Applications of Topological Information Implicit in 3-D Finite Element Meshes: Readily Computable but not by the Linear Structure of Cohomology Theory

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Abstract:

In topological aspects of 3-D problems in computational electromagnetics, it is instructive to develop a distinction between “readily computable but unintuitive” aspects (e.g. homology calculations and their use in quasi-static analysis of circuit parameters), and “intuitive but not computable” aspects (e.g. homotopy groups which articulate how a space might be uncontractible). This distinction is insignificant in 2-D but in 4-D it is easily related to problems not decidable on a Turing machine. In 3-D the Hopf exact sequence gives a framework for making sense of this distinction between homotopy and homology groups, but the non-abelian nature of the groups involved do not readily lend the framework to a computational treatment.

Category theory and the homotopy-theoretic approach to algebraic topology give a framework for developing formalism and obtaining rigorous results without regard to whether the results are computable or intuitive. In particular, it provides both an axiomatic treatment of (co)homology theory which is readily computable, and it formalizes many topological questions which arise naturally in the application of multivariable calculus and partial differential equations. The proposed talk will survey the instances where topological questions in 3-D computational electromagnetics, not amenable to treatment by the linear structure of cohomology theory, can be (or have been) settled computationally either by appealing to either the ring structure in standard cohomology theory or by generalized cohomology theories. Examples will focus on:

1) Information pertaining to homotopy groups which comes as a byproduct of computationally realizing cuts as orientable embedded manifolds.
2) The “Alexander basis” for systems of cuts in and out of conducting regions, applications of the resulting symplectic structure induced on the first homology group of the boundary of a region, and applications to eddy-current problems and applications to self-adjoint curl operators.
3) Massey products and ties to “configuration spaces”.