Convection, Stability, and Turbulence

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National Science Foundation
WHERE DISCOVERIES BEGIN
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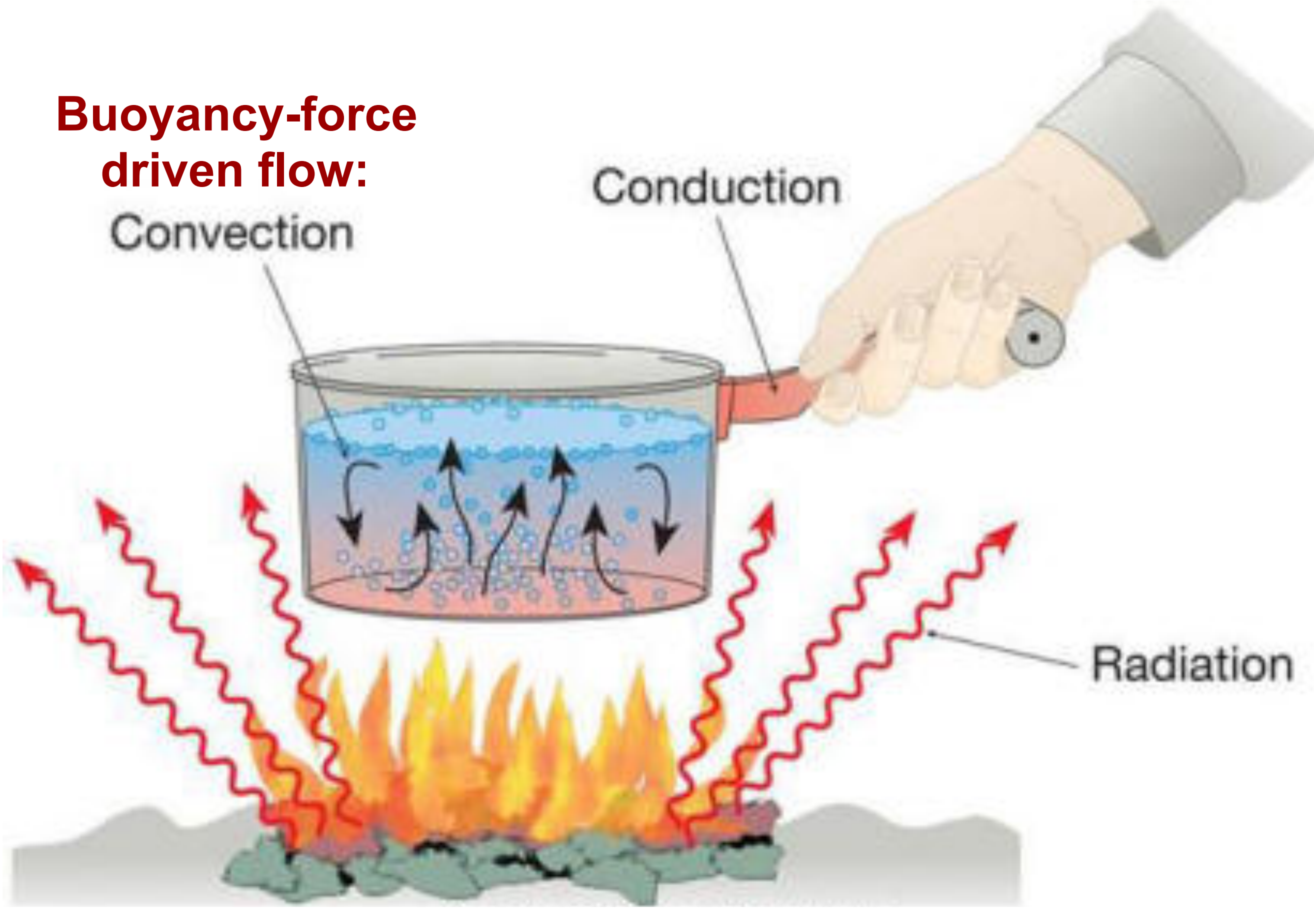
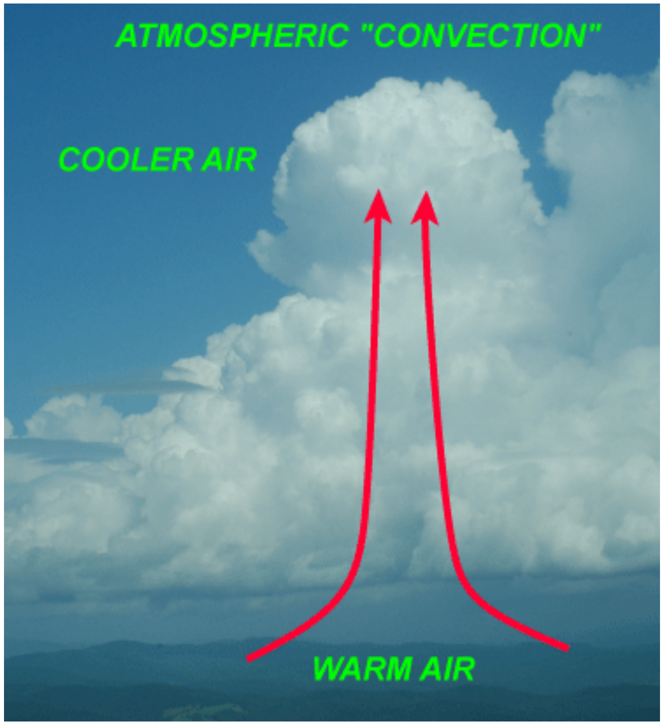
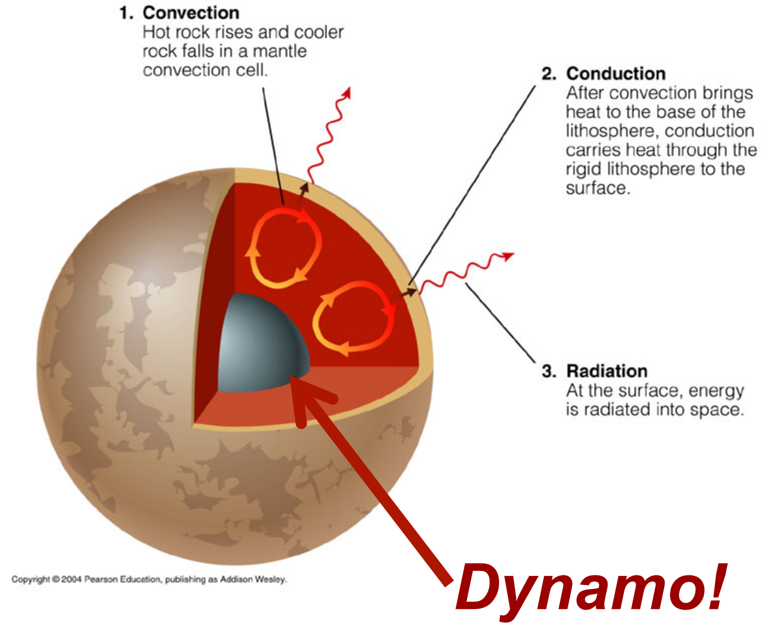
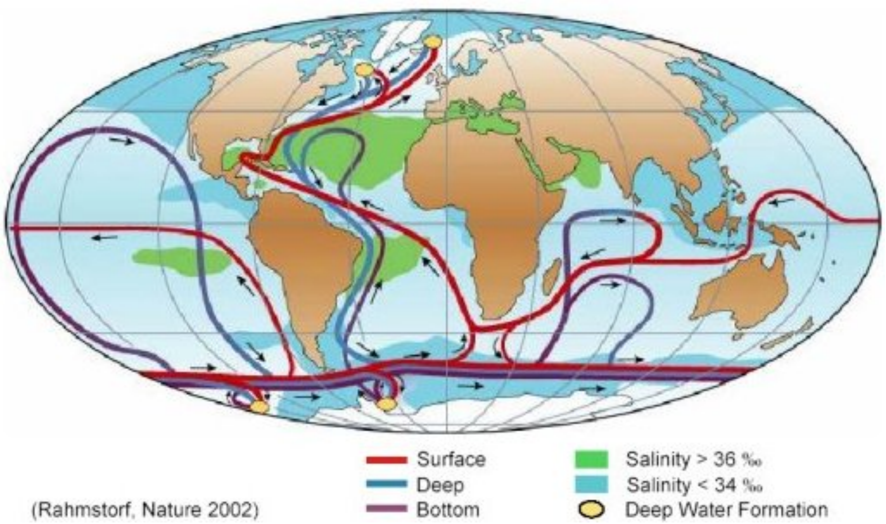
Heat transport mechanisms:

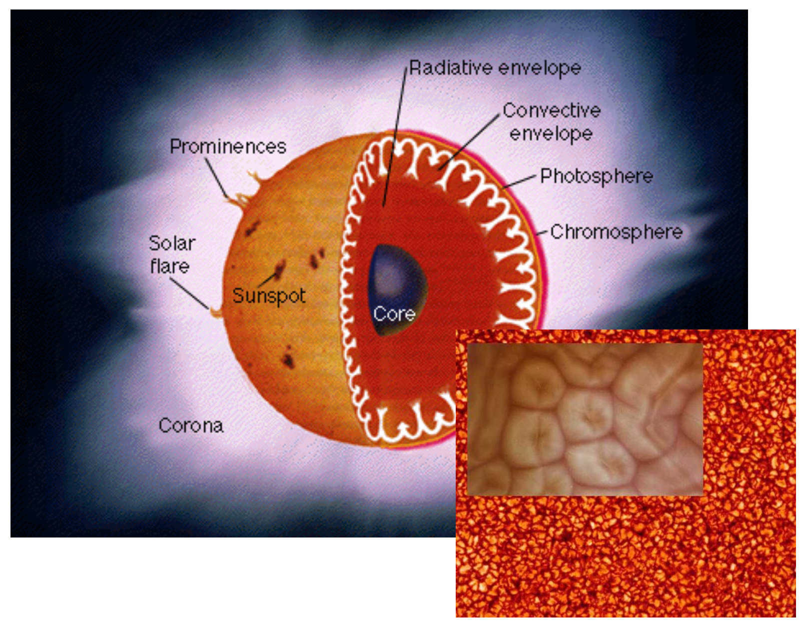








(Rahmstorf, Nature 2002)



Bénard 1900:

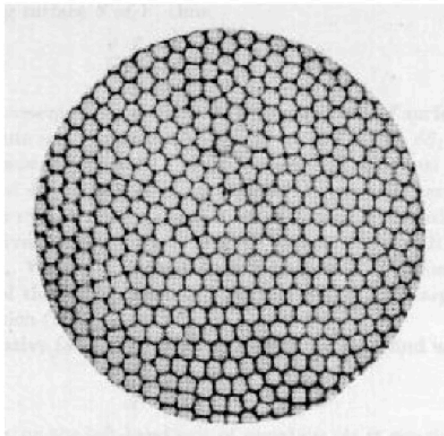


FIG. 1. Bénard cells in spermaceti. A reproduction of one of Bénard's original photographs.

The layer rapidly resolves itself into a number of *cells*, the motion being an ascension in the middle of a cell and a descension at the common boundary between a cell and its neighbours.

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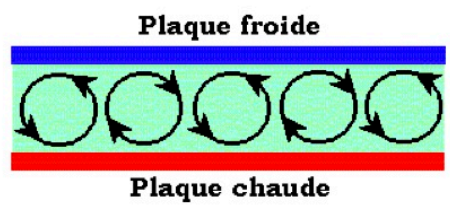
LIX. *On Convection Currents in a Horizontal Layer of Fluid, when the Higher Temperature is on the Under Side.* *By* Lord RAYLEIGH, *O.M., F.R.S.**



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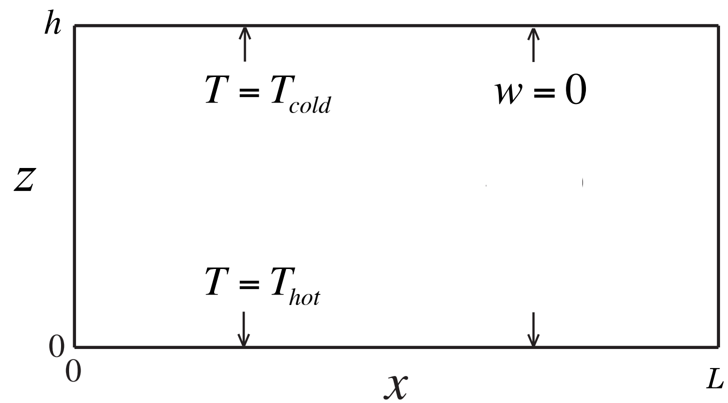
The calculations which follow are based upon equations given by Boussinesq, who has applied them to one or two particular problems. The special limitation which characterizes them is the neglect of variations of density, *except in so far as they modify the action of gravity.* Of course, such neglect can be justified only under certain conditions, which Boussinesq has discussed. They are not so restrictive as to exclude the approximate treatment of many problems of interest.

Rayleigh's minimal mathematical model:



Dynamical variables: Temperature field $T(\vec{x},t)$

Velocity field $\vec{u}(\vec{x},t) = \hat{i}u + \hat{j}v + \hat{k}w$

Pressure field $p(\vec{x},t)$

Boundary conditions: $T = T_{hot}$ and $\hat{k}\cdot\vec{u} = w = 0$ at $z = 0$

$T = T_{cold}$ and $\hat{k}\cdot\vec{u} = w = 0$ at $z = h$

Boussinesq equations:

$$\dot{T} + \vec{u} \cdot \vec{\nabla} T = \kappa \Delta T$$

$$\dot{\vec{u}} + \vec{u} \cdot \vec{\nabla}\, \vec{u} + \frac{1}{\rho} \vec{\nabla} p = \nu \Delta\, \vec{u} + g \alpha\, \hat{k}\, \left(T - T_0\right)$$

$$0 = \vec{\nabla} \cdot \vec{u}$$

Space & time average

We want to know the vertical heat flux :

$$J_z = \left\langle \rho\, c \left(-\kappa \frac{\partial T}{\partial z} + w T \right) \right\rangle$$

$$= \rho\, c\, \kappa \frac{T_{hot} - T_{cold}}{h} + \rho\, c \left\langle w T \right\rangle$$

Boussinesq equations:

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$$= \underbrace{\rho\, c\, \kappa \frac{T_{hot} - T_{cold}}{h}}_{J_{conduction}} + \rho\, c \left\langle wT \right\rangle$$

Boussinesq equations:

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$$0 = \vec{\nabla} \cdot \vec{u}$$

We want to know the

vertical heat flux : $J_z = \left\langle \rho\, c \left(-\kappa \frac{\partial T}{\partial z} + wT \right) \right\rangle$

$$= \rho\, c\, \kappa \frac{T_{hot} - T_{cold}}{h} \;+\; \underbrace{\rho\, c \langle wT \rangle}_{J_{convection}}$$

due to buoyancy

Boussinesq equations:

$$\dot{T} + \vec{u}\cdot\vec{\nabla}T = \kappa\Delta T$$

$$\dot{\vec{u}} + \vec{u}\cdot\vec{\nabla}\,\vec{u} + \frac{1}{\rho}\vec{\nabla}p = \nu\Delta\,\vec{u} + g\alpha\,\hat{k}\,\left(T - T_0\right)$$

$$0 = \vec{\nabla}\cdot\,\vec{u}$$

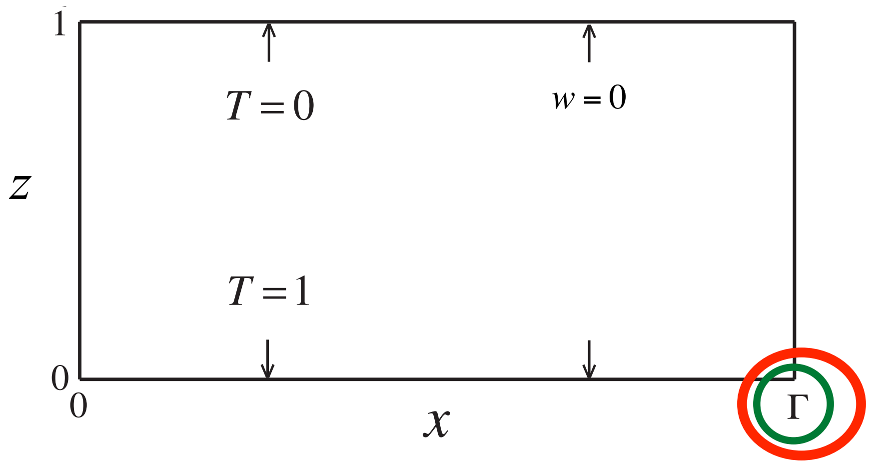
We want to know the vertical heat flux : $J_z = \left\langle \rho\, c\left(-\kappa\frac{\partial T}{\partial z} + wT\right)\right\rangle$

$$= \rho\, c\,\kappa\frac{T_{hot} - T_{cold}}{h} + \rho\, c\left\langle wT\right\rangle$$

Lots of parameters! $h,\ L,\ T_0,\ T_{hot}$ - T_{cold}, $g,\ \kappa,\ \rho,\ \nu,\ \alpha,\ c$

Dimensionless variables:

Rayleigh number: $\mathrm{Ra} = \dfrac{g\alpha(T_{hot} - T_{cold})h^3}{\nu\kappa}$ *Prandtl number*: $\mathrm{Pr} = \dfrac{\nu}{\kappa}$



$$\dot{T} + \vec{u} \cdot \vec{\nabla} T = \Delta T$$

$$\frac{1}{\mathrm{Pr}}\left(\dot{\vec{u}} + \vec{u} \cdot \vec{\nabla}\, \vec{u}\right) + \vec{\nabla} p = \Delta\, \vec{u} + \mathrm{Ra}\, \hat{k}\, T$$

$$0 = \vec{\nabla} \cdot \vec{u}$$

Nusselt number $\mathrm{Nu} \equiv \dfrac{J_z}{J_{conduction}} = 1 + \langle wT \rangle$

Challenge:

find Nu(Ra,Pr,Γ)

$$\text{Facts}: \quad \mathrm{Nu} = \left\langle \left| \vec{\nabla} T \right|^2 \right\rangle = 1 + \frac{1}{Ra} \left\langle \left| \vec{\nabla} \vec{u} \right|^2 \right\rangle \geq 1$$

The task is to determine

(predict/compute/estimate)

$\boldsymbol{J_z}$ resulting from

buoyancy.

Not to be confused with ***Jay-Z*** and ***Beyonce***

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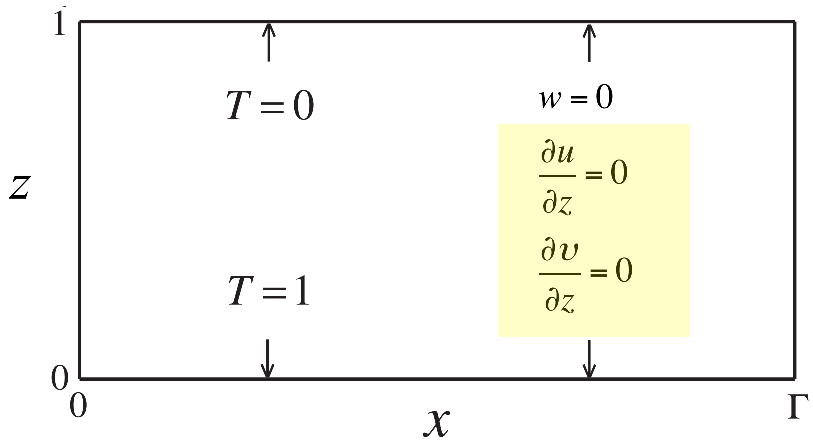
We have also to reconsider the boundary conditions at $z=0$ and $z=\zeta$. We may still suppose $\theta=0$ and $w=0$; but for a further condition we should probably prefer $dw/dz=0$, corresponding to a fixed solid wall. But this entails much complication, and we may content ourselves with the supposition $d^2w/dz^2=0$, which (with $w=0$) is satisfied by taking as before w proportional to $\sin sz$ with $s=q\pi/\zeta$. This is equivalent to the annulment of lateral *forces* at the wall.

Dimensionless variables:

Rayleigh number : $\mathrm{Ra} = \dfrac{g\alpha(T_{hot} - T_{cold})h^3}{\nu\kappa}$ Prandtl number : $\mathrm{Pr} = \dfrac{\nu}{\kappa}$



$$\dot{T} + \vec{u}\cdot\vec{\nabla}T = \Delta T$$

$$\frac{1}{\mathrm{Pr}}\left(\dot{\vec{u}} + \vec{u}\cdot\vec{\nabla}\,\vec{u}\right) + \vec{\nabla}p = \Delta\,\vec{u} + \mathrm{Ra}\,\hat{k}\,T$$

$$0 = \vec{\nabla}\cdot\vec{u}$$

Nusselt number: $\mathrm{Nu} \equiv \dfrac{J_z}{J_{conduction}} = 1 + \langle wT\rangle$

Challenge:

find Nu(Ra,Pr)

Facts : $\mathrm{Nu} = \left\langle \left|\vec{\nabla}T\right|^2 \right\rangle = 1 + \dfrac{1}{Ra}\left\langle \left|\vec{\nabla}\vec{u}\right|^2 \right\rangle \geq 1$

Stability & *instability*

Conduction solution: $\vec{u} = 0$ $T = 1 - z$ $\mathrm{Nu} = 1$

- Linear analysis → sufficient condition for *instability*.
- Write $T(x,y,z,t) = 1 - z + \theta(x,y,z,t)$ and linearize in θ ...
- $(\theta, w) \sim (\theta_k(z), w_k(z)) \cdot \mathrm{e}^{-\lambda t} \mathrm{e}^{i(k_x x + k_y y)}$ → eigenvalue problem:

$$-\lambda \hat{\theta}_k(z) = (\partial_z^2 - k^2)\hat{\theta}_k + \hat{w}_k \qquad -\lambda(\partial_z^2 - k^2)\hat{w}_k(z) = \mathrm{Pr}\,(\partial_z^2 - k^2)^2 \hat{w}_k - \mathrm{Pr}\,\mathrm{Ra}\,k^2\,\hat{\theta}_k$$

- with $\theta_k = 0$ & $w_k = 0 = \partial_z^2 w_k$ at boundaries $z = 0,1$.
- If *any* λ has real part < 0, then there is an *instability*.
- Lord R. '16: $\mathrm{Ra} > \mathrm{Ra_c} = 27\pi^4/4$ → $\lambda_{min} < 0$ → *convection*

Stability & instability

Conduction solution: $\vec{u} = 0 \qquad T = 1 - z \qquad \mathrm{Nu} = 1$

- "Energy" analysis → sufficient condition for *stability*.

- Let $T(x,y,z,t) = 1 - z + \theta(x,y,z,t)$ … then *without* linearization,

$$\frac{d}{dt}\frac{1}{2}\int\left[\theta^2 + \frac{1}{\mathrm{Pr}\,\mathrm{Ra}}\left|\vec{u}\right|^2\right]dx\,dy\,dz \;=\; -\int\left[\left|\vec{\nabla}\theta\right|^2 + \frac{1}{\mathrm{Ra}}\left|\vec{\nabla}\vec{u}\right|^2 - 2w\theta\right]dx\,dy\,dz$$

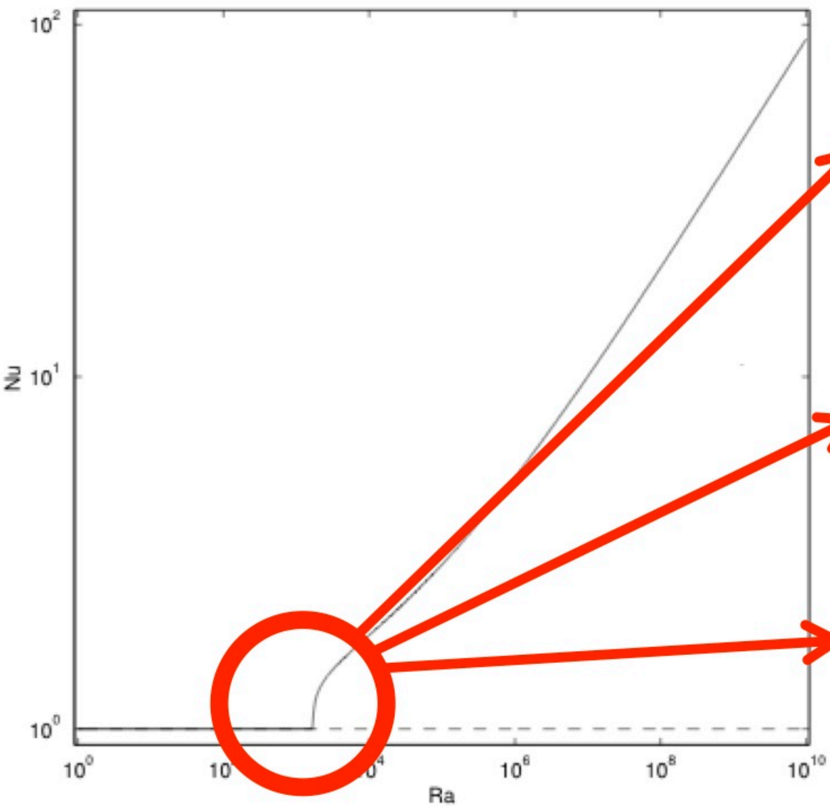
$$= \; -Q\{\theta, w\}$$

- $Q\{\theta,w\} = \int(\theta,w)\cdot S\cdot(\theta,w)$ with symmetric linear operator S.

- If $Q\{\theta,w\} > 0$, i.e., *all* $\lambda > 0$ for $S\cdot(\theta,w) = \lambda(\theta,w)$ → *stability*.

- **Fact**: $\mathrm{Ra} < \mathrm{Ra}_c = 27\pi^4/4 \rightarrow \lambda_{min} > 0 \rightarrow$ *no convection*.

Nu vs. Ra … the big picture:



J. Fluid Mech. (1958), *vol.* 4, *part* 3, *pp.* 225–260

Finite amplitude cellular convection

By W. V. R. MALKUS and G. VERONIS
Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

(*Received* 22 *November* 1957)

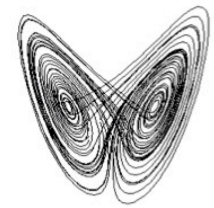
JOURNAL OF THE ATMOSPHERIC SCIENCES

Deterministic Nonperiodic Flow[1]

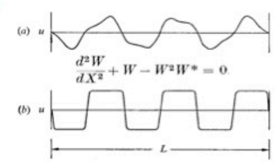
EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)



J. Fluid Mech. (1969), *vol.* 38, *part* 2, *pp.* 225–260



Finite bandwidth, finite amplitude convection

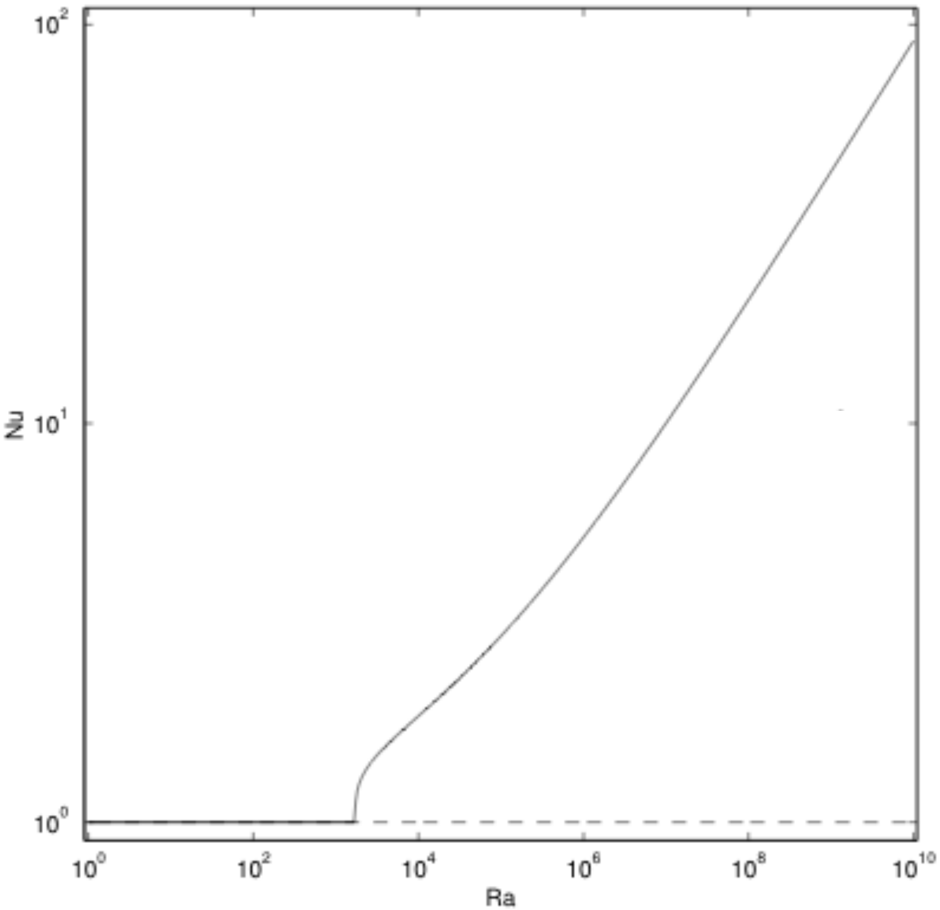
By ALAN C. NEWELL
Department of Planetary and Space Science,
Department of Mathematics

AND J. A. WHITEHEAD
Institute of Geophysics and Planetary Physics,
University of California, Los Angeles

(Received 19 July 1968 and in revised form 4 March 1969)

- **Nu ≥ 1** for all Ra
- **Nu = 1** for all Ra < $Ra_c \approx 657$
- What's the behavior of Nu for Ra > Ra_c?

Nu vs. Ra … the big picture:



Thanks to Susanne Horn, Matthias Kaczorowskii, and Olga Shishkina
http://www.lfpn.ds.mpg.de/RBC2015/

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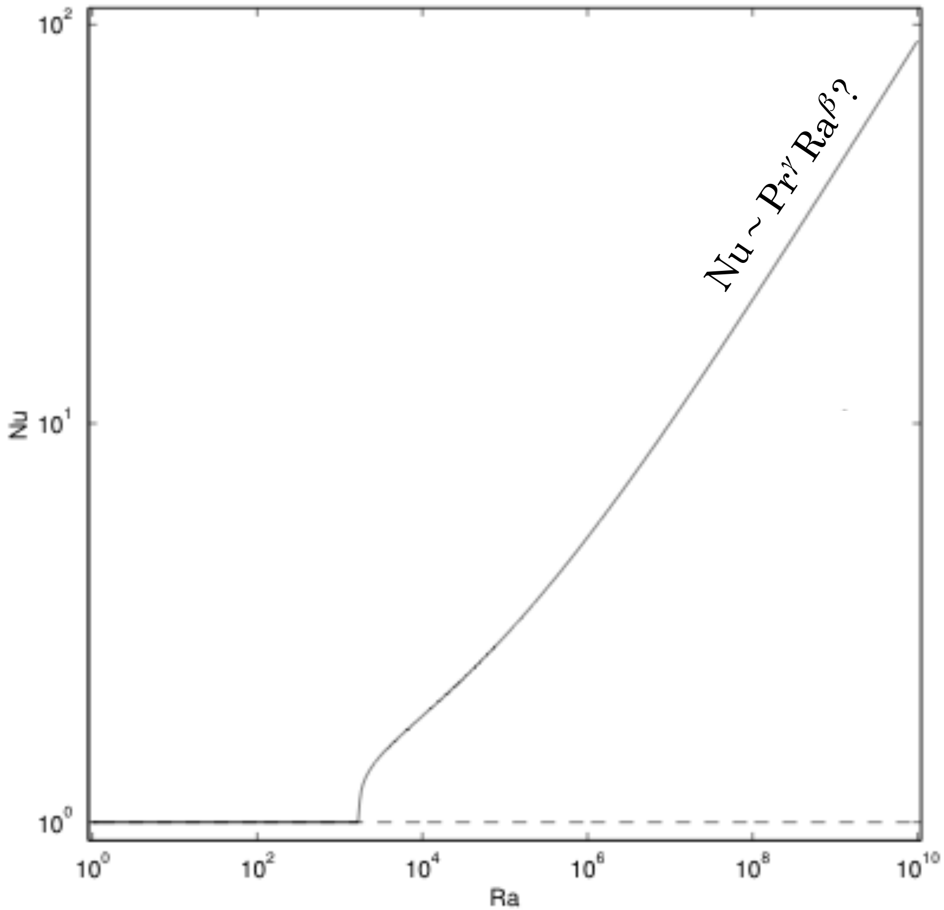
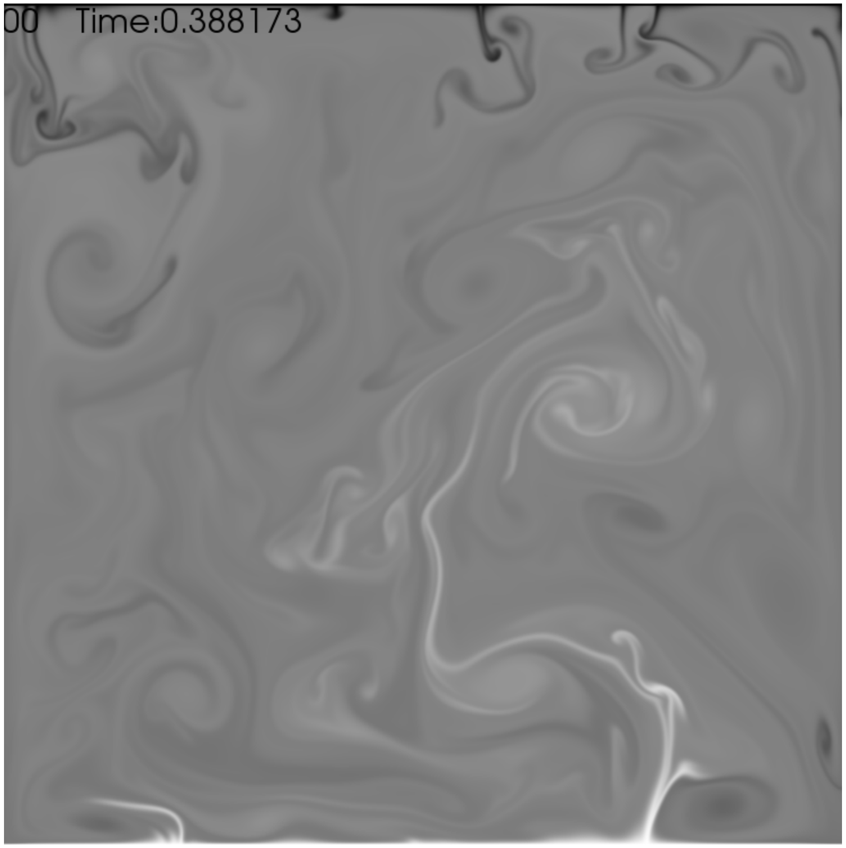




DNS of Rayleigh’s model: image courtesy David Goluskin (2014)

- **Nu ≥ 1** for all Ra
- **Nu = 1** for all Ra < $Ra_c \approx 657$
- What’s the behavior of Nu for $Ra >> Ra_c$?

$$\boxed{\mathrm{Nu} \sim \mathrm{Pr}^{\gamma}\, \mathrm{Ra}^{\beta}}$$

Theories:

Source: *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 225, No. 1161 (Aug. 31, 1954), pp. 196–212

The heat transport and spectrum of thermal turbulence*

By W. V. R. Malkus

Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

(*Communicated by S. Chandrasekhar, F.R.S.—Received* 26 *November* 1953—
Revised 13 *April* 1954)

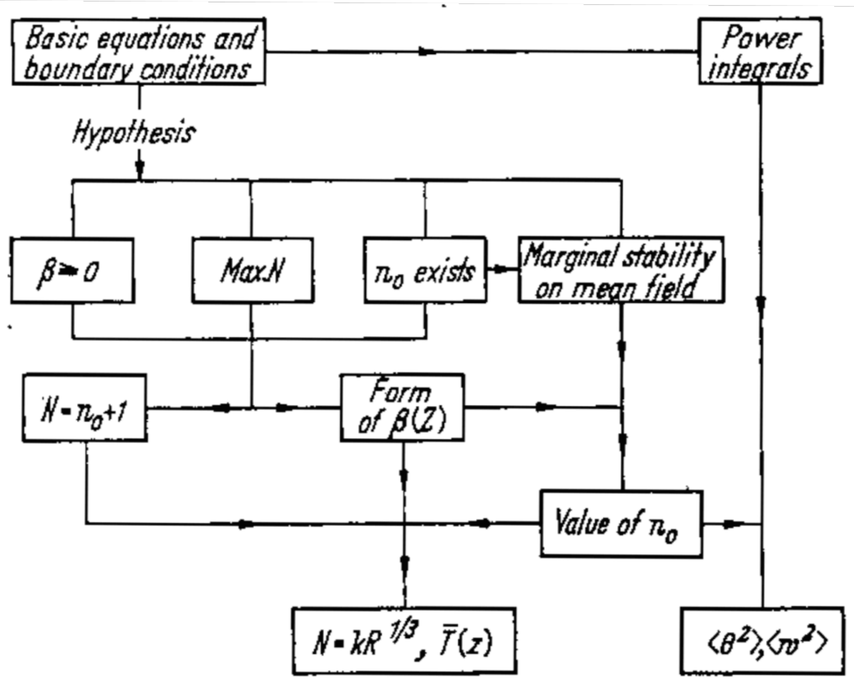
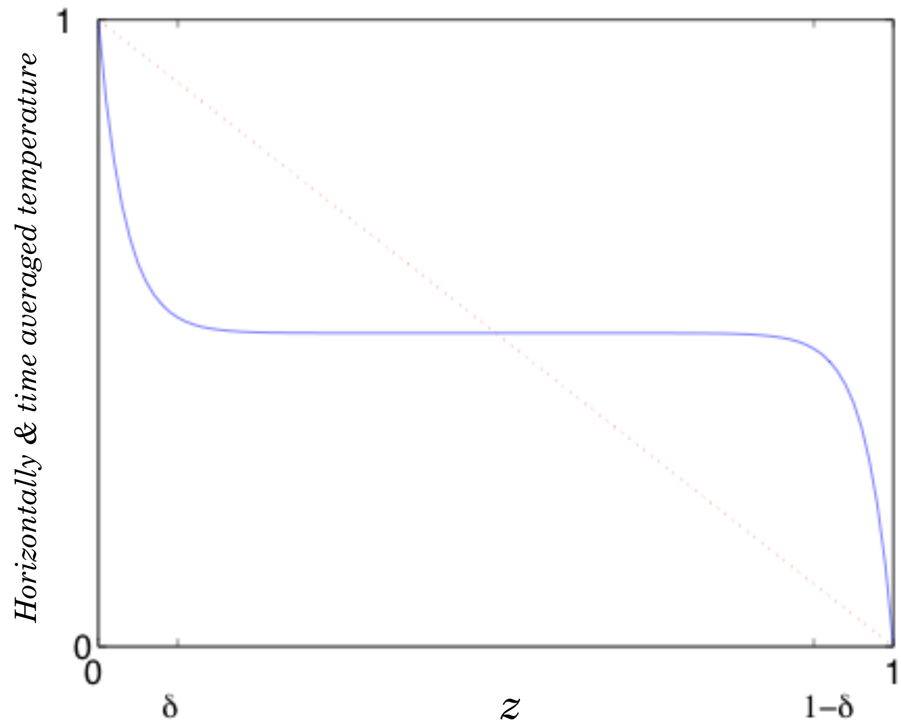


Fig. 1. Structure of Malkus' 1954 theory

Malkus' argument:

For turbulent convection, the mean temperature profile should look like:



$$\mathrm{Nu} \sim (2\delta)^{-1} \quad \ldots \quad \delta = f(\mathrm{Ra},\mathrm{Pr})$$

Assume boundary layer thickness is determined by a ***marginal stability condition***:

$$\mathrm{Ra}_c = \mathrm{Ra}_\delta = \frac{g\alpha\,\frac{1}{2}(T_{hot} - T_{cold})\,(\delta h)^3}{\nu\kappa} = \frac{1}{2}\,\frac{g\,\alpha(T_{hot} - T_{cold})\,h^3}{\nu\kappa} \times \delta^3 = \frac{1}{2}\,\mathrm{Ra}\,\delta^3$$

$$\Rightarrow \quad \delta = \left(2\mathrm{Ra}_c/\mathrm{Ra}\right)^{1/3} \quad \Rightarrow \quad \mathrm{Nu} \approx .05\,\mathrm{Ra}^{1/3}$$ ***uniformly*** in Pr

JOURNAL OF GEOPHYSICAL RESEARCH VOLUME 67, NO. 8 JULY 1962

Thermal Turbulence at Very Small Prandtl Number[1]

EDWARD A. SPIEGEL

Courant Institute of Mathematical Sciences
New York University, New York

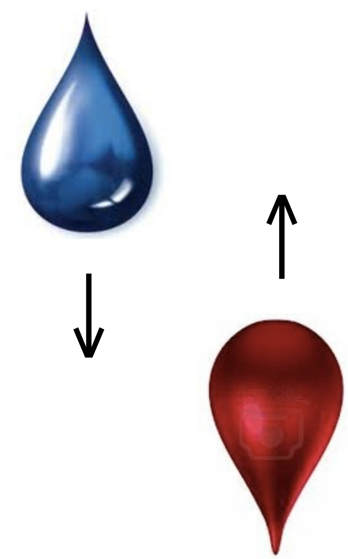
Further, in the limit as $\overset{\mathrm{Pr}}{\cancel{\sigma}} \to 0$ ...

... the well-known $R^{1/3}$ dependence of heat transfer should not hold. Yet this does not seem deducible from the Malkus approach, which shows no Prandtl-number dependence ...

Postulated "***ultimate***" high-Ra scaling: $\mathrm{Nu} \sim \mathrm{Pr}^{1/2}\,\mathrm{Ra}^{1/2}$

Spiegel's argument:

- Assume ***transport across the bulk*** is rate-limiting factor
- … fluid elements 'free-fall' w/acceleration $\sim g\alpha\Delta T$
- … vertical velocity scale is $w \sim [g\alpha\Delta T\, h]^{1/2}$
- … convective heat flux $J_{conv} \sim \rho c \Delta T \cdot w$
- … and therefore $\mathrm{Nu} = 1 + J_{conv}/J_{cond}$
- $\sim \rho c \Delta T [g\alpha\Delta T\, h]^{1/2} \div (\rho c \kappa \Delta T / h)$
- … and thus $\mathrm{Nu} \sim (\mathrm{Pr\ Ra})^{1/2}$



J. Fluid Mech. (2000), *vol.* 407, *pp.* 27–56. Printed in the United Kingdom

Scaling in thermal convection: a unifying theory

By SIEGFRIED GROSSMANN[1] AND DETLEF LOHSE[2]

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(Received 30 April 1998 and in revised form 8 November 1999)

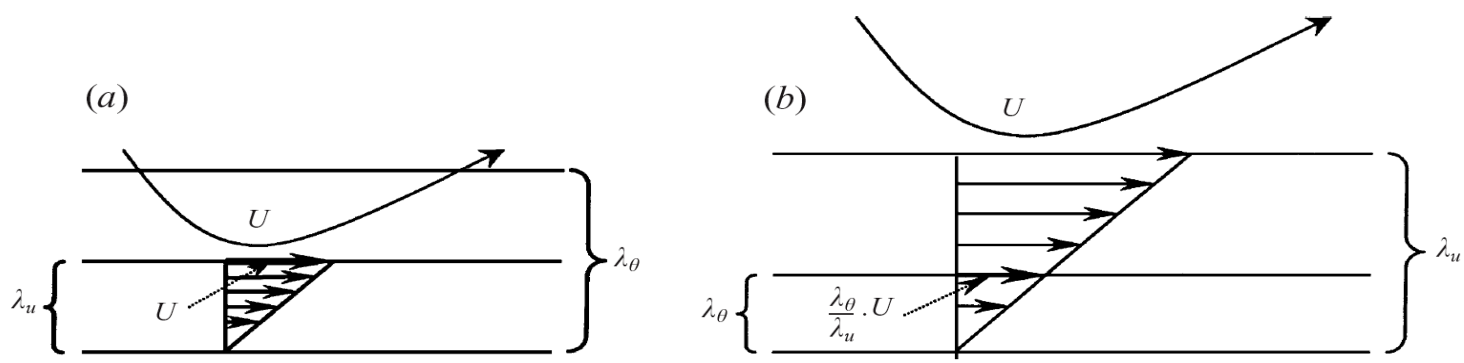
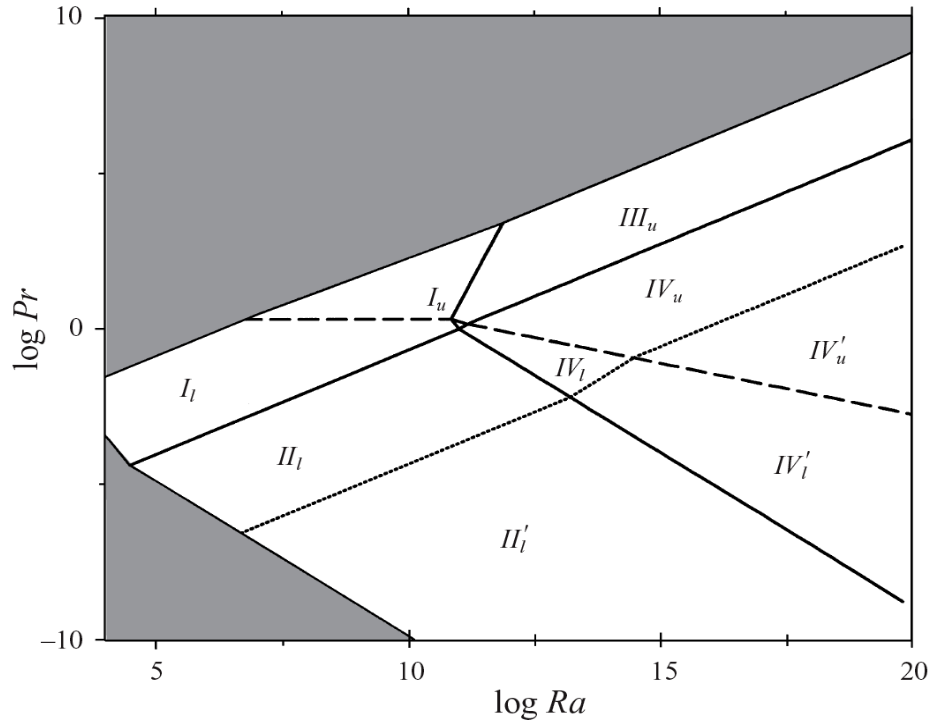


FIGURE 1. Sketch of the boundary layers, (*a*) for low *Pr* where $\lambda_u < \lambda_\theta$ and (*b*) for large *Pr* where $\lambda_u > \lambda_\theta$.

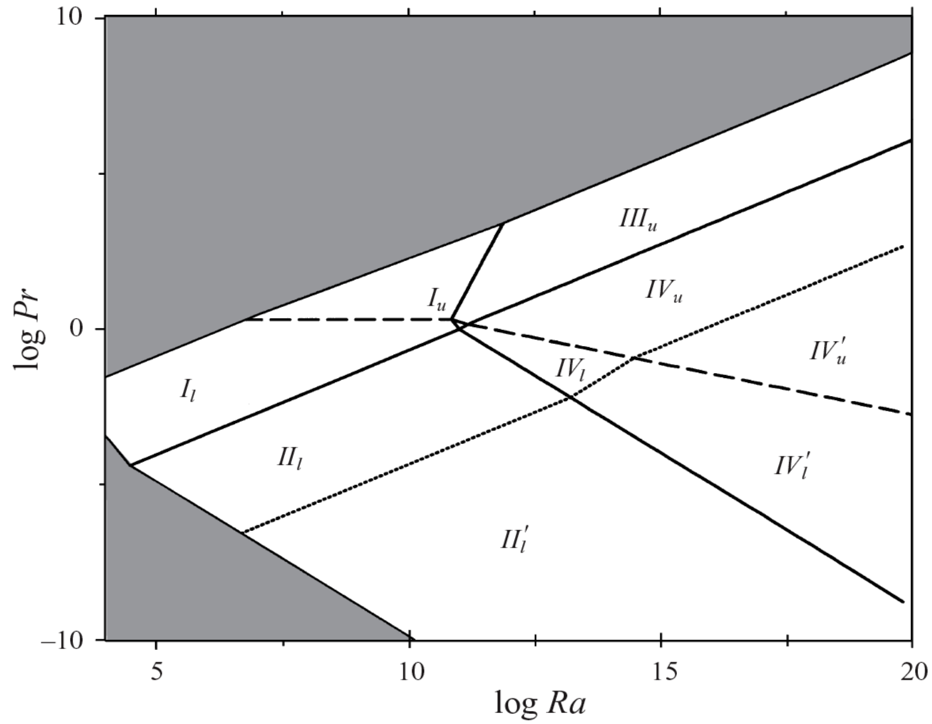
Zoo of Scaling Exponents



Regime	Dominance of	BL	Nu
I_l	$\epsilon_{u,BL}$, $\epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$0.27 Ra^{1/4} Pr^{1/8}$
I_u		$\lambda_u > \lambda_\theta$	$0.33 Ra^{1/4} Pr^{-1/12}$
II_l	$\epsilon_{u,bulk}$, $\epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$0.97 Ra^{1/5} Pr^{1/5}$
(II_u)		$\lambda_u > \lambda_\theta$	$(\sim Ra^{1/5})$
III_l	$\epsilon_{u,BL}$, $\epsilon_{\theta,bulk}$	$\lambda_u < \lambda_\theta$	$6.43 \times 10^{-6} Ra^{2/3} Pr^{1/3}$
III_u		$\lambda_u > \lambda_\theta$	$3.43 \times 10^{-3} Ra^{3/7} Pr^{-1/7}$
IV_l	$\epsilon_{u,bulk}$, $\epsilon_{\theta,bulk}$	$\lambda_u < \lambda_\theta$	$4.43 \times 10^{-4} Ra^{1/2} Pr^{1/2}$
IV_u		$\lambda_u > \lambda_\theta$	$0.038 Ra^{1/3}$

National Zoo (Washington, DC)

Zoo of Scaling Exponents



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IV_l	$\epsilon_{u,bulk}$, $\epsilon_{\theta,bulk}$	$\lambda_u < \lambda_\theta$	$4.43 \times 10^{-4} Ra^{1/2} Pr^{1/2}$
IV_u		$\lambda_u > \lambda_\theta$	$0.038 Ra^{1/3}$

$$\boxed{\mathrm{Nu} \sim \mathrm{Pr}^{\gamma}\, \mathrm{Ra}^{\beta}}$$

Theories: **Experiments:**

$$\mathrm{Nu} \sim \mathrm{Pr}^{0}\, \mathrm{Ra}^{1/3}$$

$$\Updownarrow$$

J_z independent of h

$$\mathrm{Nu} \sim \mathrm{Pr}^{1/2}\mathrm{Ra}^{1/2}$$

$$\Updownarrow$$

J_z independent of ν & κ

Search for the "Ultimate State" in Turbulent Rayleigh-Bénard Convection

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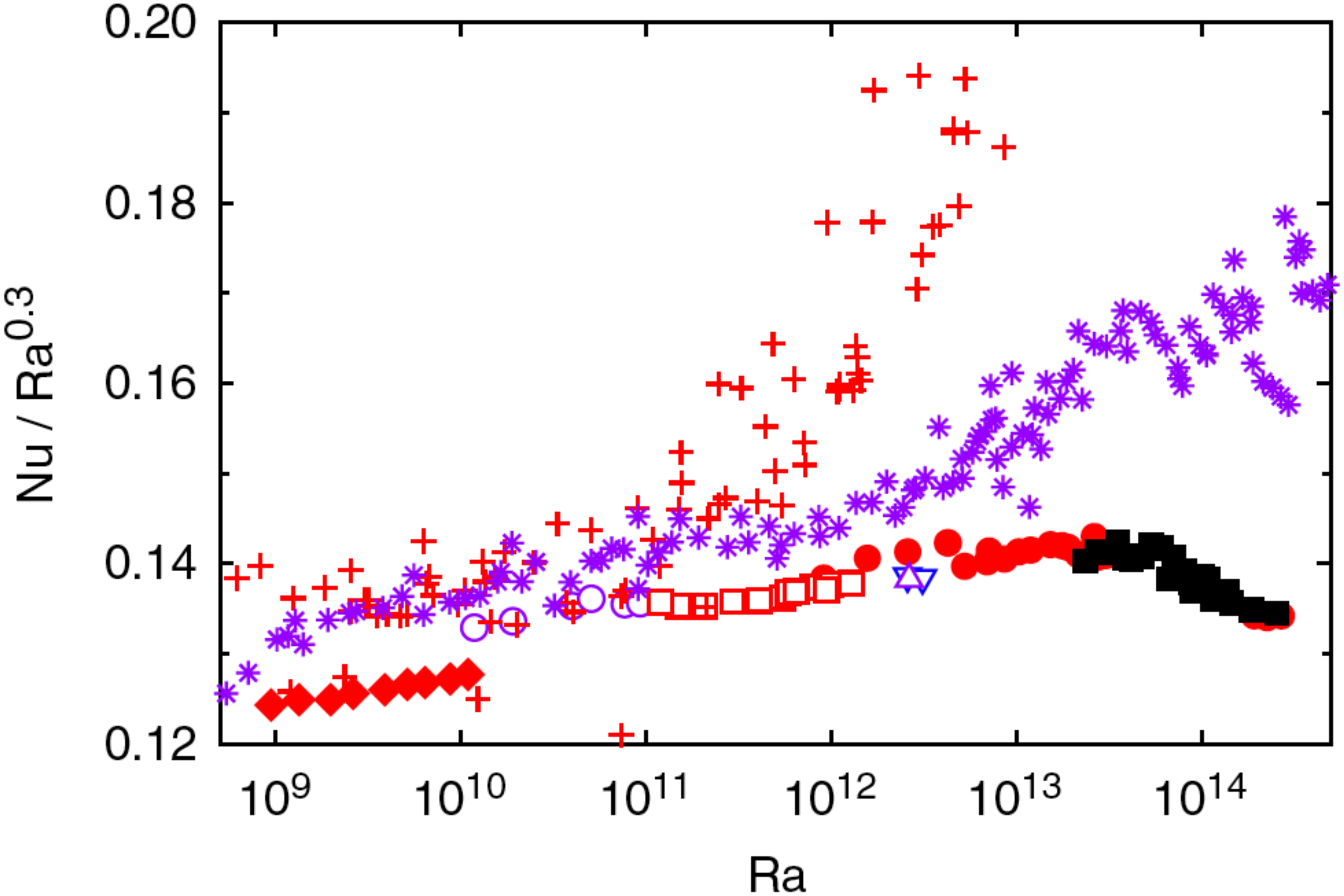
(Received 16 April 2009; published 1 July 2009)



Efficiency of Heat Transfer in Turbulent Rayleigh-Bénard Convection

P. Urban,[1] V. Musilová,[1] and L. Skrbek[2]
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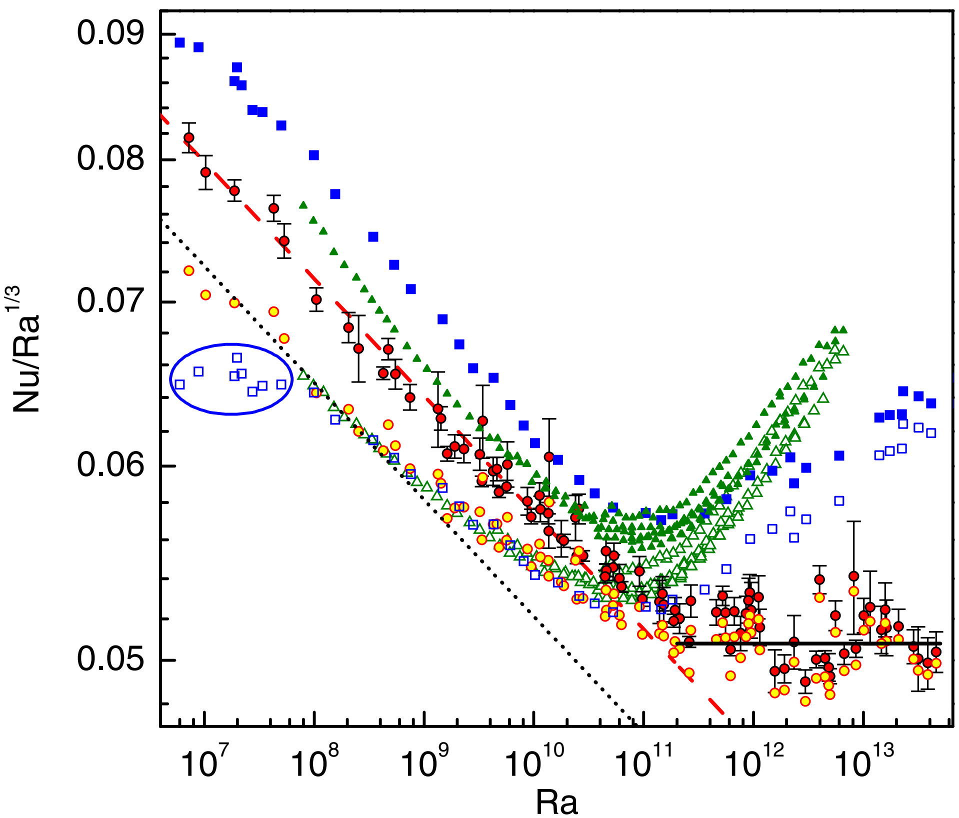

FIG. 2 (color online). The compensated NuRa$^{-1/3}$ plot versus Ra: our measured data (without wall correction) are shown as (red filled) circles with error bars representing the total uncertainty in NuRa$^{-1/3}$ caused by uncertainties in the determination of T_m (4 mK), p (0.1%), ΔT (2 mK) and heat power to the bottom plate (0.5%); (red, yellow filled) circles are our data with the wall corrections applied as described in the text; (olive) triangles and open (olive) triangles represent the uncorrected and corrected ($\Gamma = 1.14$) Grenoble data set [7]; solid (blue) squares and open (blue) squares are the uncorrected and corrected ($\Gamma = 1$) data sets from Trieste ($T_m = 5.34 \pm 0.02$ K) [9]. The dashed (red) line is functional dependence $Nu = 0.172Ra^{2/7}$, the dotted line $Nu = 0.156Ra^{2/7}$, and the solid line $Nu = 0.0508Ra^{1/3}$.

Selected for a Viewpoint in *Physics*
PRL **108,** 024502 (2012) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2012

Transition to the Ultimate State of Turbulent Rayleigh-Bénard Convection

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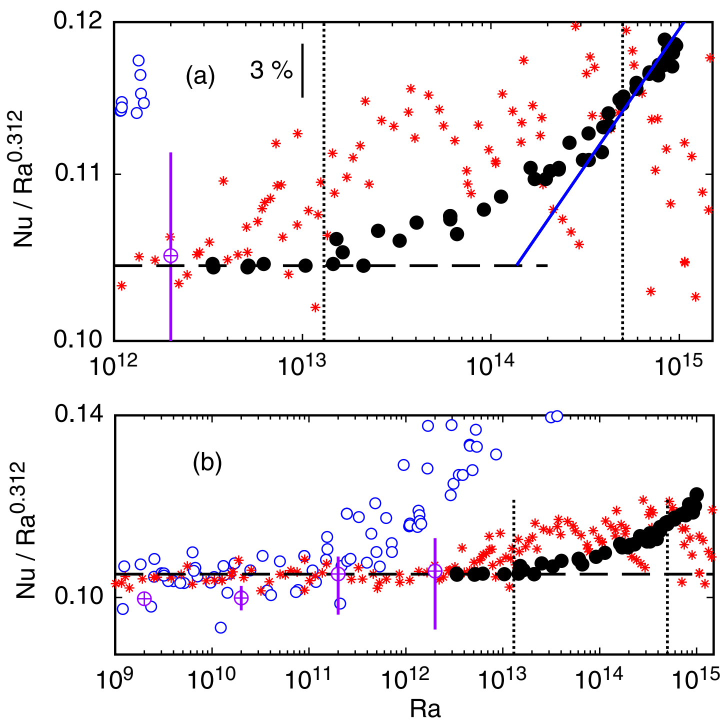

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Measurements of the Nusselt number Nu and of a Reynolds number $\mathrm{Re}_{\mathrm{eff}}$ for Rayleigh-Bénard convection (RBC) over the Rayleigh-number range $10^{12} \lesssim \mathrm{Ra} \lesssim 10^{15}$ and for Prandtl numbers Pr near 0.8 are presented. The aspect ratio $\Gamma \equiv D/L$ of a cylindrical sample was 0.50. For $\mathrm{Ra} \lesssim 10^{13}$ the data yielded $\mathrm{Nu} \propto \mathrm{Ra}^{\gamma_{\mathrm{eff}}}$ with $\gamma_{\mathrm{eff}} \simeq 0.31$ and $\mathrm{Re}_{\mathrm{eff}} \propto \mathrm{Ra}^{\zeta_{\mathrm{eff}}}$ with $\zeta_{\mathrm{eff}} \simeq 0.43$, consistent with classical turbulent RBC. After a transition region for $10^{13} \lesssim \mathrm{Ra} \lesssim 5 \times 10^{14}$, where multistability occurred, we found $\gamma_{\mathrm{eff}} \simeq 0.38$ and $\zeta_{\mathrm{eff}} = \zeta \simeq 0.50$, in agreement with the results of Grossmann and Lohse for the large-Ra asymptotic state with turbulent boundary layers which was first predicted by Kraichnan.

PRL **109,** 154301 (2012) PHYSICAL REVIEW LETTERS week ending 12 OCTOBER 2012

Effect of Boundary Layers Asymmetry on Heat Transfer Efficiency in Turbulent Rayleigh-Bénard Convection at Very High Rayleigh Numbers

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The heat transfer efficiency in turbulent Rayleigh-Bénard convection is investigated experimentally, in a cylindrical cell of height 0.3 m, diameter 0.3 m. We show that for Rayleigh numbers $10^{12} \lesssim \mathrm{Ra} \lesssim 10^{15}$ the Nusselt number closely follows $\mathrm{Nu} \propto \mathrm{Ra}^{1/3}$ if the mean temperature of the working fluid—cryogenic helium gas—is measured by small sensors directly inside the cell at about half of its height. In contrast, if the mean temperature is determined in a conventional way, as an arithmetic mean of the bottom and top plate temperatures, the $\mathrm{Nu(Ra)} \propto \mathrm{Ra}^{\gamma}$ displays spurious crossover to higher γ that might be misinterpreted as a transition to the ultimate ~~Kraichnan~~ regime.

Spiegel

PRL **110,** 199401 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Comment on "Effect of Boundary Layers Asymmetry on Heat Transfer Efficiency in Turbulent Rayleigh-Bénard Convection at Very High Rayleigh Numbers"

In turbulent Rayleigh-Bénard convection (a fluid between two parallel horizontal plates and heated from below) a transition was predicted to occur [1,2] with increasing Rayleigh number Ra from a "classical" state with laminar boundary layers (BLs) to an "ultimate" state with turbulent BLs. Recently, this transition was found in measurements of the Nusselt number Nu and Reynolds number Re as a function of Ra [3–6].

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$$\boxed{\mathrm{Nu} \sim \mathrm{Pr}^{\gamma}\,\mathrm{Ra}^{\beta}}$$

Theories: **Experiments:**

$$\mathrm{Nu} \sim \mathrm{Pr}^{0}\,\mathrm{Ra}^{1/3}$$

controversial

$$\Updownarrow$$

J_z independent of h

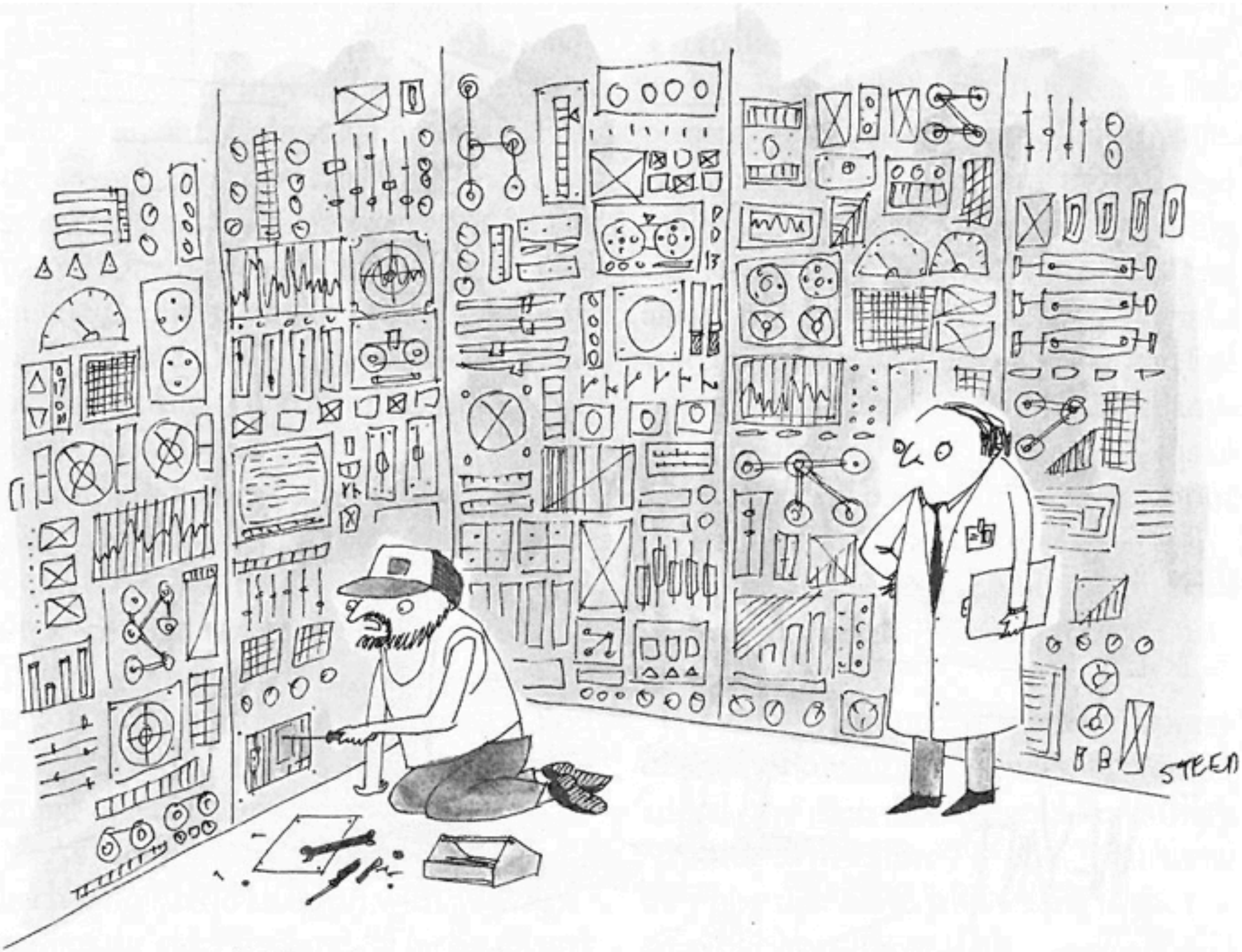
$$\mathrm{Nu} \sim \mathrm{Pr}^{1/2}\mathrm{Ra}^{1/2}$$

$$\Updownarrow$$

J_z independent of ν & κ

CONSIDER …

- Operating budget of LHC runs to $\sim \$10^9$ per year
- Total cost of finding the Higgs boson $= \$1.3 \times 10^{10}$
- Cost of an ultra-high-Γ/high-Ra RBC experiment?



"All right, pal, I'm just saying, that's what I'd do if it was my Large Hadron Collider."

$$\boxed{\mathrm{Nu} \sim \mathrm{Pr}^{\gamma}\,\mathrm{Ra}^{\beta}}$$

Theories: **Experiments:** **Theorems:**

$$\mathrm{Nu} \sim \mathrm{Pr}^{0}\,\mathrm{Ra}^{1/3}$$

controversial

$$\Updownarrow$$

J_z independent of h

$$\mathrm{Nu} \sim \mathrm{Pr}^{1/2}\mathrm{Ra}^{1/2}$$

$$\Updownarrow$$

J_z independent of ν & κ

Journal of Fluid Mechanics **17**, 405-432 (1963)

Heat transport by turbulent convection

By LOUIS N. HOWARD
Department of Mathematics, Massachusetts Institute of Technology

(Received 5 April 1963)

Upper bounds for the heat flux through a horizontally infinite layer of fluid heated from below are obtained by maximizing the heat flux subject to (*a*) two integral constraints, the 'power integrals', derived from the equations of motion, and (*b*) the continuity equation. This variational problem is solved completely, for all values of the Rayleigh number R, when only the constraints (*a*) are imposed, and it is thus shown that the Nusselt number N for any statistically steady convective motion cannot exceed a certain value $N_1(R)$, which for large R is approximately $(3R/64)^{\frac{1}{2}}$. When (*b*) is included as a constraint, the variational problem is solved for large R, under the additional hypothesis that the solution has a single horizontal wave number; the associated upper bound on the Nusselt number is $(R/248)^{\frac{3}{8}}$. The mean properties of this maximizing 'flow', in particular the mean temperature and mean square temperature deviation fields, are found to resemble the mean properties of the real flow observed by Townsend; the results thus tend to support Malkus's hypothesis that turbulent convection maximizes heat flux.