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Recent Advances in PDEs and Applications

(on occasion of Professor Hugo Beirão da Veiga's 70th birthday)

Levico Terme (Trento, Italy) – February 17–21, 2014

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BOOK OF ABSTRACTS / TALKS

TALKS

New results for the Stokes and Navier-Stokes equations with different boundary conditions

Chérif Amrouche

University of Pau (France)

We consider here elliptical systems as Stokes and Navier-Stokes problems in a bounded domain, eventually multiply connected, whose boundary consists of multi-connected components. We investigate the solvability in L^p theory, with $1 < p < \infty$, under the non standard boundary conditions

$$\boldsymbol{u} \cdot \boldsymbol{n} = g, \quad \mathbf{curl}\, \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{h} \quad \text{or} \quad \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{g}, \quad \pi = \pi_* \quad \text{on}\, \Gamma.$$

We consider also the case of Navier boundary conditions:

$$\boldsymbol{u} \cdot \boldsymbol{n} = g \quad \text{and} \quad 2\,[\mathbf{D}(\boldsymbol{u})\boldsymbol{n}]_{\boldsymbol{\tau}} + \alpha \boldsymbol{u}_{\boldsymbol{\tau}} = \boldsymbol{h} \quad \text{on}\, \Gamma$$

where α is a friction coefficient and $\mathbf{D}(\boldsymbol{u}) = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^\top)$ is the stress tenseur. The main ingredients for this solvability are given by the Inf-Sup conditions, some Sobolev's inequalities for vector fields and the theory of vector potentials satisfying

$$\boldsymbol{\psi} \cdot \boldsymbol{n} = 0, \quad \text{or} \quad \boldsymbol{\psi} \times \boldsymbol{n} = \mathbf{0} \quad \text{on}\, \Gamma.$$

Those inequalities play a fundamental key and are obtained thanks to Calderon-Zygmund inequalities and integral representations. In the study of ellpitical problems, we consider both generalized solutions and strong solutions that very weak solutions.

In a second part, we will consider the nonstationary case for the Stokes equations.

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Analysis of finite energy weak solutions for a class of systems in Quantum Hydrodynamics

Paolo Antonelli

Gran Sasso Science Institute, L'Aquila (Italy)

In this talk I will give an overwiev of some results concerning the Cauchy problem for a class of systems describing quantum fluids. Such models arise in many physical contexts, such as the description of superfluidity phenomena, the dynamics of Bose-Einstein condensates or the modelling of semiconductor devices. I will discuss the existence of finite energy weak solutions by exploiting the analogy with a class of nonlinear Schrdinger equations through the Madelung transformations. The main advantage of this approach is that it does not need to define the velocity in the vacuum region. I will conclude the exposition by discussing some further research perspectives. These results are done in collaboration with P. Marcati.

Absolute Maxwellian Eternal solutions and scattering

Claude Bardos

Université Paris-Diderot, Paris (France)

This is a report on a joint work in progress with Irene Gamaba , Francois Golse and David Levermore.

When he derived the equations

$$\partial_t F + v \cdot \nabla_x F = \mathcal{B}(F, F)$$

Boltzmann identified special solutions which are both Maxwellian and which are solution of the advection equation:

$$F = F(v, x - vt) \Rightarrow \mathcal{B}(F, F) = 0 \quad \text{and } \partial_t F + v \cdot \nabla F = 0$$

Dave Levermore dubbed these l solutions "global Maxwellian" and identified them by their global moments.

In this talk I want to show that the global Maxwellian generate global solutions both of the compressible Euler and Navier Stokes equations. Their stability and scattering properties can be studied following classical contributions of Kaniel-Shinbrot [1] or Wei and Zhang [2].

And eventually this perturbation analysis produces eternall solutions of the Boltzmann equation which do not coincide with global Maxwellian

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Concerning the existence of classical solutions to the Stokes system. On the minimal assumptions problem

Hugo Beirão da Veiga

Università Pisa (Italy)

We consider the stationary Stokes system under (for instance) Dirichlet boundary conditions. We say that a solution is *classical* if all derivatives appearing in the equations are continuous up to the boundary. It is well known that solutions are classical if the external forces belong to a Hölder space $C^{0,\lambda}(\overline{\Omega})$. It is also well known that, in general, solutions are not classical in the presence of continuous external forces. Hence, a challenging problem is to find Banach spaces, strictly containing the Hölder spaces $C^{0,\lambda}(\overline{\Omega})$, such that solutions to the Stokes problem are classical for forces in the above space. We turn back to results for external forces in a suitable functional space, denoted $\mathbf{C}_*(\overline{\Omega})$, introduced in reference [1] in connection with the Euler equations. Some unpublished proofs will appear in reference [2].

[1] H.Beirão da Veiga, *On the solutions in the large of the two-dimensional flow of a nonviscous incompressible fluid*, J. Diff. Eq., **54**, (1984), no.3, 373-389.

[2] H.Beirão da Veiga, *Concerning the existence of classical solutions to the Stokes system. On the minimal assumptions problem.* to appear.

Generalized functions beyond distributions

Vieri Benci

Università Pisa (Italy)

The theory of distribution provides generalized solutions for problems which do not have a classical solutions. However, there are problems which do not have solutions, not even in the space of distributions. As model problem you may think of

$$-\triangle u = u^{p-1}\ ,\ u > 0,\ p \geqslant \frac{2N}{N-2}$$

with Dirichelt boundary conditions in a bounded star-shaped open set.

Having this problem in mind, we construct a new class of functions called **ultrafunctions** in which the above problem has a (generalized) solution. In this construction, we apply the general ideas of Non Archimedean Mathematics (NAM) and some techniques of Non Standard Analysis.

On the construction of suitable Weak Solutions

Luigi C. Berselli

Università Pisa (Italy)

In this talk I will discuss the problem of approximations to the Navier-Stokes equations producing solutions, which are *suitable* in the sense of Scheffer and Caffarelli-Kohn-Nirenberg [2]. This notion of solution is very relevant for partial regularity results, but also the local behavior of energy seems a natural request for numerical methods. However, since the uniqueness of weak solutions of Navier-Stokes is unknown, different approximation methods may lead (or not) to a suitable weak solution. Beside the construction of suitable solutions in [2], other approximations produce solutions satisfying the local energy inequality: those constructed by Leray [5], those obtained by hyper-dissipation by Beirão da Veiga [1], or also by certain Galerkin approximations as in Guermond [4].

I will present a recent result obtained with S. Spirito, showing that solutions obtained by means of the Navier-Voigt model (cf. [3]) are suitable. The novelty is that the problem is considered in a bounded domain, with Dirichlet boundary conditions and the approximation is an inviscid one, with an hyperbolic character and with relevant applications to the numerical simulation of turbulent flows.

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Solenoidal Lipschitz truncation

Dominic Breit

Ludwig Maximilians- Universität, München (Germany)

The Lipschitz truncation method approximates a Sobolev-function by a Lipschitz-continuous function in a way that both are equal on large set which size can be controlled. Many applications in fluid mechanics require that functions are solenoidal (=divergence-free). Due to its strongly nonlinear character the Lipschitz truncation does not preserve this property of a function. We present a new method which allows the approximation of a solenoidal Sobolev-function by a solenoidal Lipschitz continuous function (stationary and non-stationary) and present several applications.

Optimal potentials for Schrödinger operators

Giuseppe Buttazzo

Università Pisa (Italy)

We consider the Schrödinger operator $-\Delta + V(x)$ on $H^1_0(\Omega)$, where Ω is a given domain of $\mathbf{R}^d$. Our goal is to study some optimization problems where an optimal potential $V \geq 0$ has to be determined in some suitable admissible classes and for some suitable optimization criteria, like the energy or the Dirichlet eigenvalues.

Vanishing theorems for the discretely self-similar solutions to the Euler and the MHD equations

Dongho Chae

Chung-Ang University, Seoul (Korea)

Discretely self-similar solution is a generalized notion of the self-similar solution, which is equivalent to the time-periodic solution to the self-similar form of the Euler equations. We deduce sufficient conditions to guarantee that the solution is identically zero. More specifically, conditions for the decays of the velocity at spatial infinity implies that the solution is zero (Liouville type theorems). Also, conditions of the velocity at the origin implies that the solution vanishes on the whole of $\mathbb{R}^n$ without decay condition at spatial infinity (unique continuation type theorems).

A maximum modulus theorem for p(t,x)-Laplacian systems

Francesca Crispo

Seconda Università di Napoli (Italy)

We consider a quasi-linear parabolic system with non-standard growth exponent $p(t,x)$, with

$$\frac{2n}{n+2} < p_- \leqslant p(t,x) \leqslant p_+ < \infty\,. \tag{0.1}$$

Reformulating in a suitable way a duality technique employed in paper [1], under the assumption (0.1) and quite standard assumptions on $p(t,x)$, we prove a L^∞-bound for the weak solution corresponding to a bounded initial data, with no investigations on high regularity properties of the solution.

1. F. C. and P. Maremonti, Higher regularity of solutions to the singular p-Laplacian parabolic system, Advances in Differential Equations, **18** (2013), 849–894.

Optimal initial values and regularity conditions of Besov space type for weak solutions to the Navier-Stokes system

Reinhard Farwig

Technische Universität Darmstadt (Germany)

In this talk we present recent results jointly obtained with Hermann Sohr (Paderborn) and W. Varnhorn (Kassel).

The first question concerns the *optimal condition* on initial values $u_0 \in L^2_\sigma(\Omega)$ to get a locally regular solution u in Serrin's class $L^s(0,T;L^q(\Omega))$ (with $\frac{2}{s}+\frac{3}{q}=1$, $s>2$, $q>3$) to the instationary Navier-Stokes system in a bounded domain $\Omega \subset \mathbb{R}^3$. It is shown that the condition

$$\int_0^\infty \|e^{-\tau A}u_0\|^s_{L^q(\Omega)}\,d\tau < \infty \tag{0.2}$$

is *necessary and sufficient* for this local in time regularity result; here A denotes the Stokes operator on $L^2_\sigma(\Omega)$. Condition (0.2) is weaker than the more classical assumptions $u_0 \in \mathcal{D}(A^{1/4})$ or $u_0 \in L^3(\Omega)$ and ensures that the solution $e^{-\tau A}u_0$ of the linear Stokes problem lies in Serrin's class $L^s(0,\infty;L^q(\Omega))$. It can be rewritten in the form

$$u_0 \in \mathbb{B}^{-2/s}_{q,s}(\Omega)$$

where $\mathbb{B}^{-2/s}_{q,s}(\Omega)$ denotes a solenoidal subspace of the usual Besov space $B^{-2/s}_{q,s}(\Omega)$.

Actually, it suffices to consider the integral in (0.2) on a finite time interval $(0,\delta)$, leading to a Besov space $\mathbb{B}^{-2/s}_{q,s;(\delta)}(\Omega)$ with equivalent norm. Using these spaces we find new regularity and uniqueness criteria for weak solutions.

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Attempts to understand the effect of noise on the 3D Navier-Stokes equations

Franco Flandoli

Università di Pisa (Italy)

When white noise is added to differential equations in finite dimensions, it has strong regularizing properties; for instance, uniqueness holds for coefficients much weaker than Lipschitz continuous. In infinite dimensions there are examples where the same happens and thus it is natural to investigate the case of weak solutions to the 3D Navier-Stokes equations. In spite of several years of effort, this problem (regularization by noise) is still open but there have been a few progresses. The talk will review some of them.

Hyperbolic-parabolic coupling and the occurrence of resonance in partially dissipative systems

Giovanni P. Galdi

University of Pittsburgh (USA)

Resonance in elastic solids is a common phenomenon in Nature that can be roughly described as follows: when the frequency of an applied time-periodic load approaches one (or a multiple of one) of the natural frequencies of oscillations of the solid, the basic kinematic and dynamic parameters of the solid, such as displacement, velocity and energy become increasingly large, and this may result into damage and even rupture of the structure. Objective of the talk is to investigate whether the interaction of an elastic solid with a dissipative agent can affect and possibly prevent the occurrence of resonance. We shall study this problem for systems whose dynamics is governed by strongly continuous semigroups of contractions and will provide sharp necessary and sufficient conditions for the absence of resonance. Finally, we shall furnish a number of applications to physically relevant problems such as thermo- and magneto-elasticity, as well as to several liquid-structure interactions models.

On countinuity of the solution map for the cubic 1d periodic NLW equation

Vladimir Georgiev

Università di Pisa (Italy)

We consider the Cauchy problems associated with the following nonlinear equation

$$(\mathrm{i}\partial_t - |D_x|)u = \sigma|u|^2u \text{ for } t \geq 0 \ , \tag{0.3}$$

where $\sigma = \pm 1$. We shall assume that $u(t,x)$ is $2\pi-$ periodic in x. If we have solutions $u(t,x) \in C([0,T];H^s(0,2\pi))$, with $s > 1/2$, then the equation have at least two conservation laws

$$\|u(t)\|_{L^2(0,2\pi)} = const$$

and

$$\frac{1}{2}\||D|^{1/2}u(t)\|_{L^2}^2 + \frac{\sigma}{4}\|u(t)\|_{L^4}^4 = const. \tag{0.4}$$

Definition 1. *The problem* (0.3) *is well - posed in* $H^s(0,2\pi)$ *with* $s \in (0,1)$ *if for any* $R > 0$ *one can find* $T = T(R) > 0$ *so that for any data* $u(0) = f \in H^s$ *with* $\|f\|_{H^s} \leqslant R$ *one can define unique solution* $u(t,x) \in C([0,T];H^s)$ *so that the solution map*

$$f \in B(R) = \{g \in H^s; \|g\|_{H^s} \leqslant R\} \to u(t,x) \in C([0,T];H^s)$$

is continuous.

A stronger property is the uniform continuity of the solution map.

Our main result is the following.

Theorem 1. For any $s \in (1/3, 1/2)$ the Cauchy problem for

$$(\mathrm{i}\partial_t - |D_x|)u = |u|^2u \text{ for } t \geq 0 \ , \tag{0.5}$$

can not have uniformly continuous solution map in H^s.

Vortex stretching and anisotropic diffusion in the 3D NSE

Zoran Grujic

University of Virginia, Charlottesville (USA)

The purpose of this lecture is to present a physically, numerically, and mathematical analysis-motivated scenario in which the transversal small scales produced by the mechanism of vortex stretching reach the threshold sufficient for the locally anisotropic diffusion to engage and control the sup-norm of the vorticity, preventing possible formation of singularities in the 3D Navier-Stokes flows.

Regularity of generalized Newtonian flows up to the boundary

Petr Kaplicky

Charles University Prague (Czech Republic)

We want to present results about regularity of weak solutions to the system describing a flow of a generalized Newtonian fluid. We focus on local estimates that are valid up to the boundary. In the real world there are many materials that exhibit a behavior different from the one of Newtonian fluids. Perhaps the closest class to Newtonian fluids are so called generalized Newtonian fluids. They are determined by the fact that their viscosity is not constant but depends on shear rate. It is interesting that even for the simplest model, stationary generalized Stokes problem, the optimal regularity is known nor in interior neither up to the boundary.

Nonlinear stability for counterrotating vortex pairs

Milton da Costa Lopes Filho

Universidade Federal do Rio de Janeiro (Brazil)

In this talk we discuss nonlinear stability for counterrotating vortex pairs, a family of travelling-wave solutions for the incompressible 2D Euler equations in the full plane. We use a variational characterization of vortex pairs as mxima of a suitable functional on vorticity rearrangement classes and apply a concentration-compactness argument to prove a weak version of nonlinear stability.

High Regularity for solutions to the p-Navier-Stokes equations

Paolo Maremonti

Seconda Università di Napoli (Italy)

We study the existence of solutions to a modified p-Navier-Stokes system. In a recent paper by F. Crispo and P. Maremonti [1], a result of high regularity of solutions is established for a modified p-Stokes problem. Although the equations are close to a model for non-Newtonian fluids, the result is not interesting from a physical point of view since the *stress tensor* is not Galilean invariant. In this note [2], we are able to prove that for a suitable body force there exists at least a solution (whose regularity is high) to the modified p-Navier-Stokes problem. More precisely without restrictions on the *size* of the body force, for p close to 2, we prove that there exist second derivatives which are integrable on the whole domain $\mathbb{R}^n$. Of course, the interest of the result is connected to the fact that for the first time a result of high regularity is deduced for solutions to a system of p-Navier-Stokes kind. It is also interesting to point out that the proof, based on the results related to the p-Stokes problem, seems to be original and applicable to other nonlinear equations.

1 F. Crispo and P. Maremonti, A high regularity result of solutions to modified p-Stokes equations, submitted.

2 F. Crispo and P. Maremonti, A high regularity result of solutions to modified p-Navier-Stokes equations, forthcoming.

Asymptotic behavior toward a multi-wave pattern for the scalar viscous conservation law

Akitaka Matsumura

Osaka (Japan)

In this talk, we show our recent works with Natusmi Yoshida on the asymptotic behavior of solutions to the Cauchy problems for the scalar viscous conservation law, where the far field states are prescribed. In particular, we treat the cases where the flux function is convex but linearly degenerate on some interval, and the corresponding Riemann solution consists of a contact discontinuity and rarefaction waves. As for the viscosity term, we treat the cases with either a standard linear viscosity or a nonlinearly degenerate viscosity term (p-Laplacian type viscosity ($p > 1$)). Then, for the standard linear viscosity (resp. the nonlinearly degenerate viscosity), the solution of the viscous conservation law is shown to asymptotically tend toward a multi-wave pattern consisting of the rarefaction waves and a corresponding viscous contact wave which is constructed by an integration of the linear heat kernel (resp. the Barenblatt solution of the porous medium equation).

On the existence of weak solution to the coupled fluid-structure interaction problem for non-Newtonian shear-dependent fluid

Šarka Nečasová

Praha (Czech Republic)

Joint work with A. Hundertmark-Zaušková and M. Lukáčová-Medviďová.

We study the existence of weak solution for unsteady fluid-structure interaction problem for shear-thickening flow. The time dependent domain has at one part a flexible elastic wall. The evolution of fluid domain is governed by the generalized string equation with action of the fluid forces. The power law viscosity model is applied to describe shear-dependent non-Newtonian fluids.

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Vortex sheets in domains with boundaries

Helena J. Nussenzveig Lopes

Universidade Federal do Rio de Janeiro (Brazil)

This talk concerns the interaction of incompressible 2D flows with compact material boundaries under vortex sheet regularity. We focus on the dynamic behavior of the circulation of velocity around boundary components and the possible exchange between flow vorticity and boundary circulation in flows with vortex sheet initial data. We recast 2D Euler evolution using vorticity and the circulation (around boundary components) as dynamic variables and provide a weak formulation. Our main results are the equivalence between the weak velocity and weak vorticity formulations of the 2D Euler equations on domains with boundary and a criterion for computing the resulting force on boundary components for flows with little regularity.

Model order reduction for cardiovascular modeling

Alfio Quarteroni

Ecole Polytechnique Fédérale de Lausanne (Switzerland)

To face the complexity of the numerical simulation of cardiovascular flows, several reduction strategies can be adopted at the modeling level. We review some recent results on the geometric multiscale model and address new methods based on the reduced basis paradigm for the solution of parametrized problems and in the context of optimal control problems. A few FSI (Fluid Structure Interaction) examples of clinical interest will be illustrated.

Variational analysis on the Sierpinski gasket

Vicentiu Radulescu

Romanian Academy, Bucharest (Romania)

We develop some recent results concerning the qualitative analysis of solutions to some nonlinear elliptic problems on fractal domains. Our analysis includes the case of nonlinear terms with oscillatory behaviour, either at the origin or at infinity. The approach combines variational arguments with the geometrical properties of the Sierpinski gasket.

On decomposition of the Hilbert space H^1 with respect to the quadratic form $< curl\, u, curl\, v >$ and applications

Reimund Rautmann

Paderborn University (Germany)

The vorticity $curl\, v$ of a viscous incompressible fluid flow with velocity v plays a prominent rôle in regularity theory as well as for approximation methods to the Navier-Stokes equations. Of special interest is the creation of vorticity for product formula approaches based on transport-diffusion splitting schemes, which had been introduced by Lighthill, Marsden, and Chorin for flow computations at higher Reynolds numbers. The main difficulty of these schemes stems from the alternating change of the boundary condition, namely from slip condition by each transport step to no-slip condition by each successive diffusion step.

In order to get bounds to the resulting change of vorticity, recently we had considered a suitable orthogonal decomposition of the Hilbert space $H^1(\Omega)$ with respect to the quadratic form $< curl\, u, curl\, v >$ on bounded 3-dimensional domains Ω with C^2-smooth boundaries. The decomposition leads immediately to a lower bound for the change of vorticity by transition from slip- to no-slip fluid flow in Ω.

In my talk, this result will be extended to domains having less regular boundaries. In addition, on domains with C^2-regular boundaries, we present a transport-diffusion splitting scheme which is consistent with the full Navier-Stokes equations under no-slip condition, and we get also an upper bound to the change of vorticity by transport-diffusion stepping.

On the Mathematical Analysis of Thick Fluids

José Francisco Rodrigues

CMAF, University of Lisbon (Portugal)

In chemical engineering models, shear-thickening or dilatant fluids may converge to the limit class of incompressible fluids with a maximum admissible shear rate, the so-called thick fluids. These non-Newtonian fluids may be obtained, in particular, as the power limit of Ostwald-deWaele fluids, and may be formulated as a new class of evolution variational inequalities. We discuss the existence, uniqueness and continuous dependence of solutions. The asymptotic stabilization in time towards steady state solutions will be also considered if times permits.

Recent results on extended thermodynamics of polyatomic gas

Tommaso Ruggeri

Università Bologna (Italy)

In an endeavor to understand ubiquitous nonequilibrium phenomena, a number of ther- modynamic theories have been proposed and developed. In particular Extended Thermo- dynamics (ET) which describe a non-equilibrium phenomena beyond the assumption of local equilibrium shows successful result and full agreement with Kinetic Theory (KT) [1] and clarify the limit of Fourier-Navier-Stokes equations and the role of objectivity principle [2]. Nevertheless the weak point of ET and KT is that the applicable range is limited to rarefied monatomic gas.

The main idea of extended thermodynamics of dense gases consists by adopting the system of field equations with a different hierarchy structure to that adopted in the previous works. It is the theory of 14 fields of mass density, velocity, temperature, viscous stress, dynamic pressure and heat flux. As a result, all the constitutive equations can be determined explicitly by the caloric and thermal equations

of state as in the case of monatomic gases. Using Maxwellian iteration we deduce as limit case the Fourier-Navier-Stokes equations including the one that involve the dynamical pressure [3]. A particular interesting sub-system is the theory of 6 moments when bulk viscosity is very high and a comparison with Meixner approach was also made [4].

In the particular physically interesting case of rarefied polyatomic gases we show a perfect coincidence between ET and the procedure of Maximum Entropy Principle (MEP) [5]. The main difference with respect to usual procedure is the existence of two hierarchies of macroscopic equations for moments of suitable distribution function, in which the internal energy of a molecule is taken into account [6].

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Problems with p-structure: analysis and numerics

Michael Ruzicka

Universität Freiburg (Germany)

We will present some recent results concerning the existence of weak solutions for steady motions of generalized Newtonian fluids an some of its versions. Moreover, we will discuss results proving convergence rates for various problems with so called p-structure. In particular we will discuss Finite Element and Local Discontinuous Galerkin methods.

Robustness of regularity of solutions of the 3D Navier-Stokes equations

Witold Sadowski

University of Warsaw (Poland)

In the talk I will review some recent results on the robustness of regularity of solutions for the 3D Navier-Stokes equations and some other parabolic nonlinear equations. I will also show how these results can be used as a key ingredient in a method of numerical verification of regularity of solutions arising from bounded sets of initial data.

$\mathcal{R}$-bounded solution operator and its application to the mathematical study of compressible viscous fluid flow

Yoshihiro Shibata

Waseda University Tokyo, (Japan)

In studying the motion of compressible viscous fluid flow with free surface, mathematically one of crucial steps is to obtain the maximal regularity of linearized problem. I talk about the method to use the $\mathcal{R}$ - boundedness solution operator to the corresponding generalized resolvent problem to obtain the maximal regularity. And, I talk about some local and global in time existence theorems for a free boundary problem of the Navier-Stokes equations in the compressible viscous fluid flow case. My talk is based on joint works with L. von Below, Y. Enomoto and D. Goetz.

On the boundary regularity of the Navier-Stokes equations

Timofey Shilkin

V.A. Steklov Mathematical Institute, St. Petersburg (Russia)

In this talk we will give an short overview of the theory of local boundary regularity for the Navier-Stokes equations. In particular, we are going to discuss the relation between "weak" and "strong" solutions to the linear Stokes problem near the boundary including both cases of flat and curved parts of the boundary.

The motion of a fluid-rigid ball system at the zero limit of the ball radius

Ana Leonor Silvestre

Technical University of Lisbon (Portugal)

We consider the limiting motion of a system of a rigid d-dimensional ball moving in a Navier-Stokes fluid flow in $\mathbb{R}^d$ $(d = 2, 3)$ as the radius of the ball goes to zero. Recently, Masoumeh Dashti and James C. Robinson solved this problem in the case $d = 2$, in the absence of rotation of the ball [Arch. Rational Mech. Anal. 200 (2011) 285-312]. This restriction was caused by the difficulty in obtaining appropriate uniform bounds on the second order derivatives of the fluid velocity when the particle can rotate. In this talk, we show how we have obtained the required uniform bounds on the velocity fields in the case $d = 3$. These estimates then allow to pass to the zero limit of the ball radius and show that the solution of the coupled system converges to the solution of the Navier-Stokes equations describing the motion of only fluid in the whole space. The trajectory of the centre of the ball converges to a fluid particle trajectory, which justifies the use of rigid tracers for finding Lagrangian paths of fluid flow. This is joint work with Takéo Takahashi (INRIA Nancy - Grand Est, France).

The Helmholtz decomposition - revisited

Christian G. Simader

Bayreuth University (Germany)

This is a joint work with H. Sohr (Paderborn) and W. Varnhorn (Kassel).

Let $1 < q < \infty$ and $q' := \frac{q}{q-1}$. Suppose that $\phi \neq \Omega \subset \mathbb{R}^n$ is a domain and that the Helmholtz decomposition holds true in $L^q(\Omega)^n$ and in $L^{q'}(\Omega)^n$ with corresponding a-priori estimates. Then we investigate the optimal constants in these estimates and their dependence on q and q'. Further we prove a new estimate for the solenoidal part of the Helmholtz decomposition.

L_p theory of free boundary problems of magnetohydrodynamics

Vsevolod A. Solonnikov

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The communication is concerned with local existence theorems of evolution free boundary MHD problems for viscous imcompressible fluids. Under some special assumptions, global solutions are constructed. They are sought in anisotropic Sobolev spaces of functions whose derivatives belong to L_p as functions of spacial variables and time.

On local strong solutions of the non-homogeneous Navier-Stokes equations

Werner Varnhorn

Kassel University (Germany)

Joint work with Reinhard Farwig and Hermann Sohr.

Consider a bounded domain $\Omega \subseteq \mathbb{R}^3$ with smooth boundary $\partial\Omega$, a time interval $[0, T)$, $0 < T \leq \infty$, and in $[0, T) \times \Omega$ the non-homogeneous Navier-Stokes system

$$u_t - \Delta u + u \cdot \nabla u + \nabla p = f,\ u|_{t=0} = u_0,\ \operatorname{div} u = k,\ u|_{\partial\Omega} = g,$$

with sufficiently smooth data f, u_0, k, g. In this general case there are mainly known two classes of weak solutions, the class of global weak solutions, similar as in the well known case $k = 0$, $g = 0$, which need not be unique, see [5], and the class of local very weak solutions, see [1], [2], [3], [4], which are uniquely determined, but need neither have differentiability properties nor satisfy the energy inequality. Our aim is to introduce a new class of local strong solutions for the general case $k \neq 0$, $g \neq 0$, satisfying similar regularity and uniqueness properties as in the known case $k = 0$, $g = 0$. For slightly restricted data this class coincides with the corresponding class of very weak solutions yielding new regularity results. Further, through the given data we obtain a control on the interval of existence of the strong solution.

References

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Relative entropy applied to shocks for Conservation Laws and applications

Alexis Vasseur

The University of Texas at Austin (USA)

We develop a theory based on relative entropy to study the stability and contraction properties of extremal shocks of conservation laws. We will present first application of the theory to the study of asymptotic limits.

On multi-dimensional compressible Navier-Stokes systems with large oscillations and vacuum

Zhouping Xin

The Chinese University of Hong Kong (Hong Kong)

In this talk, I will discuss some of recent results on the large time well-posedness of classical solutions to the multi-dimensional compressible Navier-Stokes system with possible large oscillations and vacuum. The focus will be on finite-time blow-up of classical solutions for the 3-D full compressible Navier-Stokes system, and the global existence of classicla solutions to the isentropic compressible Navier-Stokes system in both 2-D and 3-D in the presence of vacuum and possible large oscillations. Some new estimates on the pressure will be presented which are crucial for the well-posedness theory in 2-dimension.

Solvability of boundary value problems of the stationary MHD systems

Taku Yanagisawa

Nara Women's University (Japan)

In this talk, we consider the boundary value problems of the stationary MHD systems under some inhomogeneous boundary conditions. We present our recent results on the local or global solvability of these problems.
