Topological Data Analysis and Persistent Homology

Program

Lecture 1 (Harer) *Introduction to TDA 1*
In this first lecture we will introduce the basic concepts of Topological Data Analysis including:
1) Filtrations of simplicial complexes
2) Persistent homology
3) Persistence for functions
4) Cech and Rips complexes
5) Persistence diagrams

Lecture 2 (Harer) *Introduction to TDA 2*
The second lecture continues the introduction to TDA with:
1) Bottleneck and Wasserstein distance
2) Interleaving distance
3) Stability theorems
4) Zig-zag homology

Lecture 3 (Tralie) *A Zoo of TDA Software Implementations*
We will take a tour of three state of the art software packages which implement some of the algorithms discussed in the morning:
1) Ripser (available in Python, C++, Matlab, and Javascript)
Fast Rips filtrations with varying field coefficients. Demonstrations on spaces of natural image patches
2) GUDHI: (available in Python, C++)
Alpha complexes in 2D and 3D. Demonstration on 3D bone data
3) Hera: (available from command line)
Fast bottleneck and Wasserstein matching of persistence diagrams. Demonstrations of stability and clustering on synthetic data.

Lecture 4 (Harer) *Some TDA Methods and Algorithms 1*
In the fourth lecture, we will cover some basic, example algorithms that make TDA practical. These include:
1) Union-Find Algorithm
2) Matrix reduction to compute persistence in dimensions bigger than 0
3) Matching to compute Bottleneck and Wasserstein distance
4) Computing Interleaving distance

Lecture 5 (Harer) *Some TDA Methods and Algorithms 2*
In the fifth lecture we will give some examples of interesting ways that TDA can be used:
1) Delay Reconstruction for a Time Series
2) Elevation and the Persistent Homology Transform
3) Distance to a distribution

Lecture 6 (Tralie) *Applications of Sliding Window Embeddings + TDA*
We will present various applications of sliding window embeddings + TDA in multimedia data
1) Audio applications, community-accepted features (CAFs):
a) Music versus speech
b) Twisted spaces of rhythm hierarchies
2) Sliding window videos and applications:
a) Periodicity ranking in video (class activity)
b) Quasiperiodicity quantification in videos of vibrating vocal folds
c) Laplacian circular coordinates and slow motion videos

Lecture 7 (Harer) *TDA and Machine Learning 1*
In this lecture we will discuss the use of persistence diagrams as features for Machine Learning. Methods we will cover include:
1) Binning
2) Sort and Grab
3) Persistence Landscapes
4) Cover Tree Entropy Reduction (CDER)

Lecture 8 (Harer) *TDA and Machine Learning 2*
In this lecture we will give examples of the use of the methods from lecture 7, including
1) Binning - Driving Behavior
2) Sort and Grab - Brain Arteries
3) Persistence Landscapes - Financial Time Series, FMRI Motor Tasks
4) CDER - Bone data

Lecture 9 (Oudot) *Supervised learning*
This lecture will cover the use of persistence diagrams in supervised learning tasks. After reviewing the basics of supervised learning and kernel methods, we will switch to an overview of the vectorizations and kernels that have been proposed for persistence diagrams so far, together with their respective properties.

Lecture 10 (Oudot) *Topological descriptors for data*
This lecture will demonstrate how geometric filtrations (e.g. Rips filtrations) and their persistence diagrams can be used as descriptors for data. Such descriptors are interesting for at least three reasons: they are not tied to a particular type of data; they are stable with respect to small perturbations; they provide information of a complementary nature compared to purely geometric or statistical descriptors. We will illustrate this distinguished properties on a variety of examples, and give a glimpse at some of their proofs.

Lecture 11 (Oudot/Tralie) *Workshop on Classification of 3D Shapes 1*
We will explore how topological descriptors can be used to classify triangles meshes of 2D surfaces of human bodies. The lab will follow a similar format as the lab in the following link:
http://www.enseignement.polytechnique.fr/informatique/INF556/TD7-8/index.html

Lecture 12 (Oudot/Tralie) *Workshop on Classification of 3D Shapes 2: Isometry Invariance*
We will continue the lab on 3D shape clustering, this time devising descriptors based on the heat-kernel signature (HKS), a 3D descriptor related to curvature. Since the HKS is an intrinsic feature, this will allow us to cluster shapes in a way which is invariant to pose; e.g. a human that raises her arms is identical to herself at rest.

Lecture 13 (Oudot) *Categorical view on persistent homology*
After a short introduction to the basics of category theory, we will provide a more algebra-oriented viewpoint on persistent homology. In particular, we will talk about persistence modules as functors and representations of quivers, decomposition theorems, and stability from Kan extensions.

Lecture 14 (Oudot) *A glimpse at multipersistence*
The starting point of this lecture will be the general idea of using several filters at the same time on data. We will explore the dichotomy between the persistent homology transform and multiparameter persistence in this context. We will then move on to a broad overview of the current frontier in these topics, which are hot and active research subjects in the field these days.