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Cohomology of Complex Manifolds and Special Structures

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PROGRAM and BOOK OF ABSTRACTS

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ABSTRACTS

Complex symplectic manifolds satisfying the $\partial\overline{\partial}$-Lemma

Ben Anthes

Philipps-University Marburg (Germany)

I will report on recent work joint with Andrea Cattaneo, Adriano Tomassini and Snke Rollenske on compact complex manifolds which satisfy the $\partial\overline{\partial}$-Lemma and which have trivial canonical bundle or which admit a nowhere degenerate holomorphic 2-form. For the former class, we show that the Albanese map is a fibration whose fibres have trivial canonical bundle as well. For the latter class, we study the period map and prove a local Torelli theorem. We conclude with a list of related open problems.

Foliations, basic cohomology and fan combinatorics

Fiammetta Battaglia

Università di Firenze (Italy)

We establish a relationship between combinatorics of fans and the basic cohomology of LVMB manifolds, a large family of foliated complex manifolds introduced by López de Medrano, Verjovsky and Meersseman and further generalized by Bosio.

Joint work with Dan Zaffran.

Strange Fatou components of automorphisms of C^2

Filippo Bracci

Università di Roma "Tor Vergata" (Italy)

The classification of Fatou components for automorphisms of the complex space of dimension greater than 1 is an interesting and difficult task. Recent deep results prove that the one-dimensional setting is deeply different from the higher dimensional one. Given an automorphism F of C^k, the first bricks in the theory that one would like to understand are invariant Fatou components, namely, those connected open sets U, completely invariant under F, where the dynamics of F is not chaotic. Among those, we consider attracting Fatou components, that is, those components on which the iterates of F converge to a single point. Attracting Fatou components can be recurrent, if the limit point is inside the component or non-recurrent. Recurrent attracting Fatou components are always biholomorphic to C^k, since it was proved by H. Peters, L. Vivas and E. F. Wold that in such a case the point is an attracting (hyperbolic) fixed point, and the Fatou component coincides with the global basin of attraction. Also, as a consequence of works of Ueda and Peters-Lyubich, it is know that all attracting non-recurrent Fatou components of polynomial automorphisms of C^2 are biholomorphic to C^2. One can quite

easily find non-simply connected non-recurrent attracting Fatou components in C^3 (mixing a two-dimensional dynamics with a dynamics with non-isolated fixed points in one-variable). In this talk I will explain how to construct a non-recurrent attracting Fatou component in C^2 which is biholomorphic to $CxC*$. This fantastic beast is obtained by globalizing, using a result of F. Forstneric, a local construction due to the speaker and Zaitsev, which allows to create a global basin of attraction for an automorphism, and a Fatou coordinate on it. The Fatou coordinate turns out to be a fiber bundle map on C, whose fiber is $C*$, then the global basin is biholomorphic to $CxC*$. The most subtle point is to show that such a basin is indeed a Fatou component. This is done exploiting Poschels results about existence of local Siegel discs and suitable estimates for the Kobayashi distance. Since attracting Fatou components are Runge, it turns out that this construction gives also an example of a Runge embedding of $CxC*$ into C^2. Moreover, this example shows an automorphism of C^2 leaving invariant two analytic discs intersecting transversally at the origin.

The talk is based on a joint work with J. Raissy and B. Stensones.

Complex symplectic manifolds and the del-delbar lemma

Andrea Cattaneo

Université Claude Bernard Lyon 1 (France)

In this talk I will report on a joint work with A. Tomassini on complex symplectic manifolds. These manifolds posses a non-degenerate holomorphic 2-form, making each tangent space a symplectic vector space. In analogy with the case of hyperkhler manifolds, there is a quadratic form on the second cohomology space of these manifolds, whose rank can be put in relation with suitable subspaces of the cohomology. If moreover our manifolds satisfy the del-delbar lemma, then this description relates the behaviour of the quadratic form to the whole second cohomology space.

Rational curves on Calabi-Yau varieties

Simone Diverio

Università di Roma "La Sapienza" (Italy)

Whether there always are or not rational curves on Calabi-Yau varieties is an old open question in complex algebraic geometry, solved in dimension two only. The presence of such curves is relevant from various viewpoints, including string theory, mirror symmetry, algebraic geometry and last but not least Kobayashi hyperbolicity. In this talk we shall present some existence results in dimension three (joint with A. Ferretti) and as well as in higher dimension (joint with C. Fontanari and D. Martinelli), provided the Calabi-Yau satisfies some special conditions (which imply for instance that its Picard number is greater than one).

Some remarks on disc bundles

John Erik Fornaess

NTNU, Trondheim (Norway)

I will lecture about some recent joint work with Fusheng Deng. We have investigated questions like function theory and pseudoconvexity of neighborhoods.

Scalar curvature is moment map in generalized Kähler geometry

Ryushi Goto

Osaka University (Japan)

The scalar curvature arises as the moment map in Kähler geometry. In pursuit of the analogy, we develop the moment map framework in generalized Kähler geometry of symplectic type. Then we give the notion of the scalar curvature on a generalized Kähler manifold of symplectic type from the moment map view point. We also obtain the generalized Ricci form which is a representative of the first Chern class of the anticanonical line bundle. We show that infinitesimal deformations of generalized Kähler structures with constant scalar curvature are finite dimensional on a compact manifold. Explicit descriptions of the generalized Ricci form and the generalized scalar curvature are given on a generalized Kähler manifold of type (0, 0). Poisson structures constructed from a Kähler action of T^m on a Kähler-Einstein manifold give rise to intriguing deformations of generalized Kähler-Einstein structures. We also discuss the Calabi-Yau problem of generalized Kähler manifold of symplectic type.

This talk is based on my two papers on arXiv:
arXiv:1707.03143 Moduli spaces of Einstein-Hermitian generalized connections over generalized Kähler manifolds of symplectic type.
arXiv:1612.08190 Scalar curvature as moment map in generalized Kähler geometry.

Invariant and anti-invariant forms on almost-complex manifolds

Richard Hind

University of Notre Dame (USA)

We investigate the invariant and anti-invariant forms, and L^2 cohomology of almost-complex manifolds. Following Draghici, Li and Zhang in the case of closed manifolds, the degree 2 L^2 cohomology of a 4-manifold decomposes into invariant and anti-invariant parts, a generalization of the Hodge decomposition. However other rigidity statements fail in the almost-complex case. For a complete, d-bounded Kahler manifold, Gromov showed that the L^2 cohomology is trivial except in the middle dimension; this is not true for complete d-bounded almost-Kahler manifolds.

This is joint work with Adriano Tomassini and Costantino Medori.

Twistor spaces, Del Pezzo fibrations, and quartic hypersurfaces

Nobuhiro Honda

Tokyo Institute of Technology, Tokyo (Japan)

I will talk about a recent result on algebraic description of a wide class of compact twistor spaces associated to anti-self-dual metrics on 4-manifolds. Each of these spaces is birational to the total space of a Del Pezzo fibration over CP^1, and may be described by a single quartic polynomial of a particular form. Generic fibers of the fibration are (possibly singular) Del Pezzo surfaces of degree two.

Harmonic analysis techniques in several complex variables

Loredana Lanzani

Syracuse University (USA)

I will give a survey of point work with E. M. Stein (Princeteon U.) concerning the analysis of a number of singular integral operators in complex Euclidean space (the Szego projection and a family of Cauchy-like singular integrals) for domains with minimal boundary regularity.

I will show how techniques from harmonic analysis can be employed in this context to obtain optimal, or close to optimal results, which would not be achievable with the original methods.

Specifically, I will discuss boundedness in Lebesgue space and will then give counterexamples (unboundedness) that indicate the optimality of our assumptions.

Characterization of distinguished uniruled projective subvarieties in terms of geometric substructures and applications to Kähler geometry

Ngaiming Mok

The University of Hong Kong, (Hong Kong)

Together with Jun-Muk Hwang we introduced in the late 1990s a geometric theory of uniruled projective manifolds based on the variety of minimal rational tangents (VMRT), i.e., the collection of tangents to minimal rational curves on a uniruled projective manifold $(X, \mathcal{K})$ equipped with a minimal rational component. This theory provides differential-geometric tools for the study of uniruled projective manifolds, especially Fano manifolds of Picard number 1. Associated to $(X, \mathcal{K})$ is the fibered space $\pi : C(X) \to X$ of VMRTs called the VMRT structure on $(X, \mathcal{K})$. More recently, taking $(X, \mathcal{K})$ as an ambient space, with collaborators the author has been studying the geometry of germs of complex submanifolds on them in analogy to the geometry of submanifolds in Riemannian manifolds. We focus on germs of complex submanifolds $(S; x_0)$ on $(X, \mathcal{K})$ inheriting geometric substructures, to be called sub-VMRT structures, obtained from intersections of VMRTs with tangent subspaces, i.e., from $\varpi : C(S) \to S$, $C(S) := C(X) \cap \mathbb{P}T(S)$. Central to our study is the characterization of certain classical Fano manifolds such as flag manifolds G/P or distinguished uniruled projective subvarieties on them such as Schubert cycles in terms of VMRTs and sub-VMRTs. As applications I will relate the theory to the existence and

uniqueness of certain classes of holomorphic isometries into bounded symmetric domains. For uniqueness results parallel transport (holonomy), a notion of fundamental importance both in Kähler geometry and in the study of sub-VMRT structures, plays an important role.

A note on transitive geometry

Mauro Nacinovich

Università di Roma "Tor Vergata" (Italy)

Transitive geometry was the title chosen in the '60's by V. Guillemin and S. Sternberg to describe a general framework for studying locally homogeneous manifolds. Recently I obtained with my collaborators a criterion for the finite dimensionality of the group of automorphisms of homogeneous it *CR* manifolds, only involving the underlying real contact structure. The main tool was establishing a suitable filtration of the Lie algebra of infinitesimal *CR* automorphisms and applying a criterion, due to N. Tanaka, to the associated graded object. His theorem falls into that general subject and gave us a motivation to consider the topic that I plan to survey in this talk. In particular, I'll describe some new criteria for the finiteness of prolongations, recently obtained in collaboration with S. Marini and C. Medori. As it's known after E. Cartan, prolongations are related to differential invariants of geometrical structures.

Analytic extension to unbounded domains and envelopes of holomorphy

Egmont Porten

Mid Sweden University (Sweden)

In recent years, function theory on unbounded domains in Cn attracted considerable interest. There are several fundamental open problems, which have been understood for a long time for bounded domains. This applies also to the question for which domains the classical Hartogs-Bochner extension theorem holds. In our talk we will give a characterization which links the question to the geometry of envelopes of holomorphy. Then we discuss some classes of domains where more explicit results, in terms of geometry at infinity, may be within reach. The talk presents joint work with Al Boggess and the late Roman Dwilewicz.

Dolbeault cohomology of nilmanifolds with left-invariant complex structure

Sönke Rollenske

Philipps Universität Marburg (Germany)

Classically, Dolbeault cohomology classes on compact complex tori can be represented by invariant (i.e. constant) differential forms. Conjecturally this should generalise to nilmanifolds with left-invariant complex structure and I will present a novel approach that answers the conjecture positively in complex dimension three.

The talk is based on work in progress with Anna Fino and Jean Ruppenthal.

Instantons on hyperkaehler manifolds and pre-potentials

Andrea Spiro

Università di Camerino (Italy)

Instantons on hyperkaeler manifolds (shortly "hk instantons") are connection on principle bundles with curvatures adapted to the hyperkaehler structure. They are special solutions to Yang-Mills (YM) equations and generalize the anti-self-dual solutions of YM equations on R^4 and S^4, classified by Atiyah-Drinfeld-Hitchin-Manin in the '80s using twistor bundles. We discuss an alternative approach, based on works in theoretical physics by the school of V. Ogievetsky in the '90s and by Alekseevsky, Cortes and Devchand in 2003. It provides a description of each hk instanton in terms of the so-called pre-potentials, which are holomorphic functions living on appropriate bundles over the twistor spaces, but not projectable on them. In a recent work with Devchand and Pontecorvo, we use this approach to reproduce, simplify and generalize results like Uhlenbeck's Compactness Theorem and Removable Singularity Theorem for YM fields.

Complex structures on eight dimensional nilpotent Lie algebras

Raquel Villacampa

Centro Universitario de la Defensa, Zaragoza, IUMA (Spain)

Let $\mathfrak{g}$ be an even-dimensional Lie algebra. A complex structure on $\mathfrak{g}$ is an endomorphism $J\colon \mathfrak{g} \longrightarrow \mathfrak{g}$ satisfying $J^2 = -\mathrm{Id}$ and the "Nijenhuis condition". Finding Lie algebras endowed with such structures constitutes an interesting problem with important algebraic and geometrical applications. In the last years several results on classifications of Lie algebras $\mathfrak{g}$ admitting complex structures J have been published, mainly dealing with low dimensions. In the 4-dimensional case, the solvable Lie algebras admitting a complex structure were classified by Ovando in [11]. Concerning dimension 6, Andrada, Barberis, and Dotti classified in [1] the pairs $(\mathfrak{g}, J)$ where $\mathfrak{g}$ is any Lie algebra and J is an abelian complex structure, i.e. J satisfies $[JX, JY] = [X, Y]$ for every $X, Y \in \mathfrak{g}$. The classification of 6-dimensional nilpotent Lie algebras that admit a complex structure (not necessarily abelian) was achieved by Salamon in [12]. Later, the different complex structures on each of these algebras were classified by Ceballos, Otal, Ugarte, and Villacampa in [3]. In the 6-dimensional solvable case, complex structures of certain specific types are studied in [2] and [7]. The existence of complex structures on 6-dimensional product Lie algebras have been recently investigated in [6]. However, apart from the classification by Nakamura [10] of complex paralellizable structures (that is, $J[X, Y] = [JX, Y]$ for every $X, Y \in \mathfrak{g}$) on solvable Lie algebras up to dimension 10, little is yet known in dimensions higher than six.

In [8] we develop a method for constructing all complex structures on even-dimensional nilpotent Lie algebras. In particular, we consider a partition of the space of complex structures J on a nilpotent Lie algebra $\mathfrak{g}$ into *quasi-nilpotent* and *strongly non-nilpotent* structures. The first class is given by those J's for which there exists a non-trivial J-invariant ideal in the center of $\mathfrak{g}$. We notice that this class contains those complex structures of nilpotent type [5]. The second class appeared for the first time in [4], and we will simply refer to it as the class of *SnN* complex structures. Moreover, SnN complex structures are the essentially new complex structures that arise in each even real dimension.

Focusing on this particular type of complex structures, our method allows us to obtain several restrictions on the terms of the ascending central series of $\mathfrak{g}$. Among them, we prove an upper bound on the dimension of the center of $\mathfrak{g}$ as well as a structural theorem in eight dimensions, together with a parametrization of the space of complex structures.

Based on the study carried out in [8], in [9] we prove that there exist infinite non-isomorphic nilpotent Lie algebras in dimension 8 admitting complex structures. In addition, we show that there are infinitely many real homotopy types of 8-dimensional nilmanifolds admitting a complex structure. Moreover, the nilmanifolds that we construct can be endowed with both generalized Gauduchon and balanced Hermitian metrics.

In this talk, I will present the main results of [8] and [9] (joint works with Adela Latorre and Luis Ugarte).

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Dolbeault cohomology of some LCK manifolds

Victor Vuletescu

University of Bucharest (Romania)

In the class of locally conformally Kähler manifolds (LCK for short), the class of *LCK manifolds with potential* play a very important role. The goal of the talk will be to present several recent results concerning the cohomology of some such manifolds, and some implications of these results. The talk is based on joint work with Liviu Ornea and Misha Verbitsky.

Vertical-horizontal decomposition of Laplacians

Xu Wang

NTNU, Trondheim (Norway)

I would like to report a recent joint work with Adriano Tomassini. The main result is a vertical-horizontal decomposition formula of Laplacians for local nilpotent fibrations. We will also discuss its applications in torus fibrations, which include a proof of a theorem of Nomizu and one of its complex analogies without using spectral sequence.

On geometry of 2-nondegenerate CR structures of hypersurface type via bigraded Tanaka prolongation

Igor Zelenko

Texas A & M University (USA)

The talk is devoted to the local geometry of 2-nondegenerate CR manifolds M of hypersurface type. An absolute parallelism for such structures was recently constructed independently by Isaev-Zaitsev, Medori-Spiro, and Pocchiola in the minimal possible dimension ($dimM = 5$), and for $dimM = 7$ in certain cases by C. Porter. We develop a bigraded analog of Tanaka's prolongation procedure to construct a canonical absolute parallelism for these CR structures in arbitrary (odd) dimension with Levi kernel of arbitrary admissible dimension. In the talk we will briefly review the main notions and constructions of the standard Tanaka theory and explain why and how it should be modified in the considered case. The talk is mostly based on the joint work with Curtis Porter, some of the results that will be presented are obtained in collaboration with David Sykes.
