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Meeting

Progressi Recenti in Geometria Reale e Complessa - XI

dedicato alla memoria di Paolo de Bartolomeis

Levico Terme (Trento) – 24-27 settembre 2018

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Web-page: `http://www.science.unitn.it/cirm/GeometriaReale2018.html`

Sponsor

- *FBK-CIRM (Centro Internazionale per la Ricerca Matematica)*
- *INdAM-GNSAGA*
- *HEVO: Holomorphic Evolution Equations. ERC Starting Grant n. 277691*
- *Comune di Levico Terme*
- *A.P.T. Valsugana*

BOOK OF ABSTRACTS /

SENIOR TALK

Some results on the Hard Lefschetz Condition and related questions

Adriano Tomassini

Università di Parma

We will discuss the Hard Lefschetz Condition in various contexts, e.g., almost-Kähler structures, complex symmetric structures. A general Demailly-Griffiths-Kähler identity is also given. Several examples will be presented.

JUNIOR TALKS

Laplacian cut-offs

Davide Bianchi

Università dell'Insubria, Como

Joint work with Alberto Setti.

Many analytic results in Euclidean setting require the use of compactly supported cut-off functions, essentially to localize differential equations or inequalities or to perform integration by parts arguments. A key feature of d-dimensional Euclidean space is that it is possible to construct cut-offs $\{\phi_R\}$ such that $\phi_R = 1$ on the ball $B_R(o)$, they are supported in the ball $B_{\gamma R}(o)$ and have controlled derivatives up to second order:

$$|\nabla \phi_R| \leqslant \frac{C}{R}, \qquad |\Delta \phi_R| \leqslant \frac{C}{R^2},$$

where C is a constant depending only on γ and the dimension. Indeed, such cut-offs can be defined in terms of the distance function r from 0, $r(x) = (\sum_i x_i^2)^{1/2}$, as $\phi_R(x) = \psi(r(x)/R)$ where $\psi : \mathbb{R} \to [0,1]$ is smooth, identically 1 in $(-\infty, 1]$ and vanishes in $[\gamma, +\infty)$, and the properties of ϕ_R listed above depend crucially on the fact that the distance function is proper and satisfies

$$|\nabla r(x)| \leqslant C, \quad |\Delta r(x)| \leqslant \frac{C}{r(x)}.$$

The existence of Euclidean cut-offs with the above properties is then a consequence of the fact that distance is a well-behaved proper function on $\mathbb{R}^d$.

In many significant situations it is actually vital to have an explicit uniform decay of $\Delta\phi_R$ in terms of R. We quote, for example, spectral properties of Schrödinger-type operators and the approximation procedures used in the proof of existence, uniqueness and qualitative and quantitative properties of solutions to the Cauchy problem for the porous and fast diffusion equations.

We construct exhaustion and cut-off functions with controlled gradient and Laplacian on manifolds with Ricci curvature bounded from below by a (possibly unbounded) nonpositive function of the distance from a fixed reference point, without any assumptions on the topology or the injectivity radius. Along the way we prove a generalization of the Li-Yau gradient estimate which is of independent interest.

References

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Static spacetimes with positive cosmological constant

Stefano Borghini

Università di Trento

In General Relativity, the study of *static solutions* of the Einstein Field Equations with positive cosmological constant is equivalent to the study of triples (M, g, u), where (M, g) is a compact 3-dimensional Riemannian manifold with nonempty boundary ∂M, and $u : M \to \mathbb{R}$ is a smooth function satisfying the equations

$$\begin{cases} u\,\mathrm{Ric} = D^2 u + 3\,u\,g\,, & \text{in } M\,,\\ \Delta u = -3\,u\,, & \text{in } M\,,\\ u = 0\,, & \text{on } \partial M\,. \end{cases}$$

A celebrated result, proved by Boucher-Gibbons-Horowitz, states that, if ∂M is connected, then $|\partial M| \leqslant 4\pi$, and the equality holds if and only if (M, g) is an hemisphere and u is rotationally symmetric. We improve and extend this sharp inequality to the case where ∂M has several connected components (*horizons*). Building on this, we show some characterization results for the model solutions. Time permitting, we will also discuss how similar techniques can be applied to the study of the torsion problem.

Approximation and Loewner Theory of holomorphic covering maps

MATTEO FIACCHI

Università di Roma "Tor Vergata"

We give conditions in order to approximate locally uniformly holomorphic covering maps of the unit ball of Cn (with respect to an arbitrary norm) with entire holomorphic covering maps. The results rely on a generalization of the Loewner theory for covering maps.

Applications of monotonicity formulas to manifolds with non-negative Ricci curvature

MATTIA FOGAGNOLO

Università di Trento

I will discuss monotonicity formulas for harmonic functions defined on exterior domains of manifolds with non-negative Ricci curvature. We apply them to deduce a Willmore-type inequality, a sharp isoperimetric inequality on 3-manifolds and to recover aspects of minimal submanifolds. These results will appear in a joint work with V. Agostiniani and L. Mazzieri.

Special coadjoint orbits of simple Lie groups

ALICE GATTI

Università di Pavia

Starting with Calabi's seminal works, Kähler-Einstein metrics turned out to be a very powerful tool for studying Kähler manifolds from many points of view. Following the alternative approach of Donaldson and Fujiki, Kähler-Einstein metrics may be investigated by keeping the symplectic form fixed and varying the almost-complex structure on the symplectic manifold. Hence, it is very natural to look for symplectic manifolds admitting *special* almost-complex structures, meaning that the Chern-Ricci form of the almost-complex structure is a multiple of the symplectic form. This condition may be considered as a generalization of the Kähler-Einstein condition to the non-integrable case.

A broad class of examples for which the speciality condition holds is made up of symplectic homogeneous manifolds admitting special almost-complex structures. In this seminar will be shown some existence results for special almost-complex structures and a classification program in the case of (co)adjoint orbits of simple Lie groups.

Backward iterations in the unit ball

LORENZO GUERINI

University of Amsterdam (The Netherlands)

We show that, if $f : \mathbb{B}^q \to \mathbb{B}^q$ is a holomorphic self-map of the unit ball and $\zeta \in \mathbb{B}^q$ is a boundary repelling fixed point with dilation $\lambda > 1$, then there exists a backward orbit converging to ζ with step $\log \lambda$. Moreover, any two backward orbits converging at the same boundary repelling fixed point stay at finite distance.

Exotic black holes and the Penrose inequality

SAMUELE LANCINI

Scuola Normale Superiore, Pisa

In this talk, I will present a class of special solutions for the Einstein's constraint equations. These solutions with constant negative scalar curvature can be associated to space-times with exotic black holes. After recalling briefly some physical definitions, we will observe that our examples satisfy the (Riemannian) Penrose inequality conjecture.

This talk has been extracted from my PhD thesis (supervisors C. Arezzo and L. Mazzieri).

Complex analytic properties of minimal Lagrangian submanifolds

ROBERTA MACCHERONI

Università di Ferrara

In this seminar I will give examples of minimal Lagrangian submanifolds in Kaehler-Einstein ambient spaces and I will describe some topological and complex analytic properties: in particular about the non-existence of filling by holomorphic discs. Then I will compare geometric properties of minimal Lagrangian submanifolds varying the sign of the ambient curvature.

(This is part of my PhD thesis contained in arXiv:1805.09651; my supervisor is Prof. Tommaso Pacini (Univ. di Torino))

Compact CMC surfaces in S^3 via integrable methods

BENEDETTO MANCA

Università di Cagliari

In this talk i will show some recent results about CMC embeddings of compact Riemann surfaces into the 3-sphere using the integrable systems method. Heller, Heller and Schmit, considering the family of Lawson surfaces of genus > 1 were able to find a way to recover the CMC immersions from the associated family of flat connections using the Riemann sphere with four punctures as an auxiliary surface. I will explain how to generalise these methods for every CMC immersion of a symmetric compact Riemann surface in S^3.

A finiteness criterion in Tanaka theory

Stefano Marini

Università di Parma

A G-structure, described as the datum of a Lie algebra of infinitesimal transformations acting as linear maps on the tangent space at a point, was historically introduced to treat in a unified manner various interesting differential geometrical structures, e.g. subriemannian, subconformal and CR structures. When its maximal effective Tanakas prolongation turns out to be finite dimensional, Cartan geometry method can be used to study the automorphism group and the equivalence problem for the corresponding G-structure. A key point is to find under which conditions the infinitesimal automorphisms of the structure form a finite dimensional Lie algebra. In a recent joint work with C.Medori and M.Nacinovich we discuss a necessary and sufficient conditions for the finiteness of Tanakas maximal effective prolongation of fundamental graded Lie algebras.

Riemannian geometry of Engel structures

Nicola Pia

Università di Cagliari

I will present some results that are part of my thesis in the theory of metrics associated to Engel structures, i.e. maximally non-integrable 2-distributions in 4-manifolds. I will talk about the basic theory of these objects, which are strongly related to contact structures. Then I will present some results on the Riemannian geometry of Engel structures. Finally I will introduce the concept of K-Engel manifold, analogous to the one of K-contact manifold.

Translating solitons of the mean curvature flow in the Heisenberg group

Giuseppe Pipoli

Università de L'Aquila

Translating solutions of the mean curvature flow are special hypersurfaces that move under mean curvature flow preserving their shape and translating in a fixed direction. They have a crucial role in understanding the singularities of the flow and provide interesting explicit examples of solutions. In this talk we present the construction of infinitely many new translating surfaces in the Heisenberg group. We discuss similarities and differences with the analogous examples in the Euclidean space.

Hermitian curvature flow and special Hermitian metrics on complex Lie groups

Mattia Pujia

Università di Torino

We discuss the behaviour of the Hermitian curvature flow (HCF) on complex Lie groups. We show that long-time existence of the solutions always holds on complex unimodular Lie groups and a suitable normalization of the flow converges to a soliton. Further, we investigate existence of static Hermitian metrics and solitons on complex Lie groups. Finally, we give a structural result concerning expanding semi-algebraic solitons on complex Lie groups.

Closed $G2$-structures and the $G2$-Laplacian flow

Alberto Raffero

Università di Torino

The $G2$-Laplacian flow is a geometric flow evolving a closed $G2$-structure in the direction of its Hodge Laplacian. In this talk, after reviewing some recent results on closed $G2$-structures, I shall discuss the behaviour of the Laplacian flow starting from a closed G2-structure whose induced metric satisfies suitable curvature properties.

This is based on joint works with F. Podestà (Firenze) and A. Fino (Torino).

Automorphisms of $\mathbb{C}^2$ with non-simply attracting connected Fatou components

Josias Reppekus

Università di Roma "Tor Vergata"

I will present examples of automorphisms of complex space of dimension 2 admitting any (finite) number of non-recurrent attracting Fatou components biholomorphic to a product $\mathbb{C} \times \mathbb{C}^*$. This generalises the same result for a single such Fatou component by Bracci, Raissy and Stensones that first showed the existence of such strange Fatou components which are fundamentally different from one-dimensional examples. Since attracting Fatou components are known to be Runge, it also shows the existence of a Runge embedding of $\mathbb{C} \times \mathbb{C}^*$ into $\mathbb{C}^2$.

The proof closely mimics that of the single-component result: Combining a local construction by Bracci and Zaitsev with a globalisation result of Forstnerič one obtains a Fatou coordinate on a global basin of attraction. A local construction of coordinates for the remaining dimension shows that the Fatou coordinate is a fibre bundle map over $\mathbb{C}$ with typical fibre $\mathbb{C}^*$. Such a bundle is trivial, showing the basin to be isomorphic to $\mathbb{C} \times \mathbb{C}^*$. To show that the global basin is actually a Fatou component, Bracci, Raissy and Stensones introduced a new technique that involves careful analysis of rates of convergence of each coordinate, a result by Pschel ensuring the existence of local Siegel discs, and estimates on the Kobayashi distance between individual coordinates.

A Quantitative Version of Alexandrov's theorem

Alberto Roncoroni

Università di Pavia

Let M be the Euclidean space or the hyperbolic space or the hemisphere. The celebrated Alexandrov's Soap Bubble Theorem states that spheres are the only closed (i.e. compact and without boundary) constant mean curvature hypersurfaces embedded in M. The talk maily focuses on the following quantitative version of the Alexandrov Theorem

Theorem 1. *Let S be an n-dimensional, C^2-regular, connected, closed hypersurface embedded in M. There exist constants ε, $C > 0$ such that if*

$$osc(H) \leqslant \varepsilon,$$

then there are two concentric balls B_{r_i} and B_{r_e} such that

$$S \subset B_{r_e} \setminus \overline{B}_{r_i} \quad \text{and} \quad r_e - r_i \leqslant C osc(H)\,.$$

The constants ε and C depend only on n and upper bounds on the C^2-regularity and the area of S.

The proof of the Theorem makes use of a quantitative study of the method of the moving planes and the result implies a new pinching Theorem for hypersurfaces. Furthermore, the Theorem is optimal in a sense that will be specified in the talk. The last part of the talk will be about an on-going study on the generalization of the result.

This is a joint work with G. Ciraolo and L. Vezzoni.

Riemannian and pseudo-Riemannian nilsolitons

Federico A. Rossi

Università di Milano Bicocca

The structure of Riemannian Einstein solvmanifolds is well understood: by the works of Heber and Lauret, all such metrics are obtained by taking a standard extension of a nilpotent Lie group that carries a nilsoliton metric (meaning that the Ricci operator has the form $\mathrm{Ric} = c\mathrm{Id} + D$ for some derivation D; in modern language, D must coincide with the Nikolayevsky derivation); conversely, any nilsoliton has an Einstein solvable extension defined by the *Nikolayevsky derivation*. Thus, the classification of Riemannian Einstein solvmanifolds is reduced to the classification of nilsolitons, but this classification exists up to dimension 7 and in some special cases for dimension 8 (e.g. the works of Payne).

In the first part of the talk, we will explain how to classify Riemannian nilsolitons on dimension grater that 7, by using the classification of nice nilpotent Lie algebras (those algebras were introduced by Lauret).

In the second part, we will study the relations between nilsolitons and construction of Einsten solvmanifolds for pseudo-Riemannian metrics, which is very different: for example, there exist nilpotent Lie algebras with indefinite Einstein metrics of nonzero scalar curvature. Whilst these are nilsoliton metrics with $D = 0$, the standard metric induced on the (trivial) solvable extension defined by D is not Einstein. This suggests that the interplay between indefinite Einstein solvmanifolds and the geometry of nilmanifolds is more complicated compared to the Riemannian case. In the end, we will give some examples and a constructive method to build special Einstein pseudo-Riemannian metrics.

This is a joint work with Diego Conti.

The CR strain functional for Legendrian curves in the 3-sphere

Filippo Salis

Politecnico di Torino

After a brief overview of the CR geometry of Legendrian curves in the three dimensional sphere endowed with the canonical CR structure, we are going to discuss some recent results about closed Legendrian curves extremizing the total strain functional.

This is a joint work with E. Musso.

Andersen-Lempert theory and tame sets

Riccardo Ugolini

University of Ljubljana (Slovenia)

The Andersen-Lempert theory begins with the observation that all holomorphic self-maps of C^2 of the form $(z, w + f(z))$ with f holomorphic in one variable are in fact biholomorphisms. In this talk we will discuss some basic facts about the group of automorphisms of C^n and see how these results apply to more general complex manifolds, namely Stein manifolds with the density property. Particular attention will be devoted to tame sets, infinite closed discrete sets with good properties with respect to the group of automorphism of the underlying manifold.

Instantons on asymptotically hyperbolic 4-manifolds

Marco Usula

Università di Pisa

Instantons on closed 4-manifolds have played a central role in the study of 4-dimensional geometry, since the pioneering work of Donaldson in the 80s. In this talk Ill focus on instantons on asymptotically hyperbolic (AH) 4-manifolds. An AH 4-manifold is an open, complete Riemannian 4-manifold which resembles the hyperbolic metric at infinity. On such spaces, instantons do not necessarily extend smoothly at infinity: rather, they can develop a simple pole. Some partial results about moduli spaces of instantons with a simple pole at infinity will be discussed.
