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The 39th International Conference on Quantum Probability and Infinite Dimensional Analysis, dedicated to Prof. Takeyuki Hida for his contributions to Infinite Dimensional Analysis and Quantum Probability

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BOOK OF ABSTRACTS

ABSTRACTS

A mathematical definition of Bose-Einstein Condensation (BEC)

Luigi Accardi

Centro Vito Volterra, Roma (Italy)

Talk based on the paper: Luigi Accardi, Carlo Pandiscia: Mathematical aspects of Bose-Einstein condensation in equilibrium and local equilibrium regimes, Infin. Dimens. Anal. Quantum Probab. Relat. Top., (IDA-QP) 20 (04)(2017) 1750024-1–22

The usual mathematical definition of BEC is unsatisfactory for many reasons that will be discussed in the talk.

A new, purely deductive definition is proposed.

This definition suggests the possibility of non–trivial statistical correlations between the condensate and the non–condensate parts of the gas.

On the unitary groups of operator algebras

Ahmed Saleh Al-Rawashdeh

UAE University, Al Ain (UAE)

In von Neumann algebras, H. Dye showed that an isomorphism between the unitary groups in two von Neumann factors is implemented by a linear $*$-isomorphism of the factors. We show that if the unitary groups of two simple unital AH-algebras with real rank zero are isomorphic as abstract groups, then their K_0-ordered groups are isomorphic. Also, using Dadarlat-Gong's classification theorem, we prove that such C^*-algebras are isomorphic if and only if their unitary groups are topological isomorphic. In the case of simple, unital purely infinite C^*-algebras, we show that two unital Kirchberg algebras are $*$-isomorphic if and only if their discrete unitary groups are isomorphic. We also study the extension problem and following Dye's approach, if φ is an isomorphism between the unitary groups of two unital C^*-algebras, it induces a bijective map θ_φ between the sets of projections. For a large class of unital finite C^*-algebras, we show that θ_φ is always an orthoisomorphism. Based on these results, we prove that if φ is a continuous automorphism of the unitary group of a UHF-algebra A, we show that φ is implemented by a linear or a conjugate linear $*$-automorphism of A.

In a certain taype of UHF-algebra, we constructed a unitary group automorphism, such that its induced map between the projections is not an orthoisomorphism.

Quantum optomechanical systems: a quantum-stochastic approach

Alberto Barchielli

Politecnico di Milano (Italy)

In this talk I shall show how use quantum stochastic differential equations to describe the dynamics of quantum opto-mechanical systems, which are very relevant systems for quantum information and quantum thermodynamics. An essential point to note is that the quantum-stochastic Hudson-Parthasarathy equation has been mainly used to construct unitary dilations of quantum dynamical semigroups, while memory effects are very important in opto-mechanics. As we shall see, these new features can be covered by using the Hudson-Parthasarathy equation and the derived quantum Langevin equation (Evans-Hudson flow) in conjunction with a suitable choice of the field state. By considering the prototypical opto-mechanical system (a vibrating micro-mirror coupled to the light by radiation pressure interaction) we firstly show how to treat both this interaction and the effects of the coupling to a phonon field with a nearly arbitrary thermal spectrum, which in particular gives rise to a non-Markovian dynamics; the whole theory is valid at arbitrarily low temperature. Then, by combining the Hudson-Parthasarathy equation with the theory of measurements in continuous time, always based on quantum stochastic calculus, we obtain the optical spectra and we apply the theory to "laser thermometry" and "laser cooling"; finally, measurement based control (closed loop control) can be introduced, which again gives rise to memory effects.

A. Barchielli, B. Vacchini, Quantum Langevin equations for optomechanical systems, New J. Phys. 17 (2015) 083004.

A. Barchielli, Quantum stochastic equations for an opto-mechanical oscillator with radiation pressure interaction and non-Markovian effects, Rep. Math. Phys. 77 (2016) 315-333.

Infinite mode quantum Gaussian states

B V Rajarama Bhat

Indian Statistical Institute, Bangalore (India)

Quantum Gaussian states on Bosonic Fock spaces are quantum versions of Gaussian distributions. They are type I quasi-free states. Here infinite mode quantum Gaussian states have been explored and to do this we need Williamsons normal form in infinite dimensions. We extend many of the results of K R Parthasarathy from finite mode to the infinite mode setting. This include various characterizations, convexity and symmetry properties. This is a joint work with Tiju Cherian John.

Positive definite functions on Coxeter groups with applications to free probability and multi-valued logic of Jan Lukasiewicz

Marek Bozejko

University of Wroclaw (Poland)

We will present the following subjects:

1. Positive definite functions on permutations(Coxeter) groups with applications to Generalized Gaussian processes.

2. Applications to free infinitely divisibility of many probability measures on line like normal law N(o,1), 1/cosh. classical Meixner laws.q-Gaussian laws and others.

3. Free projections as the model of 3 and multi-valued logic of Jan Lukasiewicz.

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Stationary states and irreducibility for open quantum random walks

Raffaella Carbone

Università di Pavia (Italy)

For a quantum channel (completely positive, trace-preserving map), we discuss some standard probabilistic properties: subjects as reducibility and irreducible restrictions, period, transience and recurrence have a well-established theory for classical Markov chains but are still evolving for quantum channels. The connections with asymptotic behavior, structure of invariant states and environmental decoherence have been central issues in our research.

Naturally, the case of open quantum random walks shows some interesting peculiarities, combining quantum features with the elementary theory of classical Markov chains.

This is a work in collaboration with A. Jencova and Y. Pautrat.

Steins method and Palm theory in random measures

Louis H.Y. Chen

National University of Singapore (Singapore)

In this talk, I will discuss the connection between Steins method and Palm theory and show how this connection can be exploited to study Poisson process approximation for point processes and normal approximation for random measures, with applications to computational biology and stochastic geometry.

Divisibility and information flow for non-invertible dynamical maps

Dariusz Chruscinski

Nicolaus Copernicus University, Torun (Poland)

We analyze the relation between CP-divisibility and the lack of information backflow for an arbitrary - not necessarily invertible - dynamical map. It is well known that CP-divisibility always implies lack of information backflow. Moreover, these two notions are equivalent for invertible maps. In this letter it is shown that for a map which is not invertible the lack of information backflow always implies the existence of completely positive propagator which, however, needs not be trace-preserving. Interestingly, for a class of image non-increasing dynamical maps this propagator becomes trace-preserving as well and hence the lack of information backflow implies CP-divisibility. This result sheds new light into the structure of the time-local generators giving rise to CP-divisible evolutions. We show that if the map is not invertible then positivity of dissipation/decoherence rates is no longer necessary for CP-divisibility.

Sums of monotone position operators: vacuum distribution and spectrum

Vitonofrio Crismale

Università di Bari (Italy)

We provide a recurrence formula to compute atoms and weights for the (discrete) vacuum distribution of sums of creation and annihilation operators in monotone Fock space. The result is obtained in a direct way, without using monotone convolution, and exploiting some properties of palindromic polynomials. Moreover, we show the law above is a basic measure on the spectrum of the unital C*-algebra generated by the sum of the first n position operators. This allows us to achieve the spectrum for such operators and, moreover, the norm for any finite sum of gaussian operators, which turns out to be equal to the right endpoint of the support of the vacuum distribution. This is a joint work with Y.G. Lu.

Potential Theory of some Quantum Markov Chains

Ameur Dhahri

Chungbuk National University (Korea)

In a joint work with Franco Fagnola, we showed that we can develop a potential theory for a class of Quantum Markov Chains including diagonal states and Entangled Quantum Markov Chains. Moreover, we studied the transience and recurrence for those Quantum Markov Chains.

Constructions of monotone increment processes

Uwe Franz

Université de Franche-Comté, Besancon (France)

I will present two constructions of quantum stochastic processes with monotonically independent increments. The first uses Belton's vacuum adapted quantum stochastic calculus, then second uses classical Markov processes. Based on joint work with Takahiro Hasebe, Ikkei Hotta, and Sebastian Schleissinger.

Structure of a covariant QMS with respect to a generic representation

Nicoló Ginatta

Università di Genova (Italy)

We study the structure of a covariant Quantum Markov Semigroup (QMS) with respect to a generic representation of a topological group. More precisely we characterize the generator of such QMS and its GKLS representation and also its decoherence free algebra. In both cases we obtain a remarkable simplification of their structure compared to the general context due to the covariance. Finally we show a connection between the number of connected components of the group and the number of factors in the decomposition of the decoherence free algebra.

Uncertainty relations and information loss for spin-1/2 measurements

Matteo Probo Siro Francesco Gregoratti

Politecnico di Milano (Italy)

We formulate entropic measurements uncertainty relations (MURs) for a spin-1/2 system. When incompatible observables are approximatively jointly measured, we use relative entropy to quantify the information lost in approximation and we prove positive lower bounds for such a loss: there is an unavoidable information loss. Firstly we allow only for covariant approximate joint measurements and we find state-dependent MURs for two or three orthogonal spin-1/2 components. Secondly we consider any possible approximate joint measurement and we find state-independent MURs for two or three spin-1/2 components. In particular we study how MURs depend on the angle between two spin directions. Finally, we extend our approach to infinitely many incompatible observables, namely to the spin components in all possible directions. In every scenario, we always consider also the characterization of the optimal approximate joint measurements.

Note on stream cipher based on non-commutative algebra and its application for authentication

Maki Kihara

Tokyo University of Science (Japan)

The homomorphic encryption which is the public key cryptography based on number theory achieves addition or multiplication upn two different encrypted sequences.

In this study, we introduce an algorithm to enable high-speed homomorphic operations on two different cipher texts using the common key encryption QP-DYN based on non-commutative algebra instead of public key encryptions based on number theory. Moreover, we propose its application for authentication, and compare performances with conventional homomorphic encryption.

Model of vibrones in quantum photosynthesis as an analog of model of laser

Sergei Kozyrev

Steklov Mathematical Institute, Moscow (Russia)

In this talk we consider a model of amplification of quantum transport using a model of dissipative quantum dynamics with GKSL generator.

This model is an analog of semiclassical model of laser. We conjecture that this model explains the mechanism of vibronic amplification of quantum transport of excitons in photosynthesis. We consider two models — a model of nonequilibrium three level system with vibronic mode, and some variant of a model of laser without inversion. We conjecture that quantum "dark" states discussed in relation to quantum photosynthesis might be related to mechanism of vibronic "laser" without inversion which amplifies the transfer of excitons.

Separability of multi-qubit states in terms of diagonal and anti-diagonal entries

Seung-Hyeok Kye

Seoul National University, Seoul, Korea

We give separability criteria for general multi-qubit states in terms of diagonal and anti-diagonal entries. We define two numbers which are obtained from diagonal and anti-diagonal entries, respectively, and compare them to get criteria. They give rise to characterizations of separability when all the entries are zero except for diagonal and anti-diagonal, like Greenberger-Horne-Zeilinger diagonal states. The criteria is strong enough to get nonzero volume of entanglement with positive partial transposes. This is based on the cowork(arXiv 1803.00175) with Kil-Chan Ha and Kyung Hoon Han.

Mathematical model of photosynthesis and its energy transmission process

Satoshi Iriyama

(Tokyo University of Science, Japan)

Mathematical model of photosynthesis and its energy transmission process Abstract: Recently, the light harvesting process in a purple bacteria has been described by quantum network, and it has been reported that the dephasing noise which disturbs entanglement causes improvement of its energy transmission efficiency. However, it assumes that there is no dissipation to avoid its computational difficulty. In this study we derive the conditions to increase the energy transmission efficiency in a general case with dephasing and dissipation. Moreover, we propose a qubit model to describe total process.

Inequalities for operators

Un Cig Ji

Chungbuk National University, Cheongju (Korea)

We start with the well-known Golden-Thompson inequality and discuss its extension for module operator on an operator algebra. Then we study Jensen's inequality for operators and Jensen's trace inequality. As an interpolation of the Golden-Thompson inequality and the Jensen's trace inequality, we study a multivariate Golden-Thompson inequality. Finally, we discuss uncertainty relations for conditional expectation.

Rate of convergence for Wong-Zakai-type approximations of Itô Stochastic Differential Equations

Alberto Lanconelli

Università di Bologna (Italy)

We consider a class of stochastic differential equations driven by a one-dimensional Brownian motion and we investigate the rate of convergence for WongZakai-type approximated solutions of Ito SDEs. In these approximations the diffusion coefficient is Wick multiplied by the regularized noise. We discover that the speed of convergence to the exact solution coincides with the speed of convergence of the smoothed noise toward the original Brownian motion. We also prove, in analogy with a well-known property for exact solutions, that the solutions of approximated It equations solve approximated Stratonovich equations with a certain correction term in the drift.

On a Gauge action on noncommutative solitons

Hyun Ho Lee

University of Ulsan, Ulsan (Korea)

Following a formalism of Lagrangian approach to noncommutative harmonic maps, we analyze energy minimizing fields from two-point space to noncommutative torus which is an Ising model due to Dabrowski, Karjewski, and Landi. In particular, we look at the gauge action on the space of solutions. In general, any two solutions are not gauge invariant since there is an obstruction. But we give a prescription how to gauge a Gaussian to a Gabor frame.

Quantum random walks and quasifree cocycles

Martin Lindsay

Lancaster University (UK)

Any faithful normal state on a 'particle algebra' $B(\mathfrak{p})$, determines an orthogonal decomposition of its GNS space:

$$HS(\mathfrak{p}) = \mathbb{C}\varrho^{1/2} \oplus \mathsf{k} \oplus \overline{\mathsf{k}} \oplus \mathsf{K}_0,$$

in terms of its density matrix ϱ. Here $HS(\mathfrak{p})$ denotes the Hilbert space of Hilbert-Schmidt operators on $\mathfrak{p}$ and $\overline{\mathsf{k}}$ the image of a certain closed subspace k under the natural conjugation on $HS(\mathfrak{p})$. Under the assumption of exponential decay of its eigenvalues, ϱ determines a quasifree state on $CCR(\mathsf{k})$. In this talk I shall show how quasifree cocycles arise as limits of quantum random walks, with particular reference to the quantum repeated interaction model of Attal and Pautrat.

This is joint work with Alex Belton, Michal Gnacik and Ping Zhong.

A classe of discrete IFS and a new independence

Yun Gang Lu

Università di Bari (Italy)

Let $\mathbb{T} \subset \mathbb{Z}$ and $\{\mu_k\}_{k\in\mathbb{T}}$ be a family of **symmetric probability measures** determined by the moments, let, for any $k \in \mathbb{T}$, $\left\{\omega_i^{(k)}\right\}_{i=1}^{\infty}$ be the Jacobi–coefficient of μ_k. We give a concrete construction of a certain interacting Fock space and such a sequence of creation–annihilation operators $\left\{a_k, a_k^+\right\}_{k\in\mathbb{T}}$, that with respect to the vacuum state

- the distribution of $a_k + a_k^+$ is determined by $\left\{\omega_i^{(k)}\right\}_{i=1}^{\infty}$;
- $\{\mathcal{A}_k\}_{k\in\mathbb{T}}$ verifies any appointed independence, where $\mathcal{A}_k$:=the algebra generated by $\left\{a_k, a_k^+\right\}$ (or in particular by $a_k + a_k^+$).

As an application, a new independence is introduced.

Exponential convergence to equilibrium for a mean field laser equation

Carlos Manuel Mora Gonzalez

Universidad de Concepción (Chile)

We address a non-linear quantum master equation that models a simple laser under the mean field approximation. The laser is described by a single mode optical cavity and a set of two level atoms that are coupled to two reservoirs. In the 36th QP Conference and CMO-BIRS Workshop 15w5086, we proved the existence of a unique regular stationary state, as well as that a family of non-constant free interaction solutions is born at the regular stationary state as a relevant parameter C passes through the critical value 1. Now, we show that the quantum system converges exponentially fast to the equilibrium in the trace norm as the bifurcation parameter C is less than 1. To this end, we obtain a variation of constants formula.

The talk is based on a joint work with Franco Fagnola (Politecnico di Milano).

On S-mixing entropy of Quantum Channels

Farrukh Mukhamedov

Department of Mathematical Sciences, United Arab Emirates University

In the present talk an S-mixing entropy of quantum channels is introduced as a generalization of Ohya's S-mixing entropy. We investigate several properties of the introduced entropy. Moreover, certain relations between the S-mixing entropy and the existing map and output entropies of quantum channels are investigated as well. These relations allowed us to find certain connections between separable states and the introduced entropy. Hence, there is a sufficient condition to detect entangled states. Moreover, several properties of the introduced entropy are investigated.

This work is done jointly with Noboru Watanabe.

Symbols of Infinite Dimensional Operators and Applications to White Noise Analysis

Habib Ouerdiane

University of Tunis El Manar (Tunisia)

In this talk,we develop the theory of operators defined on infinite dimensional holomorphic functions. Then we give a characterization theorem between this class of operators and their symbols. As application we give an explicit solution of some linear quantum white noise differential equations by applying the convolution calculus on a suitable distribution spaces. In particular we obtain an integral representation for the solution of the quantum heat equation. Finally we introduce the infinite dimensional notion of pseudo- differential operators and we give some examples.

BM-Central limit theorems associated with non-symmetric positive cones

Lahcen Oussi

Wroclaw University (Poland)

Analogues of the classical Central Limit Theorem are proved in the noncommutative setting of random variables which are bm-independent and indexed by elements of positive non-symmetric cones, such as the circular cone, sectors in Euclidean spaces and the Vinberg cone. The geometry of the cones is shown to play crucial role and the related volume characteristics of the cones is shown.

Structure of invariant states of weak coupling limit type semigroups

Roberto Quezada

UAM-Iztapalapa, Mexico City (Mexico)

We will discuss on the general structure of the stationary states of weak coupling limit type quantum Markov semigroups. Starting with the case of states supported on the interaction-free subspace, we study the structure of stationary states belonging to the annihilator of all Krauss operators, including detailed balance and local detailed balance states.

Multivariable operator theory – a brief survey

Jaydeb Sarkar

Indian Statistical Institute, Bangalore (India)

We will give an overview of some recent developments in the theory of commuting tuples of bounded linear operators on Hilbert spaces, with particular attention to reproducing kernel Hilbert spaces, invariant subspaces, Toeplitz operators, von Neumann inequality and dilation theory.

Integration with respect to the non-commutative fractional Brownian motion

René Schott

University of Lorraine, Nancy (France)

We study the issue of integration with respect to the non-commutative fractional Brownian motion, that is the analog of the standard fractional Brownian motion in a non-commutative probability setting. When the Hurst index H of the process is strictly larger than $1/2$, integration can be handled through the so-called Young procedure. The situation where $H = 1/2$ corresponds to the specific free case, for which an It-type approach is known to be possible. When $H < 1/2$, rough path-type techniques must come into the picture, which, from a theoretical point of view, involves the use of some a-priori-defined Lévy area process. We show that such

an object can indeed be canonically constructed for any H in $(1/4, 1/2)$. Finally, when H is less than or equal to $1/4$, we exhibit a similar non-convergence phenomenon as for the non-diagonal entries of the (classical) Lévy area above the standard fractional Brownian motion.

(Joint work with A. DEYA, Insitut Elie Cartan, University of Lorraine, Nancy, France).

Non-commutative stochastic independence and cumulants

Michael Schürmann

Universität Greifswald (Germany)

We consider the category $AlgP_{d,m}$, $d, m \in \mathbb{N}$ which has as objects triplets $\big(\mathcal{A}, (\mathcal{A}^{(1)}, \ldots, \mathcal{A}^{(m)}), \phi\big)$ where $\mathcal{A}$ comes with an m-tuple $(\mathcal{A}^{(1)}, \ldots, \mathcal{A}^{(m)})$ of subalgebras which freely generates $\mathcal{A}$, and ϕ is a d-tuple of linear functionals with each component a linear functional on $\mathcal{A}$. A morphism of $AlgP_{d,m}$ is given by an m-tuple of algebra homomorphisms adapted to the free decomposition of the algebras into subalgebras and which respect the components of ϕ.

A tensor product on $AlgP_{d,m}$ (called a (d, m)-product) gives rise to a non-commutative notion of stochastic independence. This includes examples of non-commutative independence given by N. Muraki, M. Bozejko/R. Speicher and T. Hasebe. Moreover, Voiculescu bi-freeness and bi-monotone independence of M. Gerhold are further examples.

A (d, m)-product defines a graded bialgebra structure on a (commutative) polynomial algebra. Using the comultiplication of this bialgebra, we define the cumulants of a d-tuple of normalized linear functionals as a convolution logarithmic series. The cumulants of the convolution product of (d-tuples of) normalized linear functionals can be computed from the single cumulants by a Campbell-Baker-Hausdorff formula. See also the work of F. Lehner on cumulants and recent results of T. Hasebe and F. Lehner.

Quantum Yang-Mills in 2 dimensions

Ambar Sengupta

University of CONN (USA)

There is a rigorous quantum field measure for Yang-Mills Gauge theory in two dimensions. In this talk we will describe several aspects of this theory including recent developments for the large-N limit of the U(N) theory.

The Martingale problem in Quantum Probability

Kalyan Sinha

PSTDS Vidyapith, Chinsurah (India)

A version of the Martingale problem (in analogy with the work of Stroock-Varadhan in Classical Stochastic Processes) for Quantum Stochastic processes , driven by a generator of an evolution or of a semigroup in a Hilbert space H , describing an $H-P$ type description or for the semigroup generated by a Lindbladian on $B(H)$ is explored.

This is a work in collaboration with J.M.Lindsay .

Convolution semigroups of states and their generators

Adam Skalski

Polish Academy of Sciences, Warsaw (Poland)

Convolution semigroups of states on *-bialgebras were first introduced by Accardi, Schürmann and von Waldenfels as noncommutative counterparts of families of distributions of Lvy processes (on compact groups). Due to 'local finite-dimensionality'of coalgebras their generators, known as generating functionals, can be easily characterised. This characterisation forms a counterstone of the theory of quantum Lévy processes on compact quantum groups. In this talk, based on joint work with Ami Viselter, I will discuss the difficulties arising in the locally compact quantum group context, where it is no longer possible to apply purely Hopf-algebraic techniques, and present some recent progress in studying this problem, partly shifting the attention to the Dirichlet form picture of the generator, but partly also studying the generating functionals themselves . Some concrete examples/applications will be indicated.

Computing the quantum operators for Meixner random variables

Aurel I. Stan

The Ohio State University, Marion (USA)

A random variable X, having finite moments of all orders, is a Meixner random variable, if and only if the commutator of the multiplication by X operator and the commutator of the multiplication by X and anyone of the two semi-quantum operators, that means a double (nested) commutator, is a linear combination of the two semi-quantum operators. On the other hand, each of the two semi-quantum operators can be written as a sum of compositions of the multiplication by X and differentiation operators D, in which all the X factors are to the left of the D factors. We use these two facts to compute first the semi-quantum operators, and then the quantum operators of each Meixner random variable. Finally, the concrete form of the semi-quantum operators can be used to recover all the six types of Meixner random variables.

The role of the atomic decoherence-free subalgebra in the study of Quantum Markov Semigroups

Veronica Umanità

Università di Genova (Italy)

We are working on the consequences of the atomicity of the decoherence free-subalgebra for a uniformly continuous Quantum Markov Semigroup. Indeed, recently, we showed (Deschamps, Fagnola, Sasso, Umanità, Rev. in Math. Phys., 28, N.1, 2016) that, if the decoherence-free subalgebra $N(T)$ of the QMS is atomic, it induces a decomposition of the system into its noiseless and purely dissipative parts, determining the structure of invariant states, as well as decoherence-free subsystems and subspaces. Now we show that, for a QMS with a faithful normal invariant state, the atomicity of the $N(T)$ and environmental decoherence are equivalent. Moreover,

we characterize the set of reversible states and explicitly describe the relationship between the decoherence-free subalgebra and the fixed point subalgebra for QMSs with the above equivalent properties. Finally we obtain also a characterization of attractive domain for an extremal invariant state.

On complexity for Quantum Dynamical Systems

Noboru Watanabe

Tokyo University of Science (Japan)

In 1989, Ohya propose a new concept, so-called Information Dynamics (ID), to investigate complex systems according to two kinds of view points. One is the dynamics of state change and another is measure of complexity. In ID, two complexities C^S and T^S are introduced. C^S is a measure for complexity of system itself, and T^S is a measure for dynamical change of states, which is called a transmitted complexity. An example of these complexities of ID is entropy for information transmission processes. The study of complexity is strongly related to the study of entropy theory for classical and quantum systems. The quantum entropy was introduced by von Neumann around 1932, which describes the amount of information of the quantum state itself. It was extended by Ohya for C*-systems before CNT entropy. The quantum relative entropy was first defined by Umegaki for σ-finite von Neumann algebras, which was extended by Araki and Uhlmann for general von Neumann algebras and *-algebras, respectively. By introducing a new notion, the so-called compound state, in 1983 Ohya succeeded to formulate the mutual entropy in a complete quantum mechanical system (i.e., input state, output state and channel are all quantum mechanical) describing the amount of information correctly transmitted through the quantum channel.

In this talk, we briefly review the complexity for quantum dynamical systems. We introduce transmitted complexity by means of entropy functionals in order to treat the transmission processes consistently. We apply the general frames of quantum entropy for quantum dynamical systems. Finally, we define a transmitted complexity (mutual entropy) by means of the generalized AOW entropy, and we prove the fundamental inequalities of the transmitted complexity for the quantum dynamical systems.

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Constructing the quantum exclusion process

Stephen Wills

University College Cork (Ireland)

I will survey recent results on existence methods for quantum stochastic differential equations. In particular I will explain how the various methods can give different approaches to constructing the quantum exclusion process, highlighting advantages and disadvantages of each construction.

Weakly monotone Fock space and monotone convolution of the Wigner Law

Janusz Wysoczanski

Wroclaw University (Poland)

Based on my joint paper with Vito Crismale and Maria Elena Griseta (University of Bari).

We study the distribution (with respect to the vacuum state) of a family of partial sums of position operators on weakly monotone Fock space. We show that any single operator has the Wigner law, and an arbitrary family of them (with the index set linearly ordered) is a collection of monotone-independent random variables. It turns out that our problem equivalently consists in finding the m-fold monotone convolution of the semicircle law. For $m = 2$, we compute the explicit distribution. For any $m > 2$, we give the moments of the measure and show it is absolutely continuous and compactly supported on a symmetric interval whose endpoints can be found by a recurrence relation.

Quadratic open quantum harmonic oscillators

Hyun Jae Yoo

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We introduce a new quantum open system model for an oscillator by means of a quantum Markov semigroup with formal Lindblad generator with operators arising in Fock representations of the sl_2 Lie algebra. We describe explicitly all the invariant states and analyze the asymptotic behavior of the evolution of the states. We study its connection with the two-photon absorption and emission processes. Finally we investigate the spectral gap of the generator in the model.

This is a joint work with Ameur Dhahri and Franco Fagnola.

POSTERS

A formulation of Markovian quantum dynamical mutual entropy and its computations

Kyouhei Ohmura

Tokyo University of Science (Japan)

Joint work with Noboru Watanabe.

Classical dynamical entropy provides the information rate of the stationary information source, and dynamical mutual entropy is interpreted as the rate of transmission information through the channel. The quantum dynamical mutual entropy was defined by Ohya, by means of an analogue of Kolmogorov-Sinai type. On the other hand, classical Markovian sources have important roles in communication theory since it constitute reasonable models for languages. The notion of quantum Markov chain was formulated with transition expectation introduced by Accardi. In this conference, we formulate the Markovian quantum dynamical mutual entropy, and show that it satisfies Shannon's fundamental inequalities. Moreover, we calculate it for several simple models.