The topology of Helmholtz domains

Riccardo Benedetti[1], Roberto Frigerio[1], and Riccardo Ghiloni[2]

The use of cuts along surfaces for the study of domains in Euclidean 3-space widely occurs in theoretic and applied literature about electromagnetism, fluid dynamics and elasticity. The aim of this talk is to discuss techniques and results of 3-dimensional topology that provide an appropriate theoretical background to the method of cuts along surfaces. We consider two classes of bounded domains that become "simple" after a finite number of cuts along disjoint properly embedded surfaces. The difference between the two classes arises from the different meanings that the word "simple" may assume, when referred to spatial domains. In the definition of Helmholtz domain, we require that the domain may be cut along disjoint surfaces into pieces such that any curl-free smooth vector field defined on a piece admits a potential. On the contrary, in the definition of weakly Helmholtz domain we only require that a potential exists for the restriction to every piece of any curl-free smooth vector field defined on the whole initial domain. We give an exhaustive description of Helmholtz domains, proving that their topology is forced to be quite elementary: in particular, the complement of any non-trivial link is not Helmholtz. The discussion about weakly Helmholtz domains is more advanced, and their classification is a more demanding task. Nevertheless, we provide interesting characterizations and examples of weakly Helmholtz domains.

[1]Department of Mathematics - University of Pisa
[2]Department of Mathematics - University of Trento