Was it worth the effort to insist on geometric formulations?

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The Finite Element Method (FEM) is probably the most flexible and general purpose technique to solve problems arising in computational engineering. Yet, in the last decade, there has been an increasing interest in reformulating physical laws directly in algebraic form. Finite Integration Technique (FIT), cell method, the discrete geometric approach (DGA), the discrete electromagnetism (DE), cochain-based formulations, etc., are instances of this philosophy. In most cases, even classical FEM fits into this framework (there are, however, a few exceptions. The inconsistency of the mass matrices for hexahedra built with the Galerkin Hodge technique is a good example.)

Even though the geometric formulations are not as mature as FEM, it is perhaps time to ask the question whether all these efforts of developing geometric formulations paid off. It would be disappointing to establish that they provide just a mere reinterpretation of old results. Geometric approaches are attractive for pedagogical reasons (at least our undergraduate students state so), but it would be nice if they would also make the computations faster and more accurate. The aim of this contribution is exactly to show that this is definitely the case, at least for some classes of problems. Of course, we do not consider structured meshes as cubical grids, where FIT already proved its superiority in most cases. Conversely, we assume to deal with unstructured meshes, where geometric approaches face direct competition with FEM. In particular, we present three examples of problems where DGA seems to be more flexible or more accurate than FEM:

1) DGA is superior to FEM both in speed and accuracy in solving stationary Schrodinger equation, that is fundamental to correctly describe the behaviour of modern electron devices. As applications, we will concentrate on modeling FinFET transistors, recently adopted by Intel as the new state-of-the-art transistor technology, and three-dimensional quantum dots.

2) DGA allows to construct symmetric, positive-definite and consistent discrete Hodge operators for arbitrary polyhedra. Moreover, they are constructed geometrically with simple closed form expressions.

3) DGA offers a technique to solve transient full wave Maxwell problems explicitly even on simplicial meshes. This is not possible with FEM, even though many people attempted to do it.

To conclude, the DGA and related methods not only shade a different light on the geometry behind physical laws, but now it can be proved that sometimes they provide more efficient schemes than FEM. In our opinion these results motivate our intention to insist in performing research on this topic.