Deterministic uncertainty quantification in Nano-Optics

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The aim of this work is to investigate how defects in the fabrication process affect the optical response of nano structures. Assuming an additional TM/TE symmetry, the physical phenomenon can be modeled by a 2D Helmholtz equation. To model uncertainties in the fabrication process, the domain is considered to depend on a large number of stochastic parameters.

In Nano-Optics application, the stochastic fluctuations of the domain cannot be considered small and a perturbative method would be inappropriate. On the other hand, the low convergence rate 1/2 of Monte Carlo sampling, respectively the so-called 'curse of dimensionality' of standard interpolation schemes in high-dimensional parameter spaces, makes this method unsuitable in our framework. Instead, the approach followed is *deterministic uncertainty quantification*: the stochastic parameter modeling the domain is represented as sum of a high number (possible infinite) of random variables; realizations of the stochastic parameter are represented by a multi-dimensional deterministic parameter taking space in the *parameter domain*, where each dimension corresponds to the image space of a random variable in the sum.

This strategy inevitably leads to an integration over a parameter-dependent domain in the variational formulation of the problem. To overcome this, a *mapping approach* is adopted: the parameter-dependent domain is mapped onto a deterministic domain through a parameter-dependent map. In this way, the problem turns out to the solution of a partial differential equation with stochastic coefficient over a fixed domain, for which theoretical results and numerical techniques are available.

Since the stochastic coefficient does depend in a nonlinear, non-affine way on the deterministic parameter representing stochasticity, a so-called nonintrusive method is used: the *stochastic collocation* method. This approach is based on a polynomial approximation via interpolation in the parameter domain. The collocation points are selected using an adaptive Smolyak algorithm, whose convergence rates depend on the sparsity of the parameter but are unbounded independently of the number of dimension activated in the parameter space, thus overcoming the 'curse of dimensionality'.

Numerical results are presented, where the 2D stochastic domain is modeled as a domain with stochastic and angle-dependent radius. An interface problem with an incoming plane wave is solved, both in the TE and TM case. The output considered is the far field in some space region.

References:

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