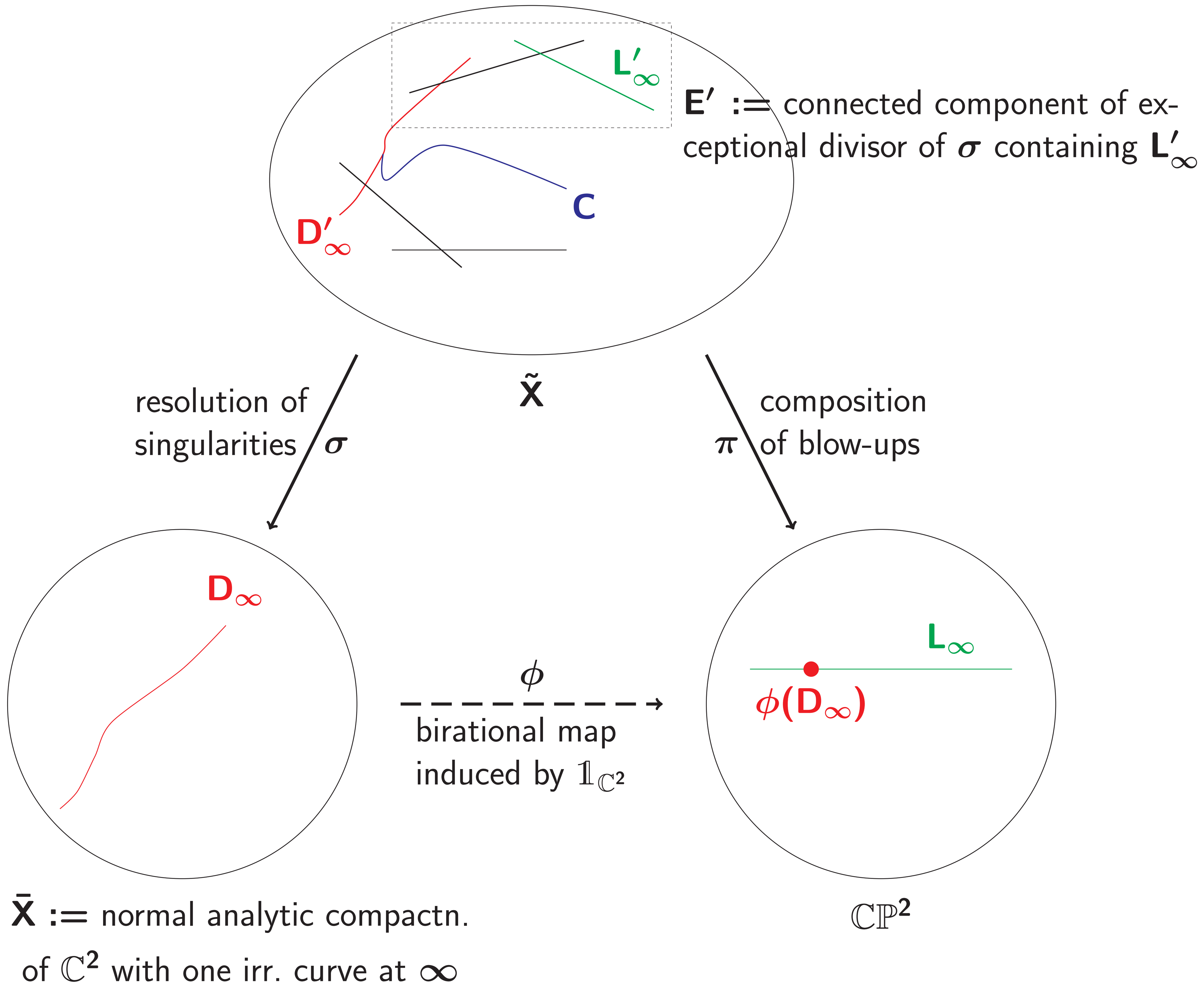
Main application: An effective criterion for algebraicity of rational normal surfaces

Pinaki Mondal, Weizmann Institute of Science

The Correspondence

Normal algebraic compactifications of $\mathbb{C}^2$ with one irreducible curve at infinity $\longleftrightarrow$ **Algebraic curves in $\mathbb{C}^2$ with one place at infinity**



Main Theorem:
$\bar{X}$ algebraic $\iff$ $\exists$ algebraic $C \subseteq \mathbb{C}^2$ such that $\bar{C} \bigcap \left(\tilde{X} \setminus \mathbb{C}^2\right) = \{P\}$ for some point $P \in \tilde{X} \setminus E'$ and $\bar{C}$ is analytically irreducible at P.

Applications

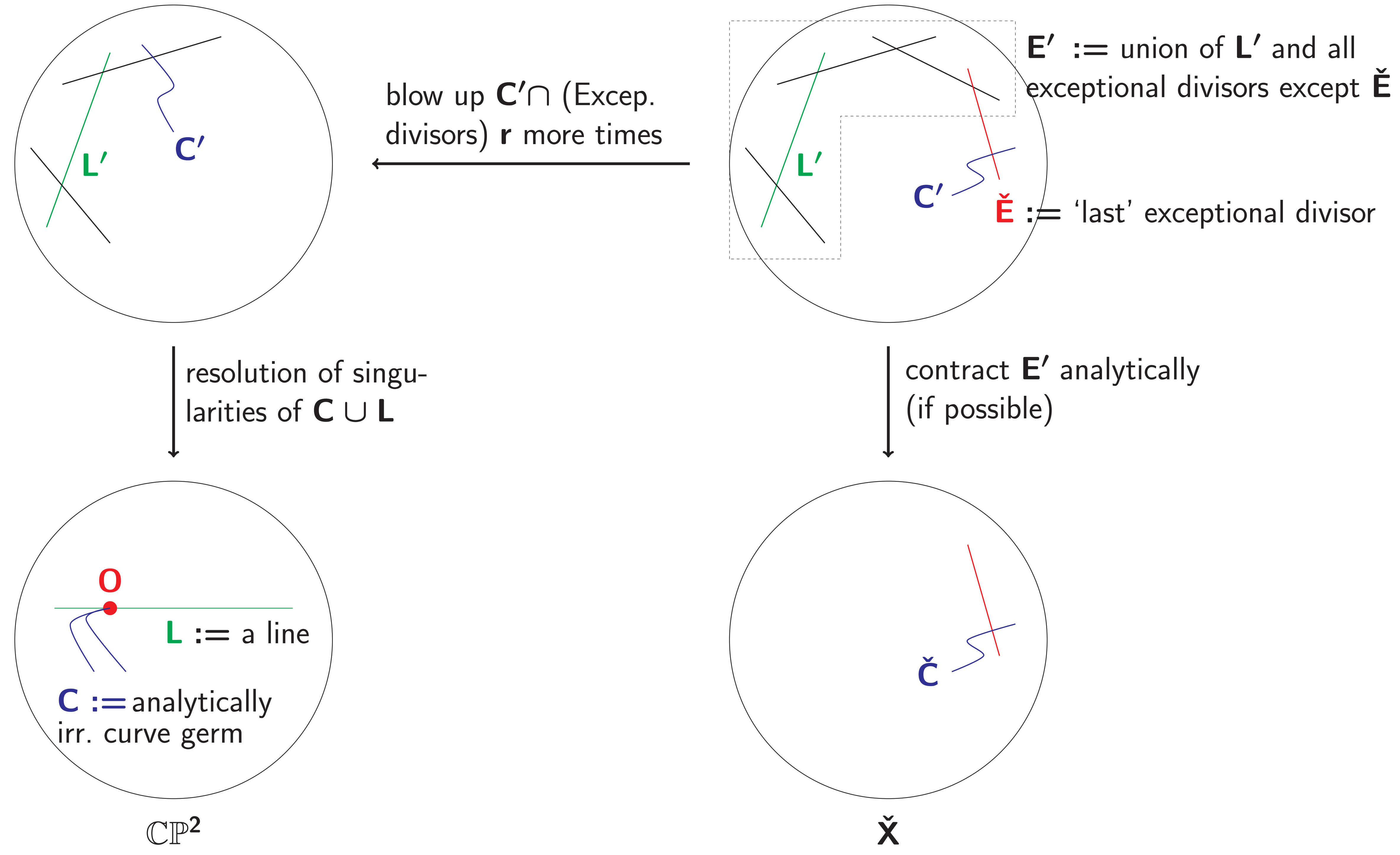
- Computations of

  1. Group of automorphisms of $\bar{X}$ when $\bar{X}$ is algebraic.
  2. Explicit equations and moduli space of normal algebraic compactifications of $\mathbb{C}^2$ with one irreducible curve at infinity.
  3. Canonical divisor of $\bar{X}$.

  .
- Algorithm to determine if a valuation is *negative* or non-positive on $\mathbb{C}[x, y] \setminus \{0\}$.

Reference

Primitive normal completions of the affine plane I and II, arxiv:1110.6905, arxiv:1110.6914 (very rough drafts).



Question: Is $\check{X}$ algebraic?
Answer: Given by an algorithm induced by the Main Theorem.

Algorithm when C has one Puiseux pair

Let $L := \{u = 0\}$, $O := \{u = v = 0\}$, where (u, v) linear coordinates on $\mathbb{P}^2$.
Assume C has Puiseux expansion $v = \phi(u)$ such that ϕ has only one characteristic exponent, namely q/p.

Algorithm:
Find the Weirstrass polynomial $F \in \mathbb{C}\{u, v\}$ in v that defines C.

Then $\check{X}$ is algebraic iff there is no monomial $u^\alpha v^\beta$ in F with non-zero coefficients such that $\alpha p + \beta q < pq + r$ and $\alpha + \beta > p$.

Example

Let $C_j := \{f_j = 0\}$ for $j = 1, 2$, where $f_1 := v^5 - u^3$ and $f_2 := (v - u^2)^5 - u^3$, and let $r := 8$.

Then $p = 5$ and $q = 3$ and $pq + r = 23$.

Therefore $\check{X}_1$ is algebraic.

Coefficient of u^2v^4 in f_2 is non-zero and therefore $\check{X}_2$ is *not* algebraic.

Both $\check{X}_j$ have the *same* dual graph for their minimal resolution of singularities.