International Winter School on Mathematical Fluid Dynamics

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SHORT COURSES

Claude Bardos
John D. Gibbon
Gregory Seregin
Steve Shkoller

INVITED LECTURES

Michele Bartuccelli
Carlo Luigi Berselli
Eduard Feireisl
Vladimir Georgiev
Zoran Grujic
Milton Lopes Filho
Koji Ohkitani
Reimund Rautmann
Tommaso Ruggeri
Alberto Valli

SHORT COMMUNICATIONS

Zachary Bradshaw
Jan Burczak
Laura Caravenna
Davide Catania
Matteo Cerminara
Marcello D'Abbicco
Rafael Granero-Belinchon
Christian Komo
Daniel Lengeler
Xian Liao
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Alessandro Morando
Ridha Selmi
Mikhail Turbin
Aibin Zang
Andrey Zvyagin
Victor Zvyagin

ABSTRACTS

M.V. Bartuccelli: Explicit estimates for the L^∞-norm of solutions of the two-dimensional Navier-Stokes equations on the torus

Abstract: Differential and integral inequalities are a basic tool in the study of the solutions of nonlinear partial differential equations (PDEs). With some exceptions, the constants on which they depend are not commonly computed explicitly. This does not present a problem in some context such as in attempting to prove regularity for the three-dimensional Navier-Stokes equations. Nevertheless, in other situations the estimation of constants is very important as they often contain geometric and number theoretic information which shed light on the overall significance and power of the inequalities themselves. In addition, knowledge of these constants can be crucial for a sharp and detailed analysis of the solutions of PDEs. In this talk we provide explicit (and accurate) values for the constants appearing in the functional estimates of the solutions of the two dimensional Navier-Stokes equations on the torus. We then use these integral inequalities to estimate the sup-norm of the solutions in explicit form.

L.C. Berselli: Pulsatile viscous flows in elliptical vessels and annuli, with application to blood and cerebrospinal fluid flow

Joint work with F. Guerra, B. Mazzolai and E. Sinibaldi (Center for Micro-BioRobotics IIT@SSSA, Italian Institute of Technology Pontedera, Italy)

Abstract: We consider the fully-developed flow of an incompressible Newtonian fluid in a cylindrical vessel with elliptical cross-section, either simply connected (ellipse) or not (annulus between two confocal ellipses). We propose a novel numerical strategy for the inverse problem, which is nonetheless grounded on analytical relations. The proposed method leads to an easily scalable algorithm, merely based on the solution of some tridiagonal linear systems. It holds promise to be more amenable to be implemented than previous approaches reported in literature, which are substantially based on the Mathieu functions.

E. Feireisl: Compressible fluid flows driven by stochastic forcing

Abstract:

Joint work with B. Maslowski (Praha) and A. Novotný (Toulon)

We consider the Navier-Stokes system describing the motion of a compressible barotropic fluid driven by stochastic external forces. Our approach is semi-deterministic, based on solving the system for each fixed representative of the random variable and applying an abstract result on measurability of multi-valued maps. The class of admissible driving forces includes the (temporal) white noise and the random kicks, considered recently in the context of incompressible fluid models.

V. Georgiev: Scalar Field equations and best Sobolev exponent

Abstract: We consider the scalar field equation

$$\Delta u + u^p = u \tag{1}$$

where $u = u(|x|)$ is a radial positive solution and $x \in \mathbb{R}^3$. The equation (1) can be rewritten as ordinary differential equation

$$ru''(r) + 2u'(r) + ru(r)^p = ru(r) \tag{2}$$

and our goal is to construct suitable series expansions representing the unique solution in far field and "near zero" domains establishing the uniform convergence of these series expansions. The problem is closely connected with the precise characterization of the best Sobolev constant in 3D.

Z. Grujic: Vortex stretching and criticality for the 3D NSE

Abstract: A mathematical evidence – in a statistically significant sense – of a geometric scenario leading to criticality of the Navier-Stokes problem is presented. It is based on the following paper http://arxiv.org/abs/1205.7080

M. Lopes Filho: Weak solutions of the incompressible Euler equations without decay at infinity

Abstract: In 1995, P. Serfati published an announcement of an existence and uniqueness Theorem for the 2D incompressible Euler equations in the full plane with bounded initial velocity and vorticity. Similar results have appeared since, but the local character and the precise scope of the idea proposed by Serfati make it interesting to revisit it in more detail. In this talk, we explore Serfati's ideas, completing the full plane proof and extending it to exterior domains.

K. Ohkitani: Phenomenology of enstrophy bounds for the Navier-Stokes and related equations

Abstract: It is well-known that the global regularity can be proved for the d-dimensional incompressible fluid dynamical equations with generalised dissipativity $(-\triangle)^\alpha$, provided $\alpha \geq \frac{d+2}{4}$ [Lions (1969)]. We are interested in obtaining lower bounds of possible blowup of the (generalised) enstrophy. Heuristic arguments are given here regarding the time evolution of the H^α-norm, by carrying out two kinds of dimensional analyses for the H^α-norm itself and its differential inequality.

On dimensional grounds, another branch $\alpha = \frac{d+2}{6}$ is identified, at which even local existence can no longer be proved by the standard method because of lack of differential inequalities. A structural similarity is noted between the 1D Burgers equation with hypo-viscosity $(-\triangle)^{1/2}$ and the 4D Navier-Stokes equations, in that both of them lie on the lower-branch. We recall that the former is known to be globally regular by [Kiselev-Nazarov-Shterenberg (2008)].

The breakdown of the differential inequality at the lower-branch is due to loss of a spatial scale in dimensional analysis. It is proposed to seek an alternative proof for the regularity of the hypo-viscous Burgers equation. A preliminary attempt is made to recover the bounds by assuming that the total energy comes into play and a one-parameter family of inequalities is determined.

Finally, numerical results on the 1D hypoviscous Burgers equations with $(-\triangle)^{1/2}$ and the 3D modified Navier-Stokes equations with $(-\triangle)^{5/6}$ are presented.

R. Rautmann: A variational aspect of the no-slip condition

Abstract: The generation of vorticity at the boundary of any solid body inside of a viscous fluid flow belongs to the important problems of hydrodynamics. Generally spoken, the no-slip condition at the boundary requires the production of vorticity, which, as we have learned from Ludwig Prandtl, is the cause of lift and drag (e.g. of any airfoil). In order to get mathematical models, Lighthill (1974), Chorin (1973, 1978), and Marsden (1974) have introduced different types of vorticity generating operators. However they had to assume sufficiently thin boundary layers without strong separation. In my talk I will show that the no-slip condition can be fullfilled in a quite natural way by means of a suitable orthogonal projection, thus by a variational principle. This orthogonal projection and the resulting orthogonal decomposition of the space H lead to simplified versions of that product formula schemes which originally had been proposed by Chorin and Marsden for flow computations at higher Reynolds numbers.

T. Ruggeri: Maximum entropy principle for rarefied polyatomic gases

Abstract: The kinetic theory and the Extended thermodynamics (ET) are important theories for rarefied non-equilibrium gas. Nevertheless the weak point is that the range is limited to monatomic gas. In this talk we want to present recent new approach to deduce hyperbolic system for real gas not necessary rarefied. In the first part of the talk we study extended thermodynamics of dense gases by adopting the system of field equations with a different hierarchy structure to that adopted in the previous works. It is the theory of 14 fields of mass density, velocity, temperature, viscous stress, dynamic pressure and heat flux. As a result, all the constitutive equations can be determined explicitly by the caloric and thermal equations of state as in the case of monatomic gas. It is shown that the rarefied-gas limit of the theory is consistent with the kinetic theory of gases. In the second part we specialized the result to the physical interesting case of rarefied polyatomic gas and we show a perfect coincidence between ET and the procedure of maximum entropy Principle. The main difference with respect to usual procedure is existence of two hierarchies of macroscopic equations for moments of suitable distribution function, in which the internal energy of a molecule is taken into account.

References:

1) I. Mueller and T. Ruggeri, (1998), Rational Extended Thermodynamics, 2nd ed., Springer Tracts in Natural Philosophy 37, Springer-Verlag, New York.

2) G. Boillat and T. Ruggeri, (1997), Moment equations in the kinetic theory of gases and wave velocities, Continuum Mech. Thermodyn., 9, 205-212.

3) T. Arima, S. Taniguchi, T. Ruggeri and M. Sugiyama, (2011), Extended thermodynamics of dense gases, Continuum Mech. Thermodyn., doi: 10.1007/s00161-011-0213-x.

4) T. Arima, S. Taniguchi, T. Ruggeri and M. Sugiyama, (2011), Extended thermodynamics of real gases with dynamic pressure: An extension of Meixners theory, Physics Letters A, 376, 2799-2803

5) M. Pavic, T. Ruggeri and S. Simic, (2012), Maximum entropy principle for rarefied polyatomic gases. Submitted to Physica A

A. Valli: Finite element approximation of the curl−div system

Abstract:
joint work with Ana Alonso Rodríguez, Enrico Bertolazzi, Riccardo Ghiloni

The aim of this talk is two-fold. First, employing edge finite elements, we construct a discrete approximation of the space of harmonic fields

$$\mathbb{H}_\mu(\Omega) = \{\mathbf{v} \in (L^2(\Omega))^3 \,|\, \mathrm{curl}\,\mathbf{v} = \mathbf{0}\,,\ \mathrm{div}(\mu\mathbf{v}) = 0\,,\ \mu\mathbf{v}\cdot\mathbf{n} = 0 \text{ on } \partial\Omega\}$$

(here Ω is a bounded three-dimensional domain with a Lipschitz boundary and μ is a symmetric matrix, uniformly positive definite in Ω and with entries in $L^\infty(\Omega)$).

In particular, we give a simple and efficient computational way for constructing the so-called *loop fields*, i.e., the irrotational vector fields $\mathbf{T}_0$ that cannot be expressed in Ω as the gradient of any single-valued scalar potential (therefore, there exists a loop in Ω such that the line integral of $\mathbf{T}_0$ on it is different from 0). These fields are of central importance for numerical fluid dynamics and electromagnetism in general topological domains. Let us also recall that a maximal set of linearly independent loop fields gives a basis of the first de Rham cohomology group of Ω.

Second, we furnish a finite element numerical solution to the curl-div system

$$\begin{array}{ll} \mathrm{curl}\,\mathbf{H} = \mathbf{J} & \text{in } \Omega \\ \mathrm{div}(\mu\mathbf{H}) = f & \text{in } \Omega \\ \mu\mathbf{H}\cdot\mathbf{n} = q & \text{on } \partial\Omega \\ \mu\mathbf{H} \perp \mathbb{H}_\mu(\Omega)\,, & \end{array}$$

where $\mathbf{J} \in (L^2(\Omega))^3$ with $\mathrm{div}\,\mathbf{J} = 0$ in Ω and $\int_{(\partial\Omega)_r} \mathbf{J}\cdot\mathbf{n} = 0$ for all the connected components $(\partial\Omega)_r$ of $\partial\Omega$, $f \in L^2(\Omega)$, $q \in H^{-1/2}(\Omega)$ and $\int_\Omega f = \int_{\partial\Omega} q$.

In particular, the vector fields satisfying $\mathrm{curl}\,\mathbf{H}_e = \mathbf{J}$ in Ω are often called *source fields* in the electromagnetic literature, and are needed for an efficient formulation of eddy current problems; on the other hand, solving the curl-div system is the standard way in incompressible fluid dynamics for reconstructing the velocity field when the vorticity is known.

The proposed method works for general topological configurations and does not need the determination of "cutting" surfaces.

Z. Bradshaw: Turbulent cascades in physical scales of 3D incompressible plasmas

Abstract: Certain cascades are uniquitous across current phenomenologically and numerically motivated theories of magnetohydrodynamic turbulence. This communication outlines an approach which, under physically reasonable assumptions, establishes the existence in physical scales of several such cascades as well as flux locality.

J. Burczak: Hölder continuity of gradients to non-diagonal parabolic systems

Abstract: The problem of local Hölder continuity of gradients for the evolutionary p-Laplace system has been shown in a series of papers by DiBenedetto, summed up in a monograph [1]. From the perspective of mathematical physics, it is interesting to replace ∇u by its symmetric part $\delta u = \frac{\nabla u + \nabla^T u}{2}$, obtaining:

$$u_{,t} - \operatorname{div} v[(\mu + |\delta u|^2)^{\frac{p-2}{2}} \delta u] = 0. \tag{3}$$

It turns out that such a supposedly harmless amendment from ∇u to δu, changes the situation dramatically, as pointwise methods used for the full gradient case turn out to be useless. We present a results on $C^{1,\alpha}$ regularity of solutions to a generalization of (3) in arbitrary dimension, which are obtained at a price either of restricting to p close to 2 or of considering partial regularity. In the second case we present an elegant version of the caloric approximation approach of [2] and generalize results of [3].

References

[1] E. DiBenedetto *Degenerate parabolic systems*, Springer, 1993
[2] F. Duzaar, G. Mingione, K. Steffen *Parabolic Systems with Polynomial Growth and Regularity*, Memoirs A.M.S. 214, 2011
[3] G. Seregin, O. Ladyzhenskaya *On partial regularity of suitable weak solutions to the three-dimensional Navier-Stokes equations*, J. Math. Fluid Mech. 1 (1999), no. 4, 356387

L. Caravenna: Continuous solution to scalar balance laws with bounded source

Abstract: We discuss the equivalence between different notion of continuous solutions to a scalar balance law with bounded source.

D. Catania: Existence for linearized current-vortex sheet solutions

Abstract: We consider the free boundary problem for current-vortex sheets in ideal incompressible magnetohydrodynamics. The problem of current-vortex sheets arises naturally, for instance, in geophysics and astrophysics. We prove the existence of a unique solution to the constant-coefficient linearized problem and an a priori estimate with no loss of derivatives. This is a preliminary result to the study of linearized varibale-coefficient current-vortex sheets, a first step to prove the existence of solutions to the nonlinear problem.

M. Cerminara: Direct numerical simulation of a compressible multiphase flow through a fast Eulerian approach

Abstract: Our work is motivated by the analysis of ash plume dynamics, arising in the study of volcanic eruptions. Such phenomena are characterized by large Reynolds number (exceeding 10^7) and a large number of polydispersed particles. Thus, the choice of the methodology to be used is straightforward: we need LES of a multiphase gas-particles flow. Since the simulation of the behavior of a large number of dispersed particles is very difficult with Lagrangian methods, we model the particles as a continuum, Eulerian fluid (dust), by using reduced models involving two fluids. Moreover, we need a robust numerical scheme to simultaneously treat compressibility, buoyancy effects and turbulent dispersal dynamics.

We analyze the turbulence properties of such models in a homogeneous and isotropic setting, with the aim of formulating a LES model. In particular, we examine the development of freely decaying homogeneous and isotropic turbulence in subsonic regime (the r.m.s. Mach number either 0.2 or 0.5) using OpenFOAM^R, which is one of the best known CFD open source software packages.

M. D'Abbicco: Semilinear structural and viscoelastic damped waves

Abstract: We study the Cauchy problem for the semi-linear structural and viscoelastic damped wave equations with source term $|u|^p$. We are interested in the critical exponent, the threshold between global existence in time of small data solutions and blow-up behavior.

R. Granero-Belinchon: Singularities in the confined Muskat problem

Abstract: In this talk we explain some recent results concerning the evolution of the interface between two fluids in a confined porous medium. In particular we study the contribution of the boundaries to the singularity formation.

C. Komo: Optimal initial value conditions for local strong solutions of the Navier-Stokes equations in exterior domains

Abstract: (Joint work with Reinhard Farwig).

Let u be a weak solution of the Navier-Stokes equations in an exterior domain $\Omega \subseteq \mathbb{R}^3$ and a time interval $[0,T[\,, 0 < T \leq \infty$, with initial value u_0 and external force $f = \operatorname{div} F$. We address the problem to find the optimal (weakest possible) initial value condition in order to obtain a strong solution $u \in L^s(0,T;L^q(\Omega))$ in some time interval $[0,T[\,, 0 < T < \infty$, where s, q with $3 < q < \infty$ and $\frac{2}{s} + \frac{3}{q} = 1$ are so-called Serrin exponents.

D. Lengeler: Global weak solutions for an incompressible fluid interacting with a linearly elastic Koiter shell

Abstract: Tba

X. Liao: Well-posedness of low Mach number limit system

Abstract: We will show some w.p. results for the low Mach number limit system, which is the full N-S-F system when the Mach number goes to zero.

V. Macha: Partial regularity of solution to the Navier-Stokes system with a pressure dependent viscosity

Abstract: In this contribution we show partial Hölder continuity of solutions to equations describing ow of non-Newtonian uids in bounded domains, in particular uids whose viscosity depends on pressure and shear rate. We show that the set of singular points is small, namely its Hausdorff dimension is less than or equal to $d-2$.

A. Morando: Well-posedness of the linearized plasma-vacuum interface problem in ideal incompressible MHD

Abstract: We study the free boundary problem for the plasma-vacuum interface in ideal incompressible magnetohydrodynamics (MHD). In the vacuum region the magnetic field is described by the div-curl system of pre-Maxwell dynamics, while at the interface the total pressure is continuous and the magnetic field is tangent to the boundary. Under a suitable stability condition, satisfied at each point of the plasma-vacuum interface, we prove the well-posedness of the linearized problem in Sobolev spaces.

R. Selmi: Long time decay to the Leray solution of the two-dimensional Navier-Stokes equations

Abstract: We give a new proof of the zero limit to the solution of the two-dimensional Navier-Stokes equations, as time goes to infinity. This proof is done in the frequency space; it is simpler and shorter compared to the existing proofs. Based on this limit, we derive some analytic properties of the solution. Mainly, it becomes infinitely differentiable with respect to time and has value in all Sobolev spaces. Moreover, its regularity grows in an exponential way and its $L^2(R^2)$ norm decays exponentially fast, as time tends to infinity. We describe the long time behavior of its homogeneous Sobolev norm for any positive, real exponent, by comparing to usual functions and we ameliorate some existing results. We establish that the Leray solution is stable as time increases.

M. Turbin: Existence of weak solution for nonsteady Herschel-Bulkley fluid

Abstract: We prove existence of weak solution for initial-boundary value problem for nonsteady Herschel-Bulkley fluid motion.

A. Zang: Smooth solutions for motion of a rigid body of general form in an incompressible perfect fluid

Abstract: In this talk, I will show the interactions between a rigid body of general form and the incompressible perfect fluid surrounding it. Local well-posedness in the space $C([0,T); Hs$ is obtained for the fluid-rigid body system.

A. Zvyagin: Solvability of the mathematical model fluid motion with the objective derivative

Abstract: It is well known the motion of an incompressible fluid with the constant density filling a bounded domain $\Omega \subset^n, n = 2, 3$, on a time interval $[0, T]$, $T > 0$, is described by the system of differential equations. This system describes flows of all kinds of fluids, but it contains the deviator of the stress tensor which is not expressed explicitly via the unknowns of the system. As a rule, relation between the deviator of the stress tensor and the tensor of deformation velocities and their time derivatives is used. This relation is called the rheological relation and is usually obtained by the mechanistic model method.

In this rheological relation we have a time derivative. Unfortunately, method of mechanistic models does not indicate which derivation we have to use (total, partial or any special derivative). In the recent years rational mechanics has influenced scientists in the way that they have started to investigate the rheological relations that are independent of the observer. It means to use an objective derivative.

In the talk the initial-boundary value problem for a system of equations that describes the mathematical model of motion of weak aqueous polymer solutions with the smoothed Jaumann's objective derivative is considered and its solvability in a weak sense is studied in space $E_1 = \{\nu : \nu \in L_\infty(0, T; V^1), \nu' \in L_2(0, T; V^{-1})\}$. For this investigation the approximation-topological approach to problems of hydrodynamics is used.

V. Zvyagin: Attractors of hydrodynamics equations

Abstract: Attractors for models of Newtonian hydrodynamics are well known (O.A. Ladyzhenskaya, M.I. Vishik, etc.). But models of non-Newtonian hydrodynamics require the development of a more general abstract theory. It appears as a result of non-invariance of a priori estimates solutions of initial-boundary value problems for huge number non-Newtonian models regarding on the translation operator. Also corresponding trajectory spaces are non-invariant regarding on this operator. This doesn't allow for the study of attractors of non-Newtonian models to use the design related with invariant trajectory spaces (the theory of Vishik-Chepyzhova-Cela). In the report the abstract theory of trajetory and global attractors for non-invariant trajectory spaces will be presented (see e.g. Zvyagin V.G., Vorotnikov D.A. Topological Approximation Methods for Evolutionary Problems of Nonlinear Hydrodynamics, Walter de Gruyter, Berlin-New York, 2008). On the basis of this developed theory it is supposed to consider attractors following models of non-Newtonian hydrodynamics:

1) Jeffreys models of motion viscoelastic media;
2) Models of motion viscoelastic media with memory.