Let $k$ be a field and let $p \in k[x_1, \ldots, x_n]$ be a polynomial. We say that $p$ is a coordinate polynomial, if it can be included in a generating set of cardinality $n$ of the algebra $k[x_1, \ldots, x_n]$. Of course, if $p$ is a coordinate polynomial, then it is irreducible and its zero-set is isomorphic to $k^{n-1}$.

Let $\phi : k[x_1, \ldots, x_n] \to k[x_1, \ldots, x_n]$ be a polynomial automorphism. It is easy to see that if $f$ is a coordinate polynomial, then so is $\phi(f)$. Conversely, Arno van den Essen and Vladimir Shpilrain in the paper [2] have stated the following:

**Problem 1.** Let $k$ be a field of characteristic zero. Is it true that every endomorphism of $k[x_1, \ldots, x_n]$ taking any coordinate polynomial to a coordinate one is actually an automorphism?

This is an interesting problem which has connection with the famous Jacobian Conjecture. In the paper [5] Problem 1 was solved in the affirmative for the complex field $\mathbb{C}$. Naturally, one can ask the following refined version of Problem 1:

**Problem 2.** Let $k$ be a field of characteristic zero. Is it true that every endomorphism of $k[x_1, \ldots, x_n]$ taking any linear polynomial to a coordinate one is actually an automorphism?

For $k = \mathbb{C}$ this problem is still unsolved. When $k$ is not algebraically closed, however, a counterexample to this problem was already constructed by Mikhalev-Yu-Zolotykh ([12]). This result suggests that the situation may be largely different depending on whether $k$ is algebraically closed or not. From this point of view a full solution of Problem 1 seems to be interesting and important.

Here we modify our old approach and using a recent result of Li Y. and Yu J. T. we obtain a full solution of Problem 1:

**Theorem.** Let $k$ be a field of characteristic zero. Let $\phi$ be an endomorphism of $k[x_1, \ldots, x_n]$. Assume that $\phi$ takes any coordinate polynomial to a coordinate one. Then $\phi$ is an automorphism.

**References**


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