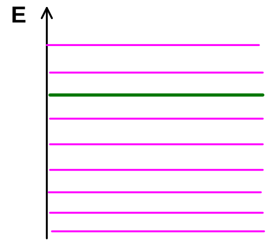
Distribuzioni statistiche

Concetti di base

- **Quantizzazione dell'energia**



1 LIVELLO = g stati

PARTICELLA SINGOLA
SISTEMA (N PARTICELLE)

$$e_k \qquad \sum n_k e_k = E_i \qquad E_i$$

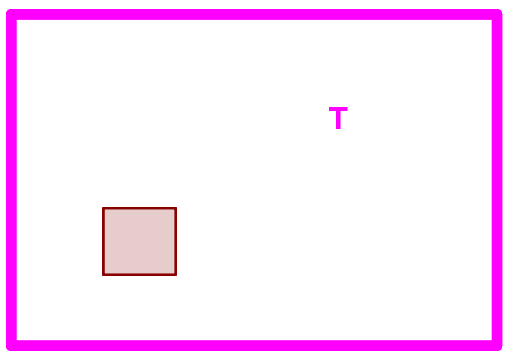
(solo particelle debolmente interagenti)

- **Due diversi approcci (per sistemi chiusi)**

SISTEMA ISOLATO

approccio microcanonico

SISTEMA TERMOSTATATO



approccio canonico

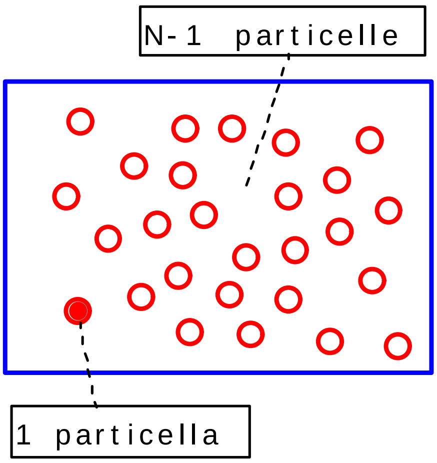
$$S = k_B \ln \Omega$$

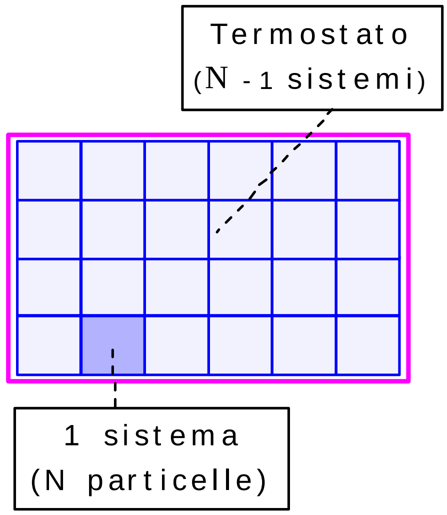
- **Statistiche ....**

1 ⟶ 10^{23} ⟶ **N**

...delle particelle
(debolmente interagenti)

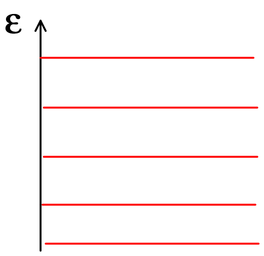
... dei sistemi



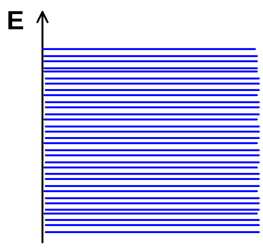


$N \approx 10^{23}$

$N \rightarrow \infty$
(Insieme canonico)



$e \ll E$



- **Ripartizioni dell'energia**

$$E = \sum n_k \, e_k$$

$$E_{tot} = \sum n_i \, E_i$$

Computo dei microstati
(per una distribuzione qualsiasi)

- **PARTICELLE (debolmente interagenti)**

distinguibili

$$\Omega_{MB} = N! \prod_j \frac{g_j^{n_j}}{n_j!}$$

indistinguibili

$$\Omega_{BE} = \prod_j \frac{(n_j + g_j - 1)!}{n_j!(g_j - 1)!}$$

$$\Omega_{FD} = \prod_j \frac{g_j!}{n_j!(g_j - 1)!}$$

$n_j << g_j$ → alta temperatura

$$\Omega_{cl} = \prod_j \frac{g_j^{n_j}}{n_j!}$$

- **SISTEMI**

$$\Omega_{can} = N! \prod_j \frac{g_j^{N_j}}{N_j!}$$

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j = indice di LIVELLO; $\mathbf{g_j}$ = degenerazione del livello

Distribuzioni predominanti

(max Ω)

- **PARTICELLE (debolmente interagenti)**

distinguibili

$$n_j = \frac{g_j}{\exp[a + be_j]}$$

indistinguibili

B.E.

$$n_j = \frac{g_j}{\exp[a + be_j] - 1}$$

F.D.

$$n_j = \frac{g_j}{\exp[a + be_j] + 1}$$

$n_j << g_j$ →
alta temp.

$$n_j = \frac{g_j}{\exp[a + be_j]}$$

- **SISTEMI**

$$N_j = \frac{g_j}{\exp[a + bE_j]}$$

N = cost.
N = cost. ⊢ α

E = cost.
E_{tot} = cost . ⊢ β

Distribuzione canonica

[livelli j ⟶ stati i (g_i = 1)]

- **La costante α**

$\Sigma n_i = N$

$\Sigma N_i = N.$

$\mapsto$ e^{α}

$$\sum \exp[-\beta \varepsilon_i] = z$$

$$\sum \exp[-\beta E_i] = Z$$

funzioni di partizione

- **La costante β**

equilibrio termico

relazione S ↔ Q

$\beta = 1/k_B T$

- **Probabilità degli stati**

$$P_i = \frac{n_i}{N} = \frac{\exp(-\varepsilon_s / k_B T)}{z}$$

$$P_i = \frac{N_i}{N} = \frac{\exp(-E_i / k_B T)}{Z}$$

- **Funzioni di partizione**

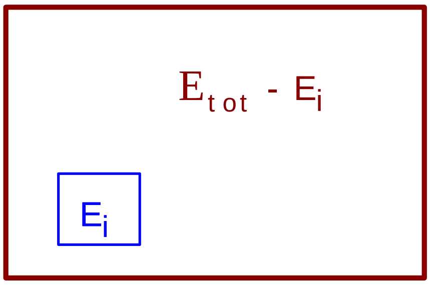
molecolare ⟶ canonica

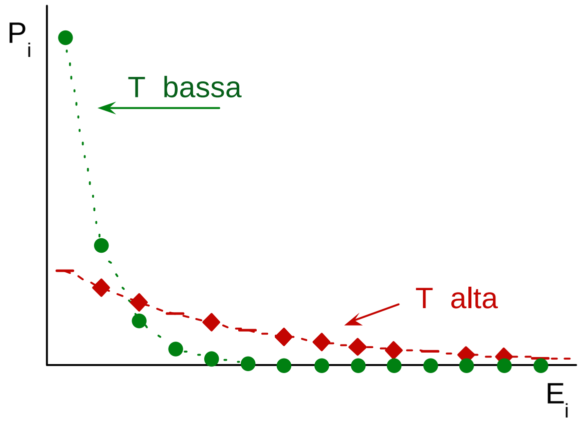
particelle distinguibili: z^N → Z

particelle **in**distinguibili (limite classico): $\frac{1}{N} z^N$ → Z

• Significato della distribuzione canonica

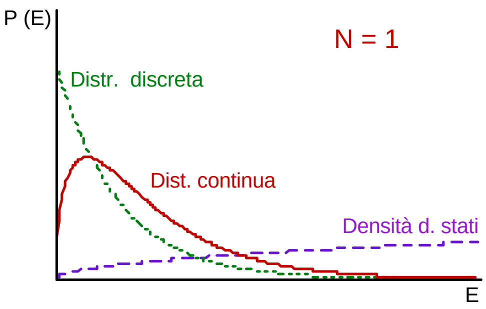
in funzione dell'energia

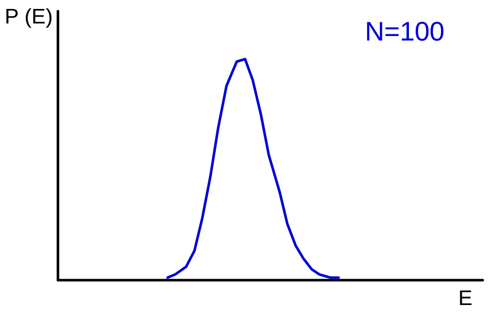


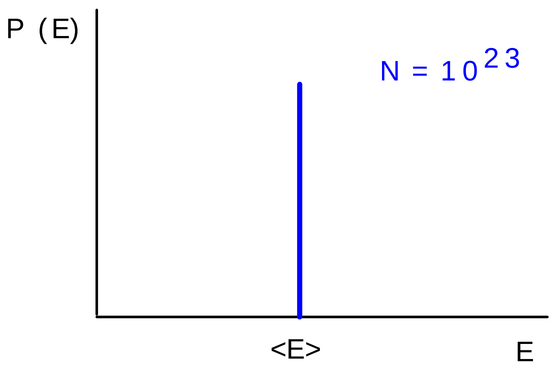


in funzione della temperatura

• Densità degli stati







Larghezza relativa

$$\propto \frac{1}{\sqrt{N}}$$