp256k1

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Bitcoin Curve: Secp256k1 Unsafe

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Bitcoin Curve: Secp256k1 Unsafe Monero Curve:

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Bitcoin Curve: Secp256k1 Unsafe Monero Curve: Curve25519

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Bitcoin Curve: Secp256k1 Unsafe Monero Curve: Curve25519 Safe

| Curve | Field | Equation | Base | $\rho$ | Transfer | CM Discr. | Rigid. | Ladder | Twist | Complete | Indistin. | Safe? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Curve25519 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Secp256k1 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $x$ | $\checkmark$ | x | $\checkmark$ | x | x | x |

## GRAZIE PER L'ATTENZIONE!!

