Discrete Exterior Calculus Approach to Discretization of Port-Hamiltonian Systems

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ACE'13 Abstract

Hamiltonian systems are at the foundation of many current physical theories, including quantum and relativistic mechanics, electromagnetism, optics, solid and fluid mechanics. Geometry as the study of observable symmetries and dynamical invariants is *de facto* the *lingua franca* of the Hamiltonian theories. The prevailing paradigm in modeling of the complex large-scale physical systems is network modeling. In many problems arising from modern science and engineering, such as multi-body systems, electrical networks and molecular dynamics, the port-based network modeling is a natural strategy of decomposing the overall system into subsystems, which are interconnected to each other through pairs of variables called ports and whose product is the power exchanged between the subsystems.

The formalism that unifies the geometric Hamiltonian and the port-based network modeling is the *port-Hamiltonian,* which associates with the interconnection structure of the network a geometric structure given by a Poisson, or more generally, a Dirac structure. The generalized Hamiltonian dynamic is then defined with respect to this Poisson, or Dirac, structure by specifying the Hamiltonian representing the total stored energy, the energy-dissipating elements and the ports of the system. Apart from enunciating a remarkable structural unity, Poisson and Dirac geometry offers a mathematical framework that gives important insights into dynamical systems. Moreover, the geometric formalism transcends the finite-dimensional scenario and has been successfully applied to study of a number of distributed-parameter systems, systems described by a set of partial differential equations.

In this talk I plan to address the issue of structure-preserving discretization of open distributed-parameter port-Hamiltonian systems on bounded domains.

Employing the formalism of discrete exterior calculus, I will introduce *simplicial Dirac structures* as discrete analogues of the infinite-dimensional Dirac structures for classical field theories. I will demonstrate how these simplicial Dirac structures provide a natural framework for deriving finite-dimensional port-Hamiltonian systems that emulate the behaviors of their infinite-dimensional counterparts.

I plan to illustrate general considerations on a number of physical examples, including Maxwell's equations on a bounded domain, the telegraph equations, and reaction-diffusion systems, where the structure-preserving discretization recovers the standard compartmental model.