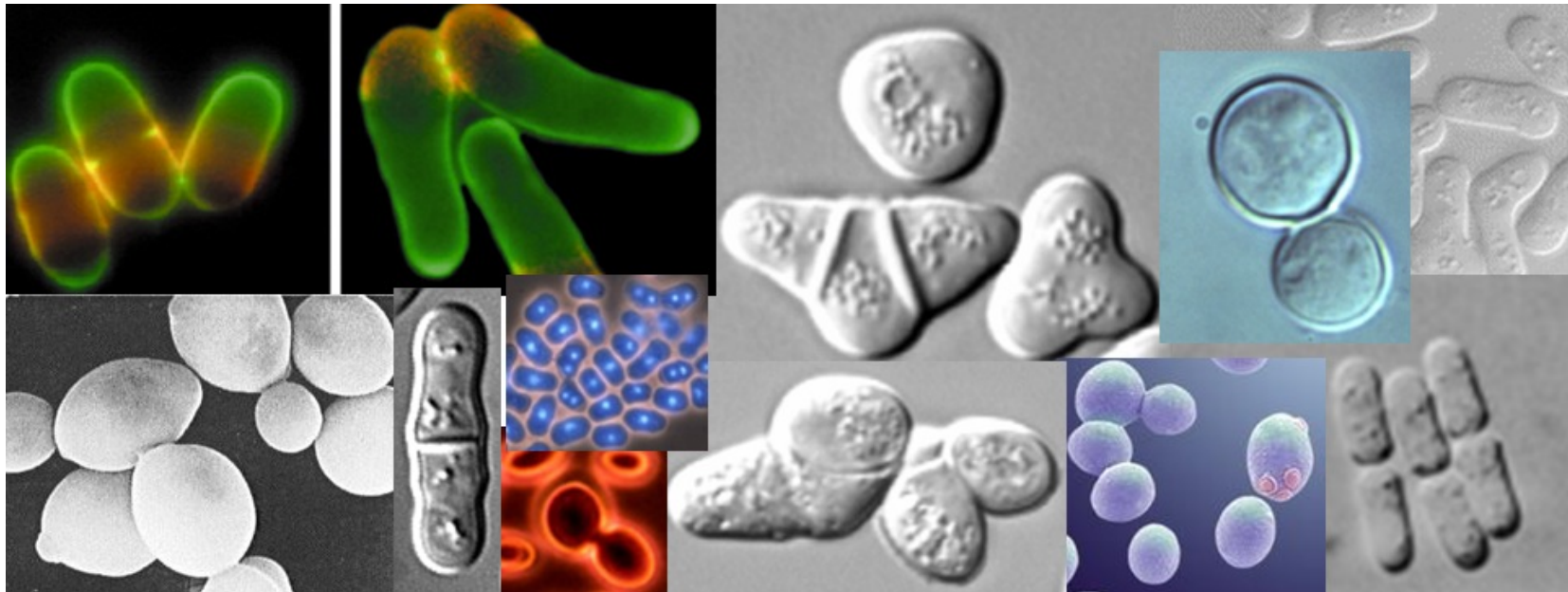
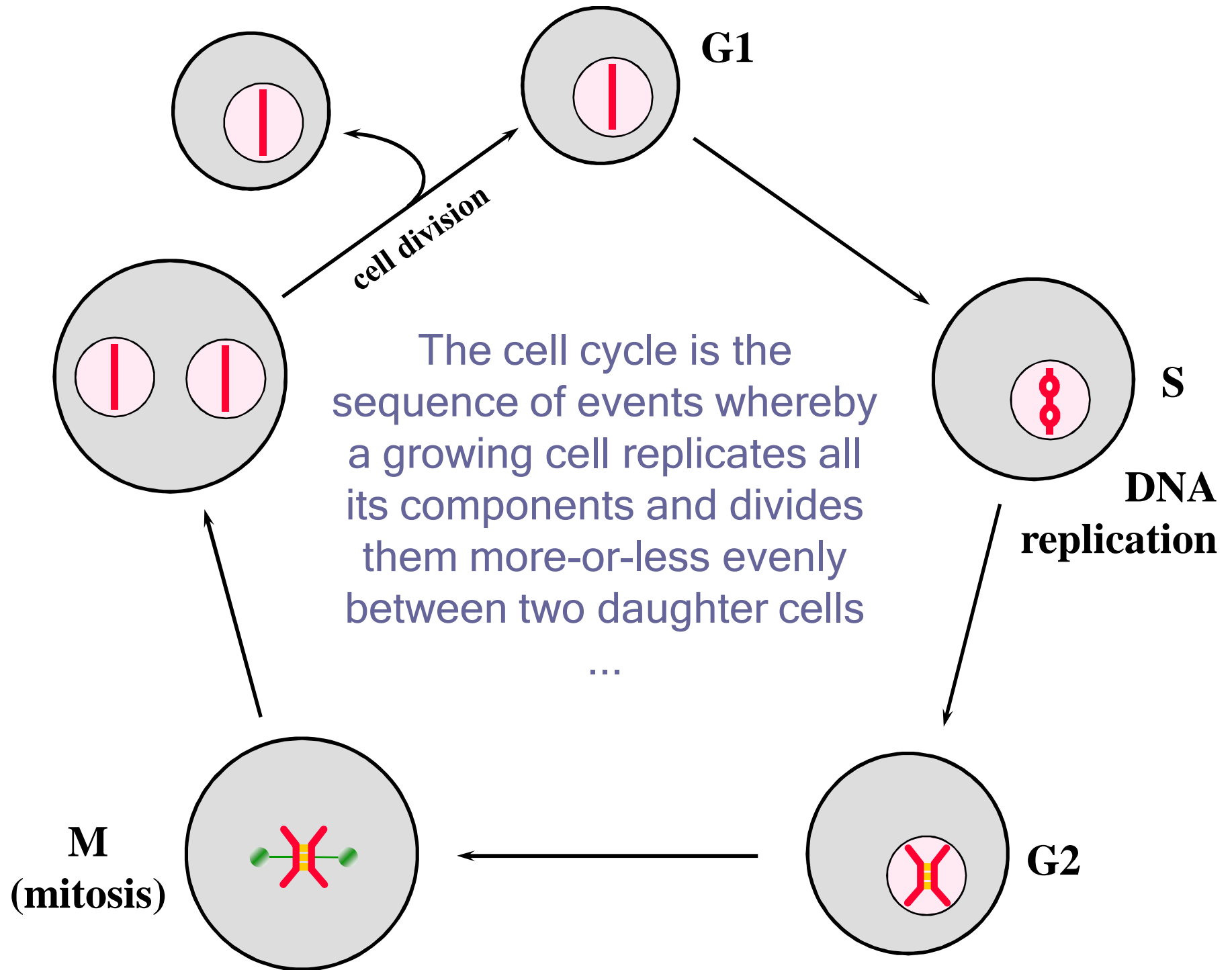


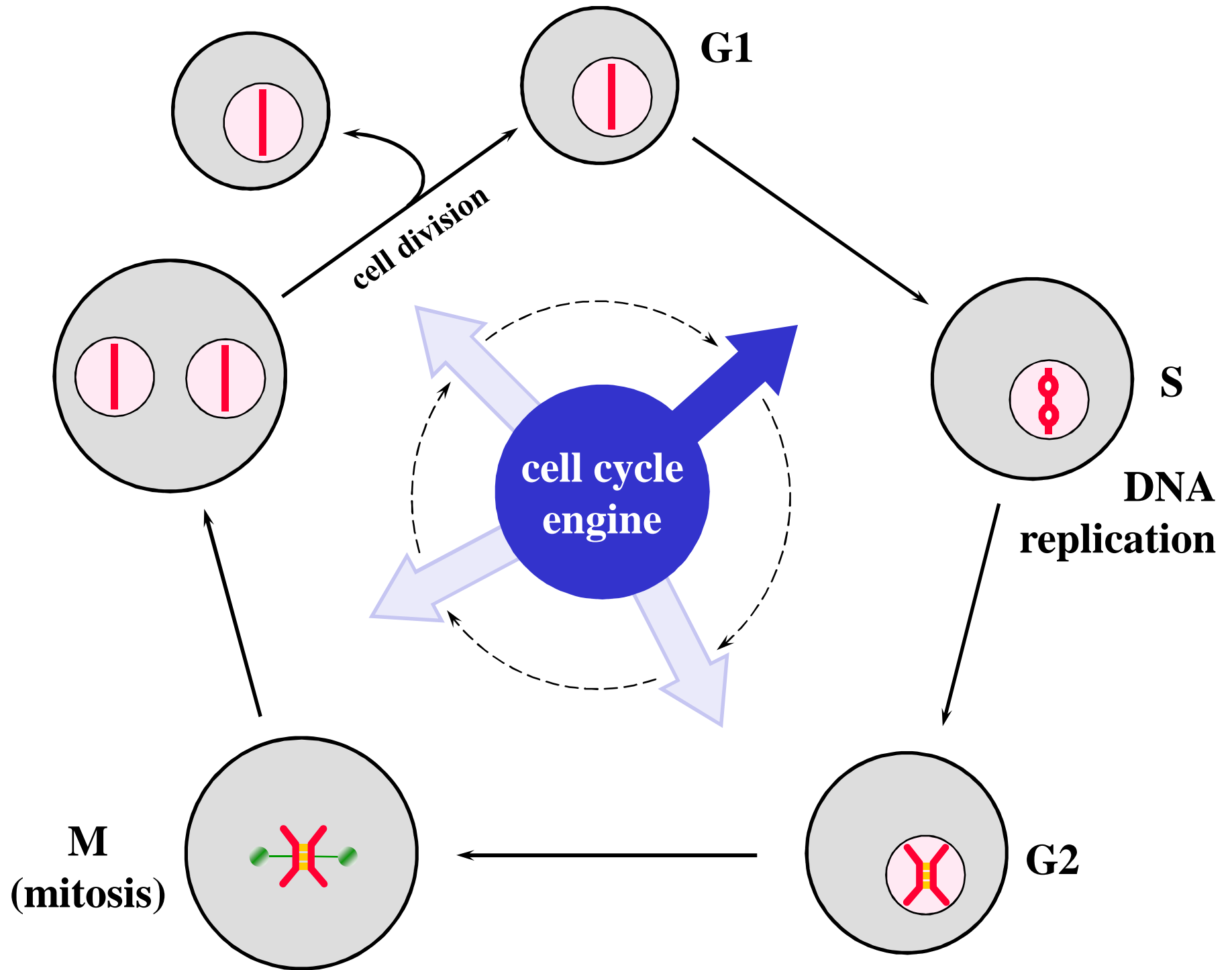
# Introduction to cell cycle modeling



**Attila Csikász-Nagy**









## The Nobel Prize in Physiology or Medicine 2001

"for their discoveries of key regulators of the cell cycle"



photo: Ralf Petterson

**Leland H. Hartwell**



USA

Fred Hutchinson  
Cancer Research  
Center  
Seattle, WA, USA

1939 -



photo: Ralf Petterson

**R. Timothy (Tim) Hunt**



Great Britain

Imperial Cancer  
Research Fund  
London, Great  
Britain

1943 -



photo: Ralf Petterson

**Paul M. Nurse**

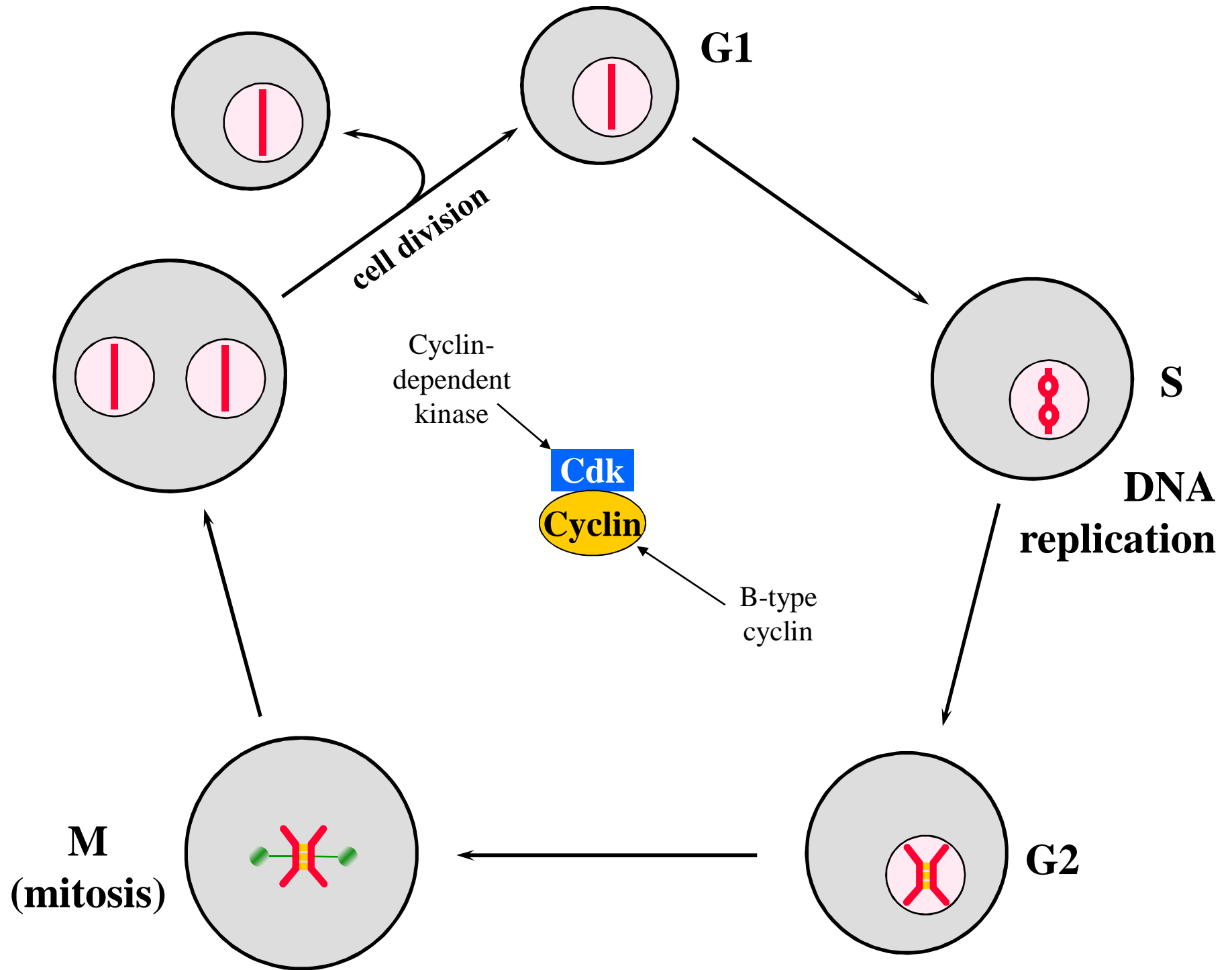


Great Britain

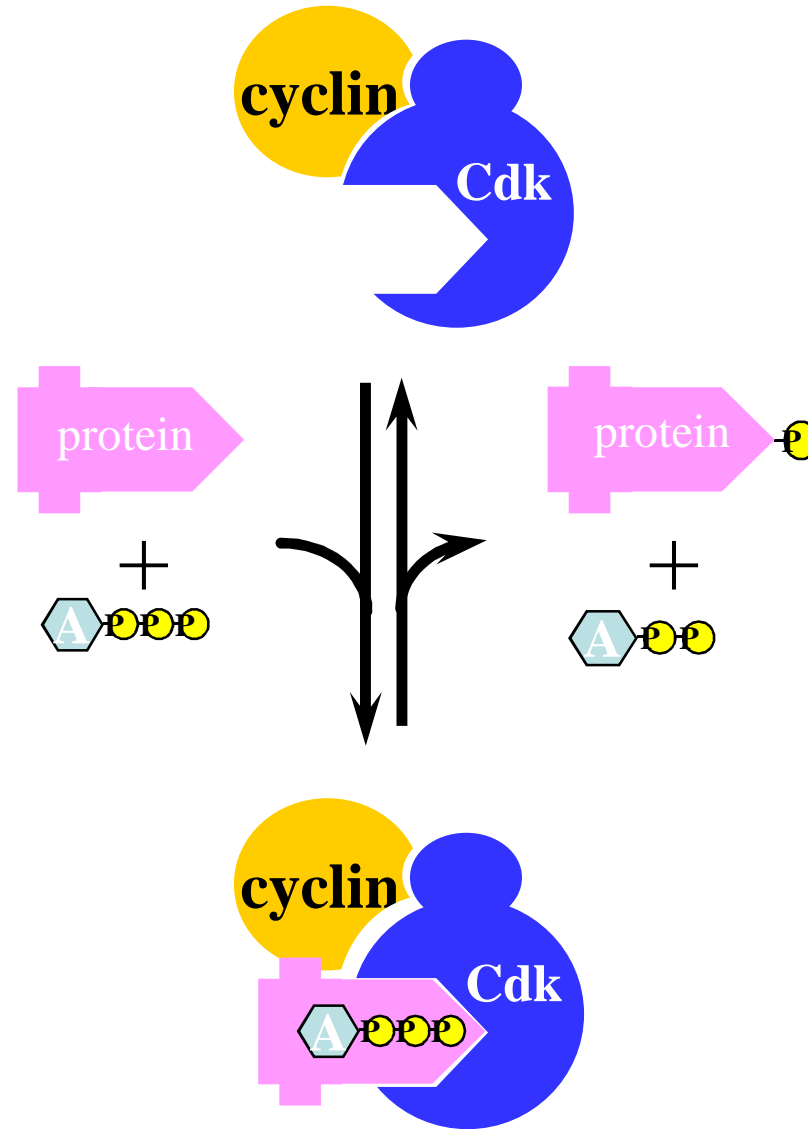
Imperial Cancer  
Research Fund  
London, Great  
Britain

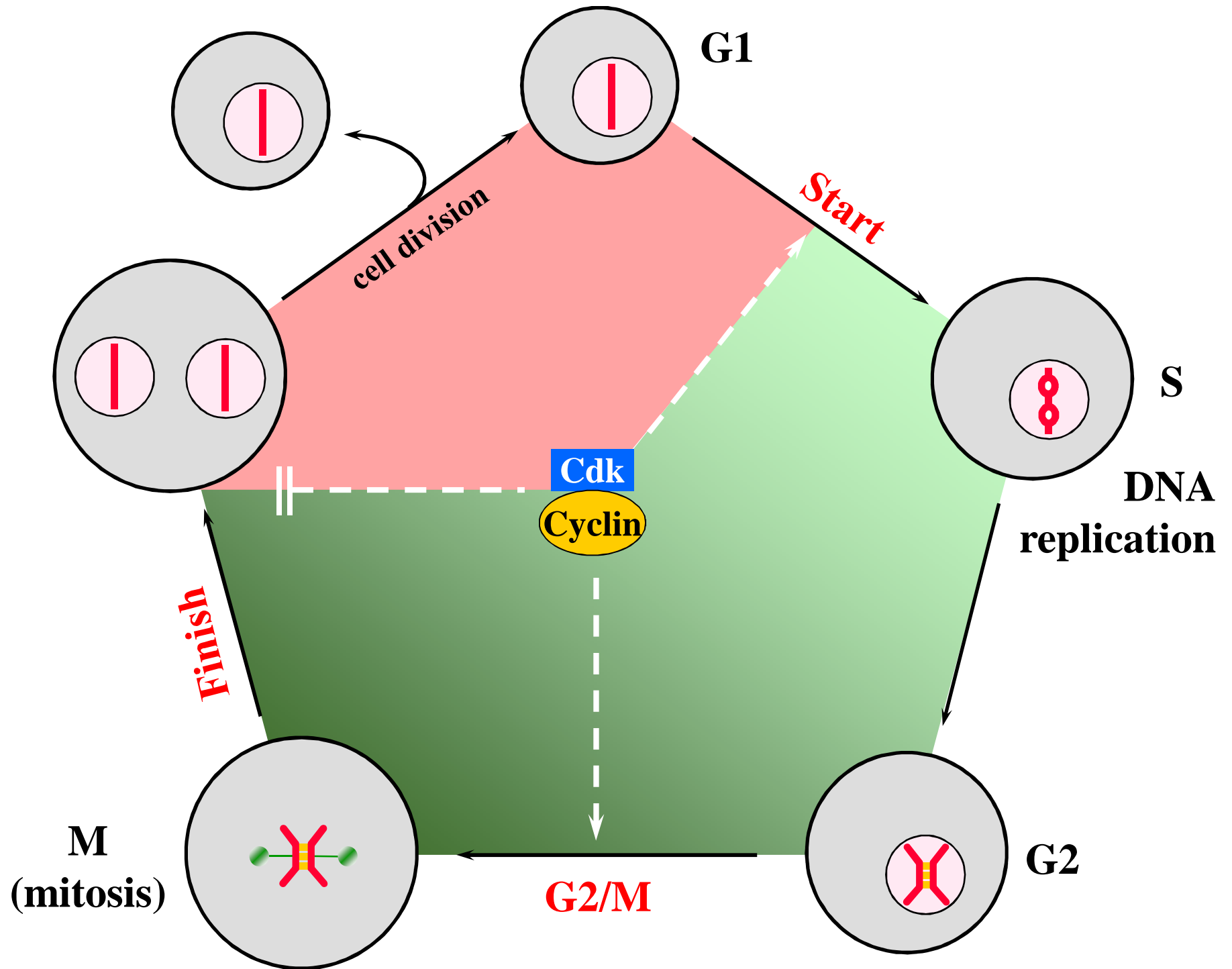
1949 -

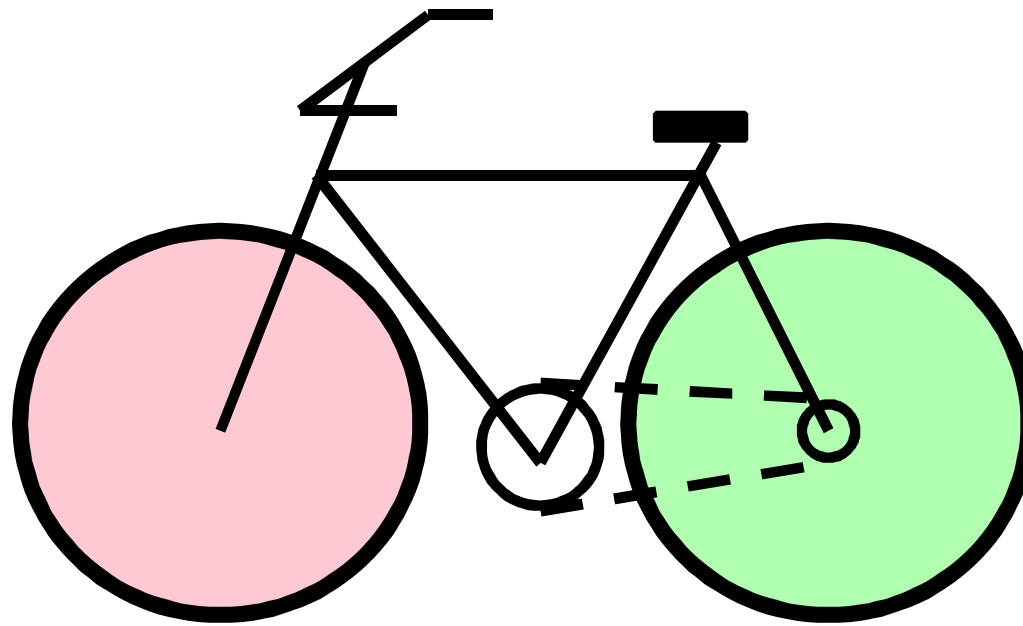




# Cyclin dependent protein kinase





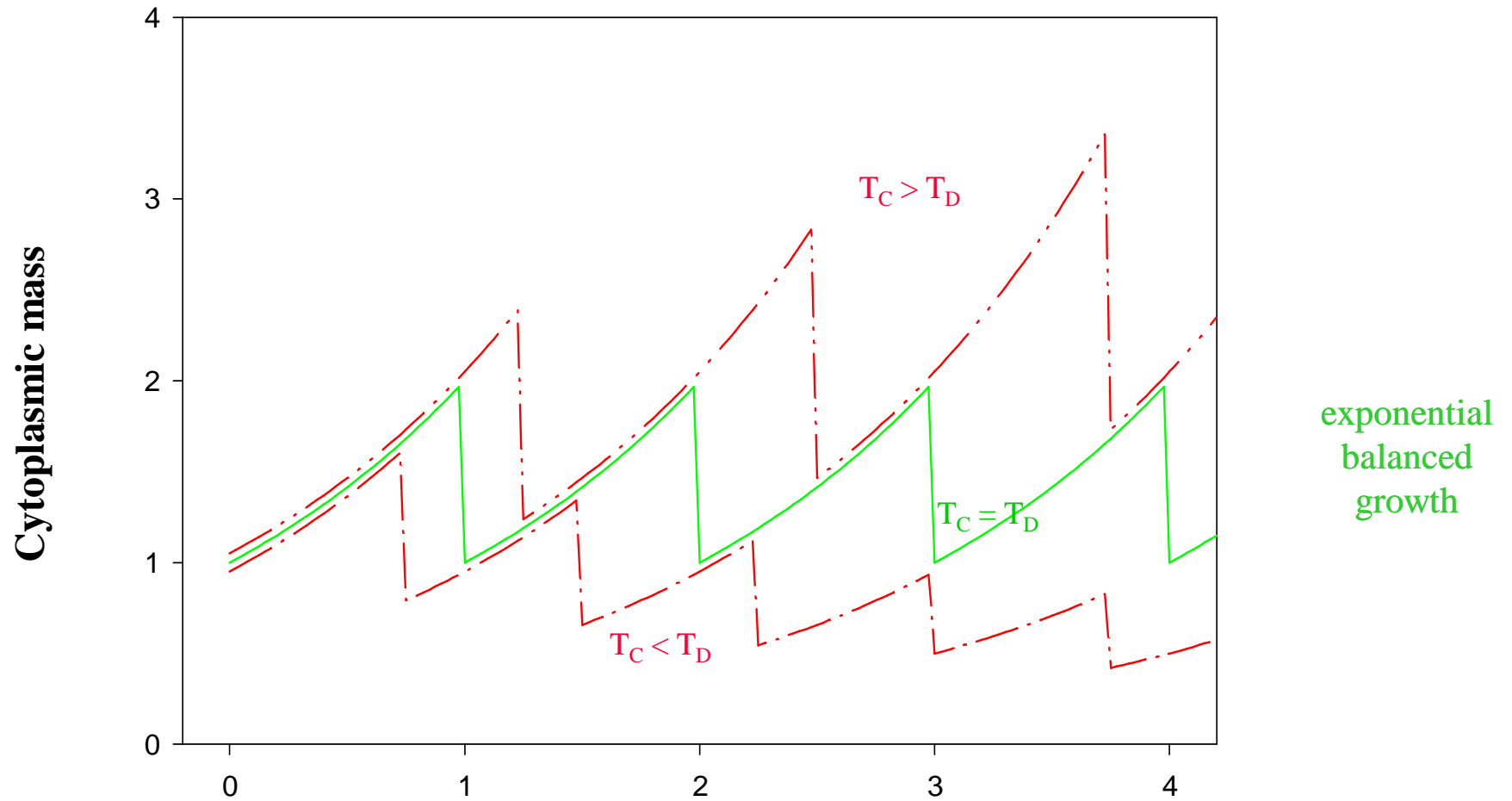
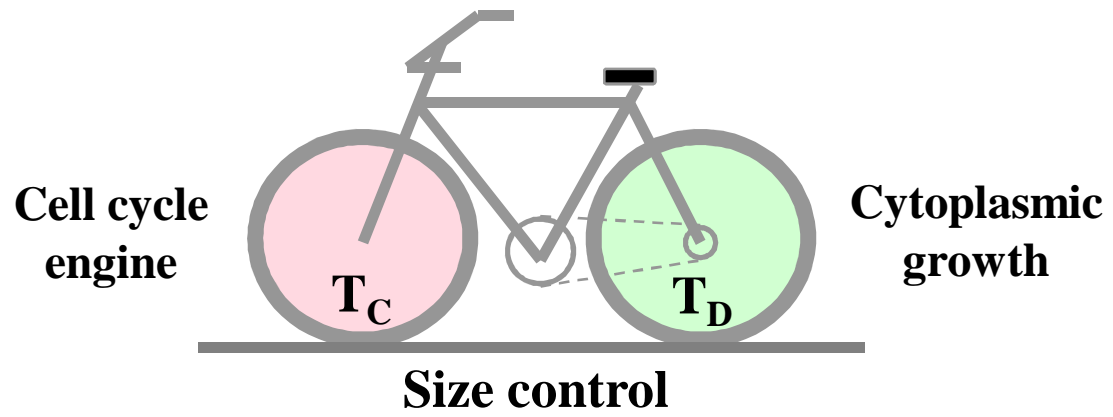


### **chromosome cycle**

- DNA replication
  - mitosis
- (precise doubling and halving)

### **growth cycle**

- cell growth
  - cell division
- (approx. doubling and halving)



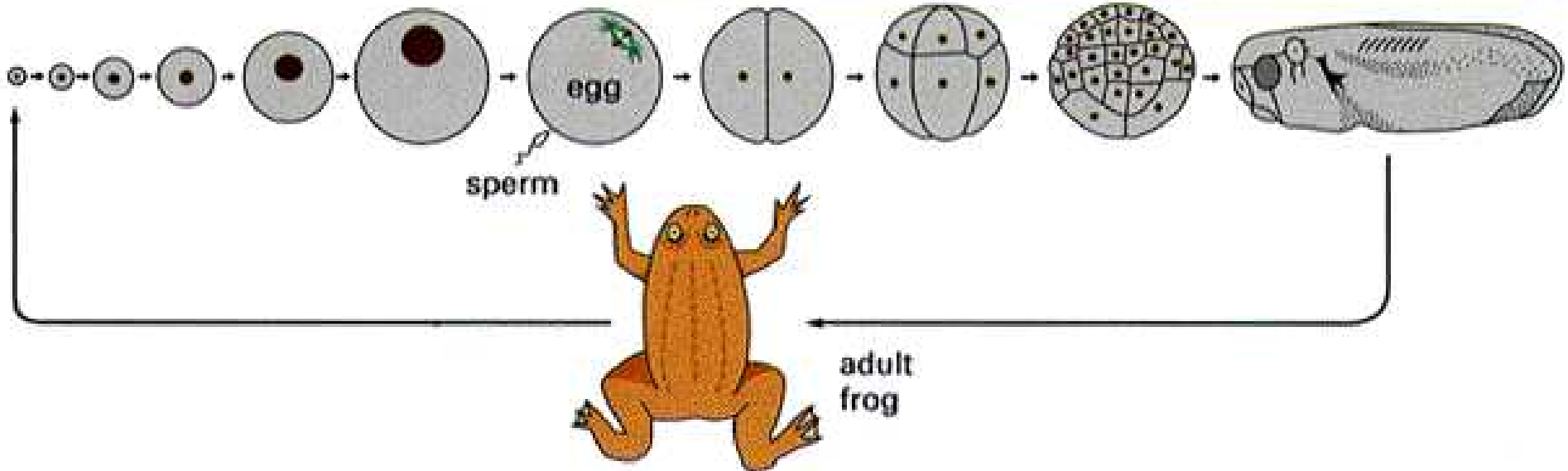
# Uncoupling of the growth and chromosomal cycles

## **oogenesis**

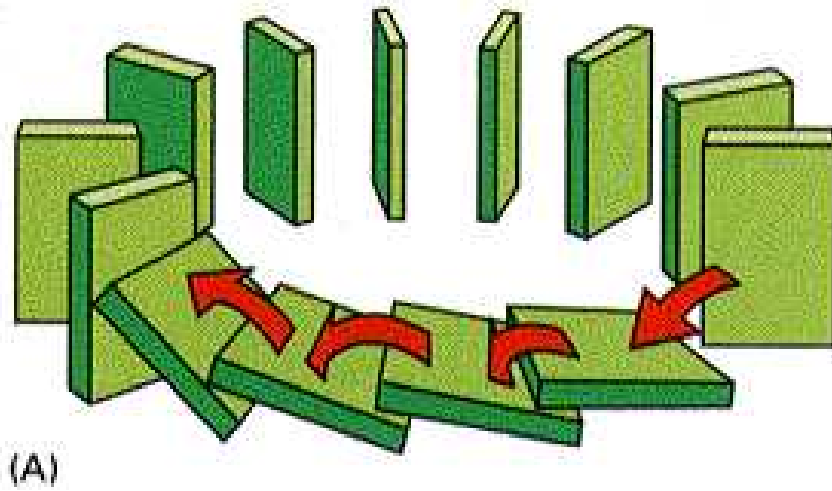
## **embryogenesis**

**oocyte grows without dividing  
(months)**

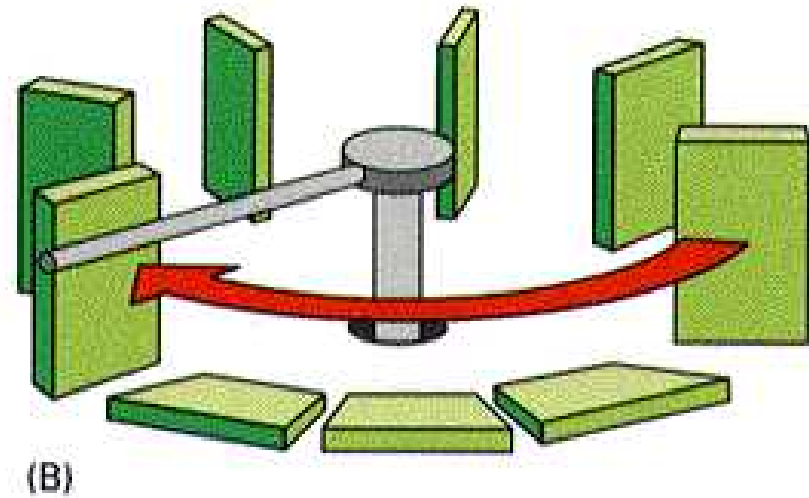
**fertilized egg divides without growing  
(hours)**



**growing (somatic) cells**



**early embryos**



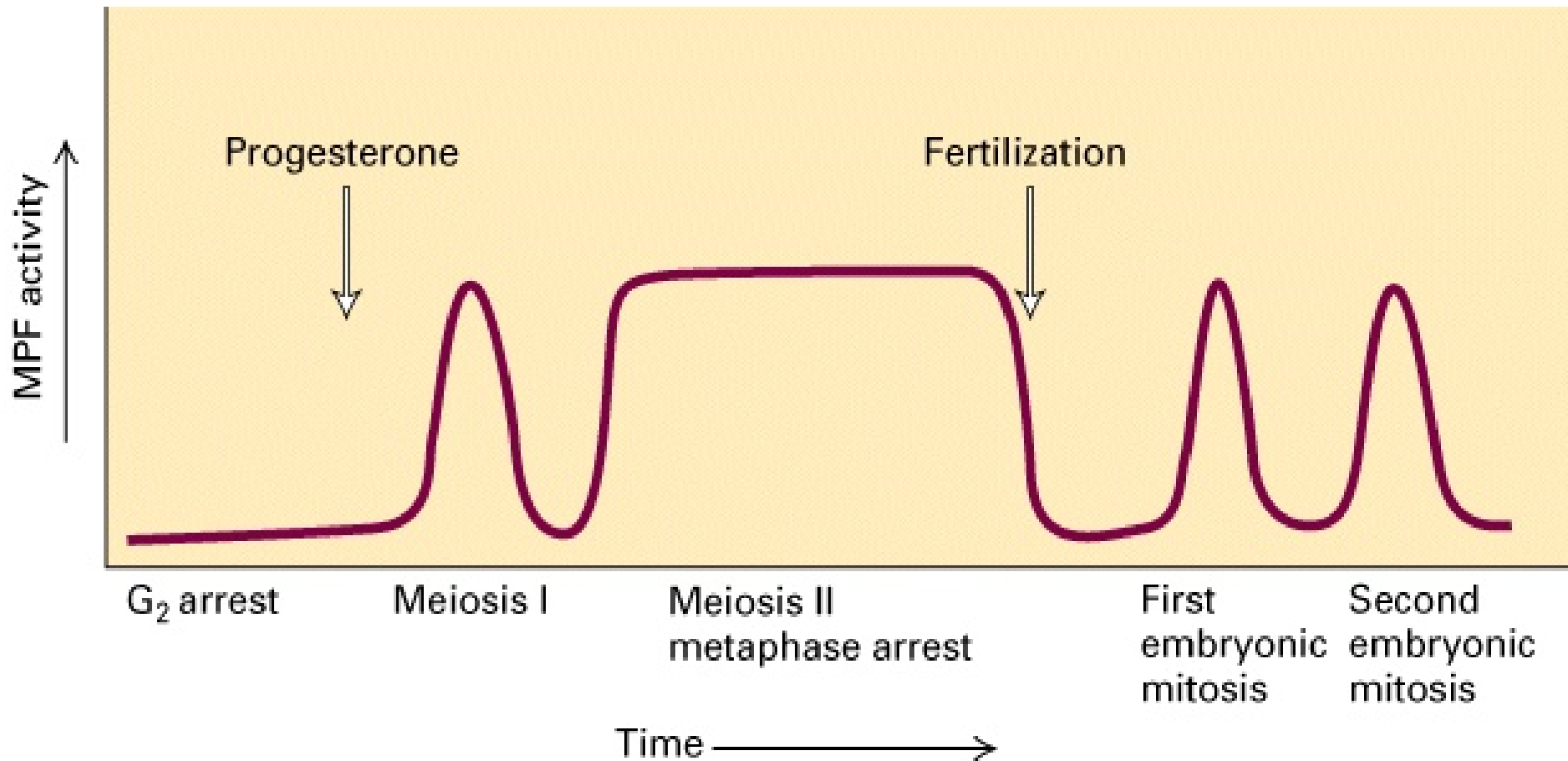
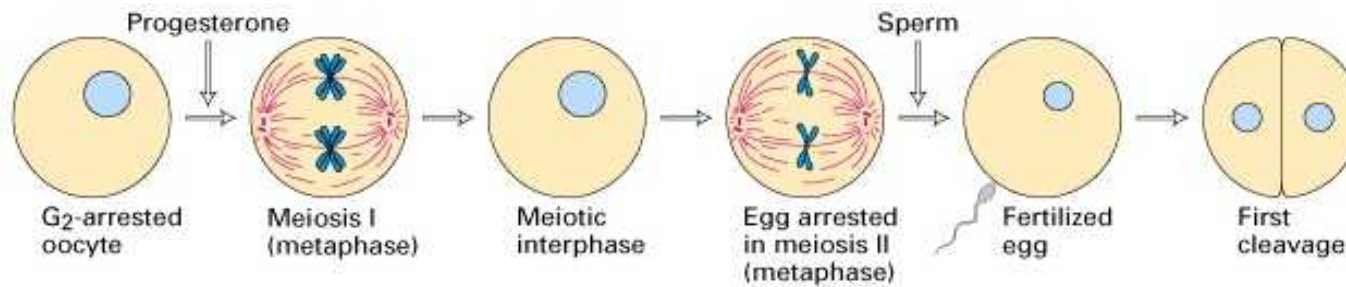
# **Modeling cell cycle regulation**

## **I. Embryonic cycles**

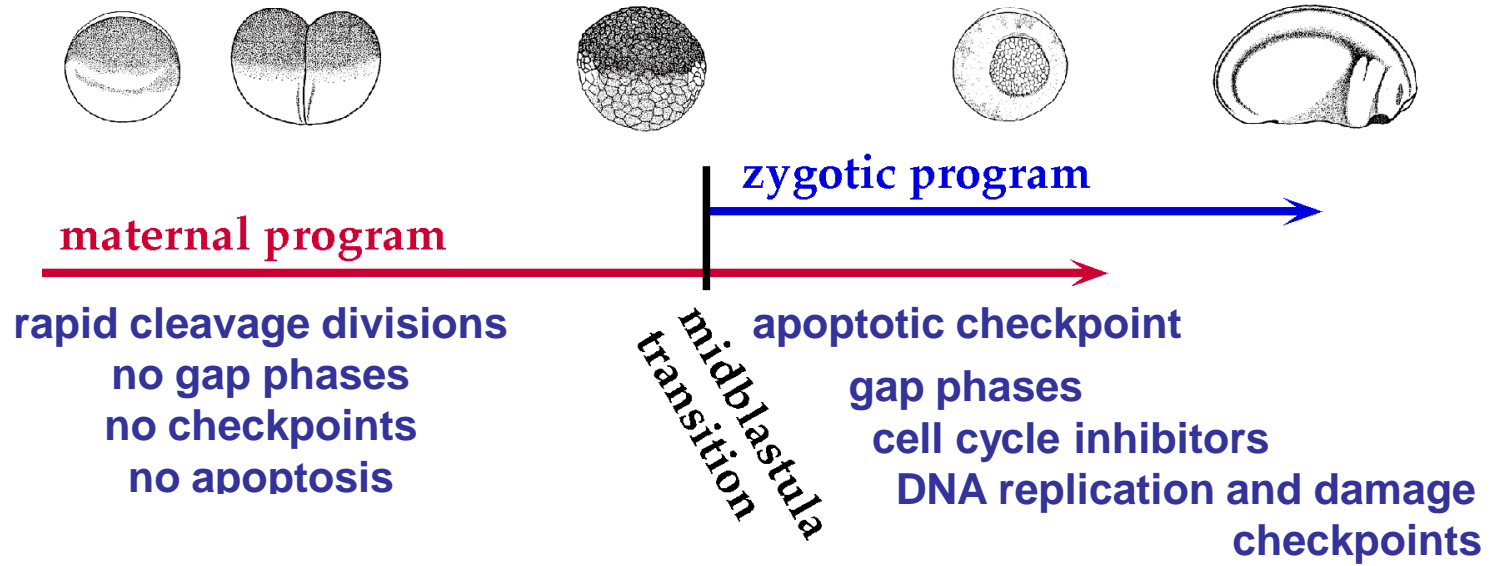




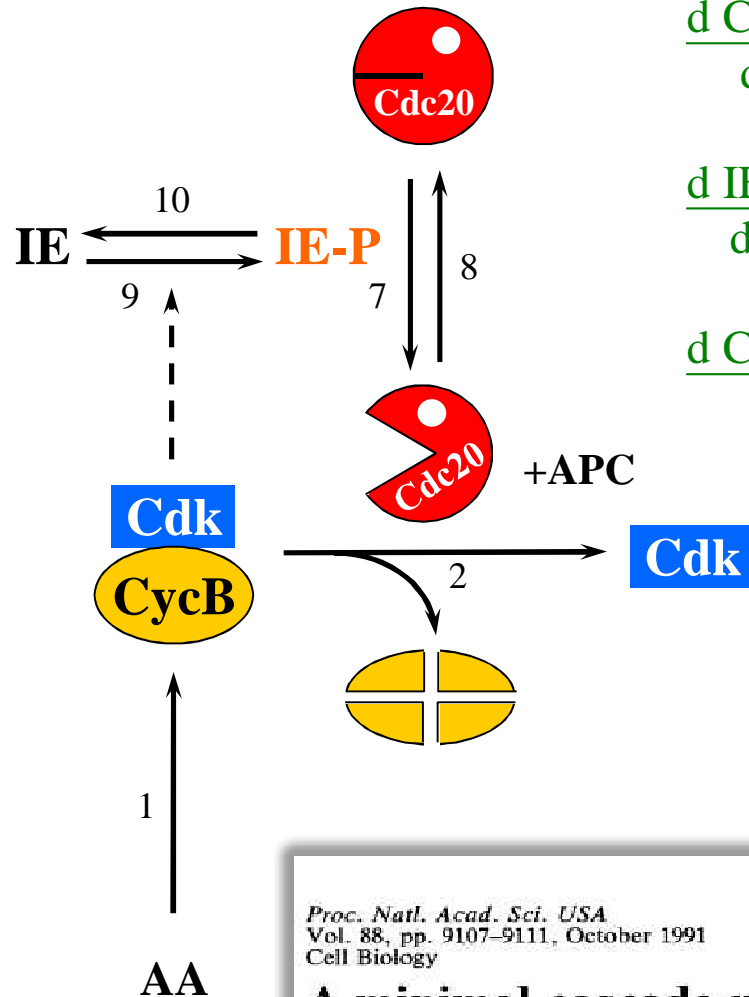
Yoshio Masui



# Cell cycle remodeling begins at the MBT.



# The negative feedback loop



$$\frac{d \text{CycB}}{dt} = k_1 - (k_2' + k_2''') \cdot \text{Cdc20}_A \cdot \text{CycB}$$

$$\frac{d \text{IE-P}}{dt} = k_9 \cdot \text{CycB} \cdot \frac{1 - \text{IE-P}}{J_9 + 1 - \text{IE-P}} - k_{10} \cdot \frac{\text{IE-P}}{J_{10} + \text{IE-P}}$$

$$\frac{d \text{Cdc20}_A}{dt} = k_7 \cdot \text{IE-P} \cdot \frac{1 - \text{Cdc20}_A}{J_7 + 1 - \text{Cdc20}_A} - k_8 \cdot \frac{\text{Cdc20}_A}{J_8 + \text{Cdc20}_A}$$



*Proc. Natl. Acad. Sci. USA*  
Vol. 88, pp. 9107-9111, October 1991  
Cell Biology

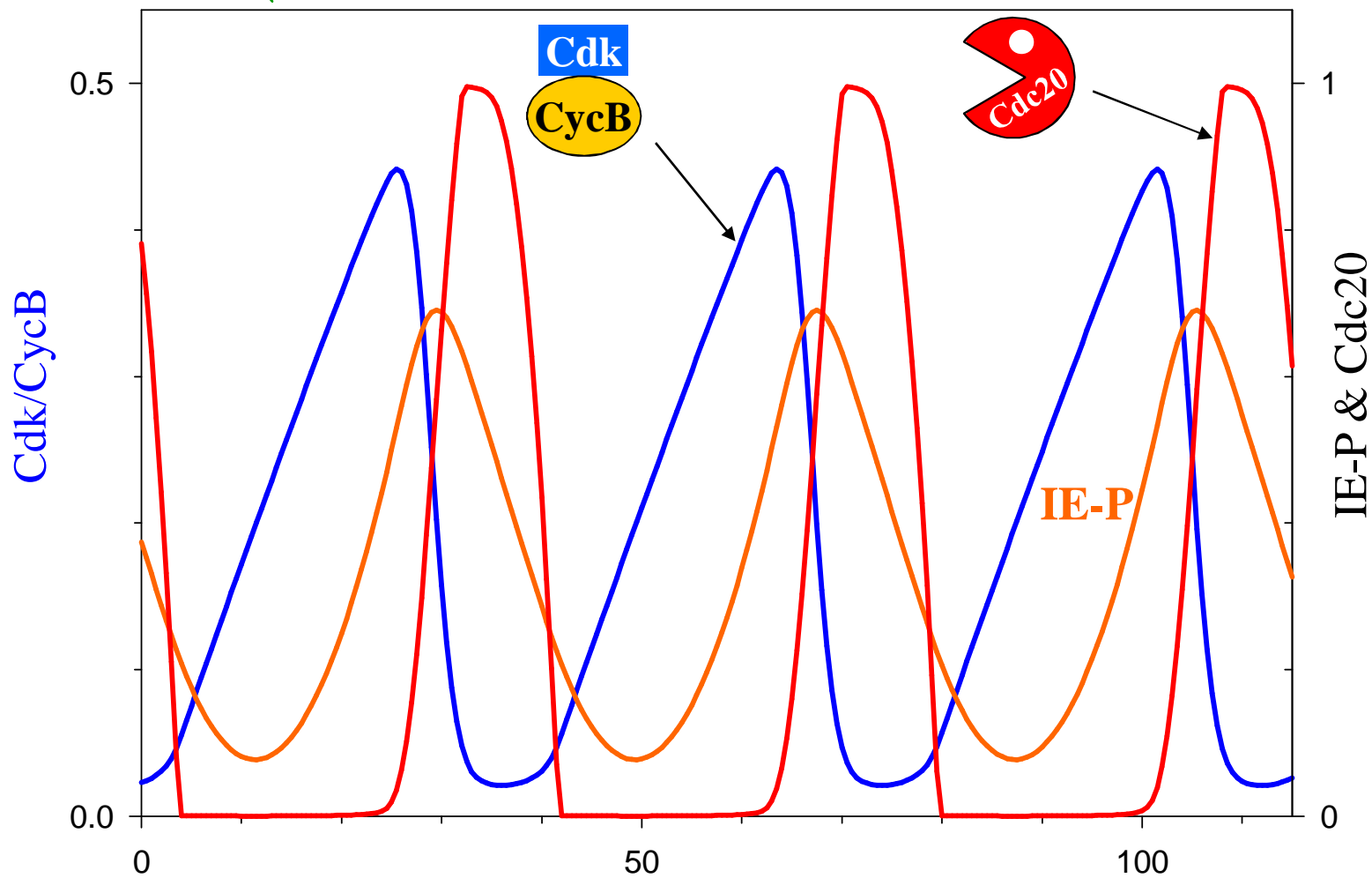
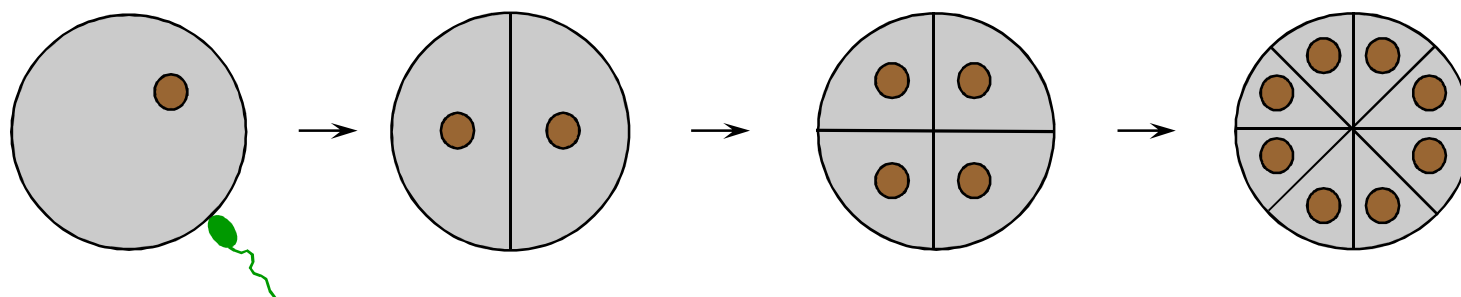
## A minimal cascade model for the mitotic oscillator involving cyclin and cdc2 kinase

(cell cycle/maturation-promoting factor/phosphorylation cascade/thresholds/biochemical oscillations)

ALBERT GOLDBETER

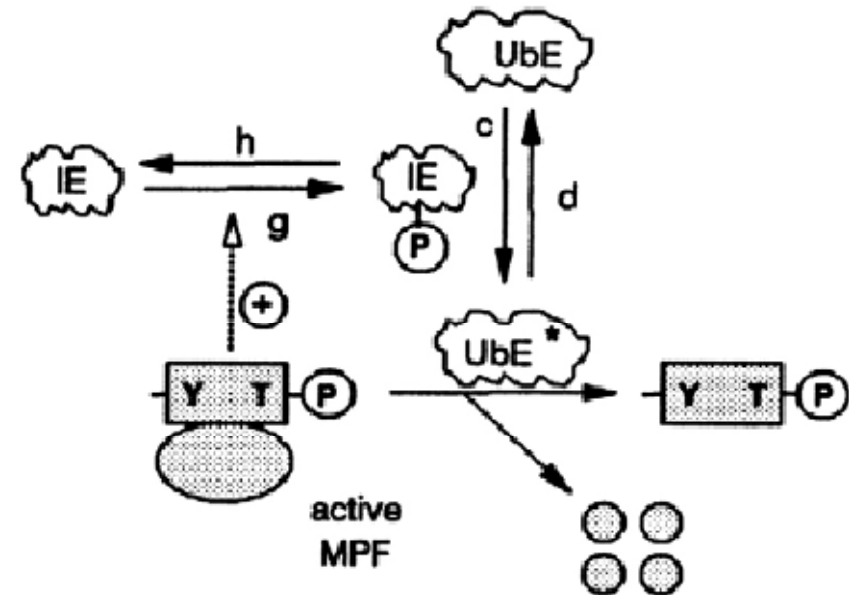
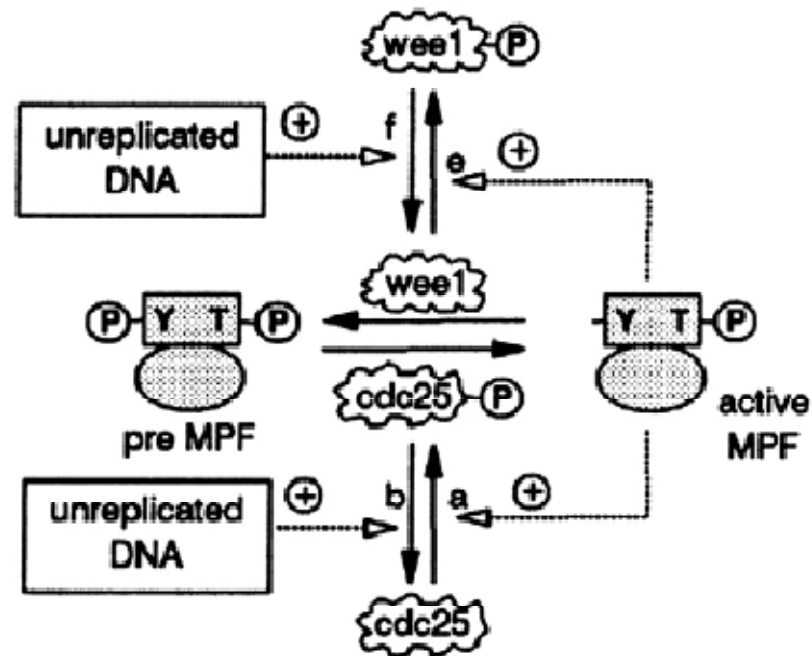
Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine, C.P. 231, B-1050 Brussels, Belgium

# Early embryonic cell cycles

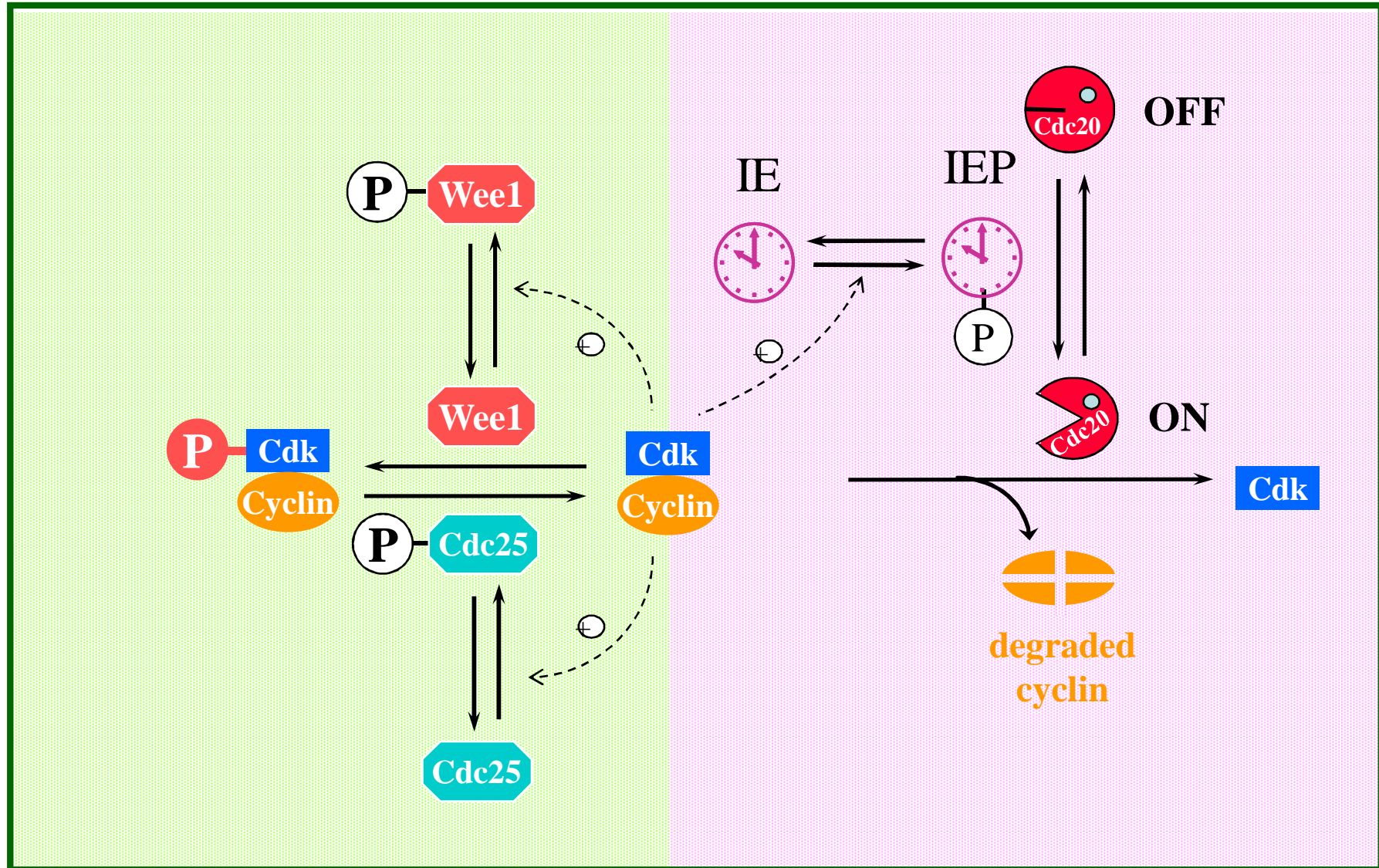


# Numerical analysis of a comprehensive model of M-phase control in *Xenopus* oocyte extracts and intact embryos

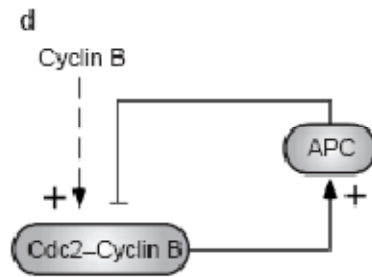
Bela Novak<sup>†</sup> and John J. Tyson<sup>†</sup>



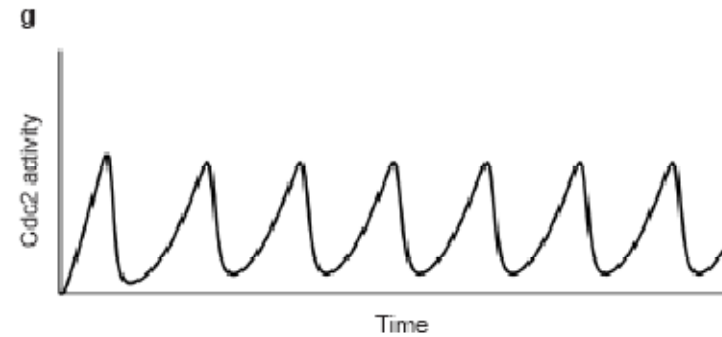
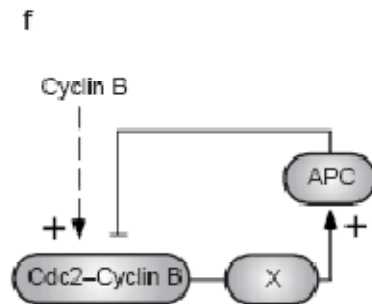
# MPF is regulated by positive and negative feedback loops



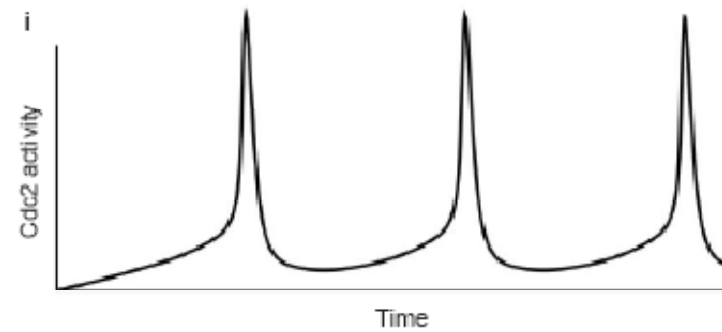
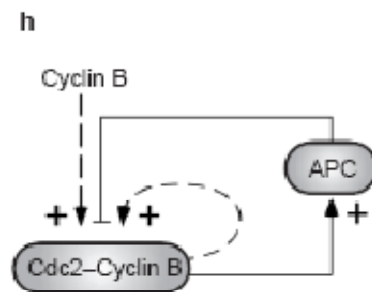
Direct negative feedback:  
**NO oscillations**



Delayed negative feedback:  
**sinusoid oscillations**



Positive & negative  
feedbacks:  
**relaxation oscillations**



# Differential Equations in Novak-Tyson Model

$$\frac{d[\text{Cyclin}]}{dt} = k_1 - k_2[\text{Cyclin}] - k_3[\text{Cdk1}][\text{Cyclin}]$$

$$\frac{d[\text{YT}]}{dt} = k_{pp}[\text{MPF}] - (k_{wee} + k_{cak} + k_2)[\text{YT}] + k_{25}[\text{PYT}] + k_3[\text{Cdk1}][\text{Cyclin}]$$

$$\frac{d[\text{PYT}]}{dt} = k_{wee}[\text{YT}] - (k_{25} + k_{cak} + k_2)[\text{PYT}] + k_{pp}[\text{PYTP}]$$

$$\frac{d[\text{PYTP}]}{dt} = k_{wee}[\text{MPF}] - (k_{pp} + k_{25} + k_2)[\text{PYTP}] + k_{cak}[\text{PYT}]$$

$$\frac{d[\text{MPF}]}{dt} = k_{cak}[\text{YT}] - (k_{pp} + k_{wee} + k_2)[\text{MPF}] + k_{25}[\text{PYTP}]$$

$$\frac{d[\text{Cdc25P}]}{dt} = \frac{k_a[\text{MPF}](\text{total Cdc25} - [\text{Cdc25P}])}{K_a + \text{total Cdc25} - [\text{Cdc25P}]} - \frac{k_b[\text{PPase}][\text{Cdc25P}]}{K_b + [\text{Cdc25P}]}$$

$$\frac{d[\text{Wee1P}]}{dt} = \frac{k_e[\text{MPF}](\text{total Wee1} - [\text{Wee1P}])}{K_e + \text{total Wee1} - [\text{Wee1P}]} - \frac{k_f[\text{PPase}][\text{Wee1P}]}{K_f + [\text{Wee1P}]}$$

$$\frac{d[\text{IEP}]}{dt} = \frac{k_g[\text{MPF}](\text{total IE} - [\text{IEP}])}{K_g + \text{total IE} - [\text{IEP}]} - \frac{k_h[\text{PPase}][\text{IEP}]}{K_h + [\text{IEP}]}$$

$$\frac{d[\text{APC}^*]}{dt} = \frac{k_c[\text{IEP}](\text{total APC} - [\text{APC}^*])}{K_c + \text{total APC} - [\text{APC}^*]} - \frac{k_d[\text{Anti IE}][\text{APC}^*]}{K_d + [\text{APC}^*]}$$

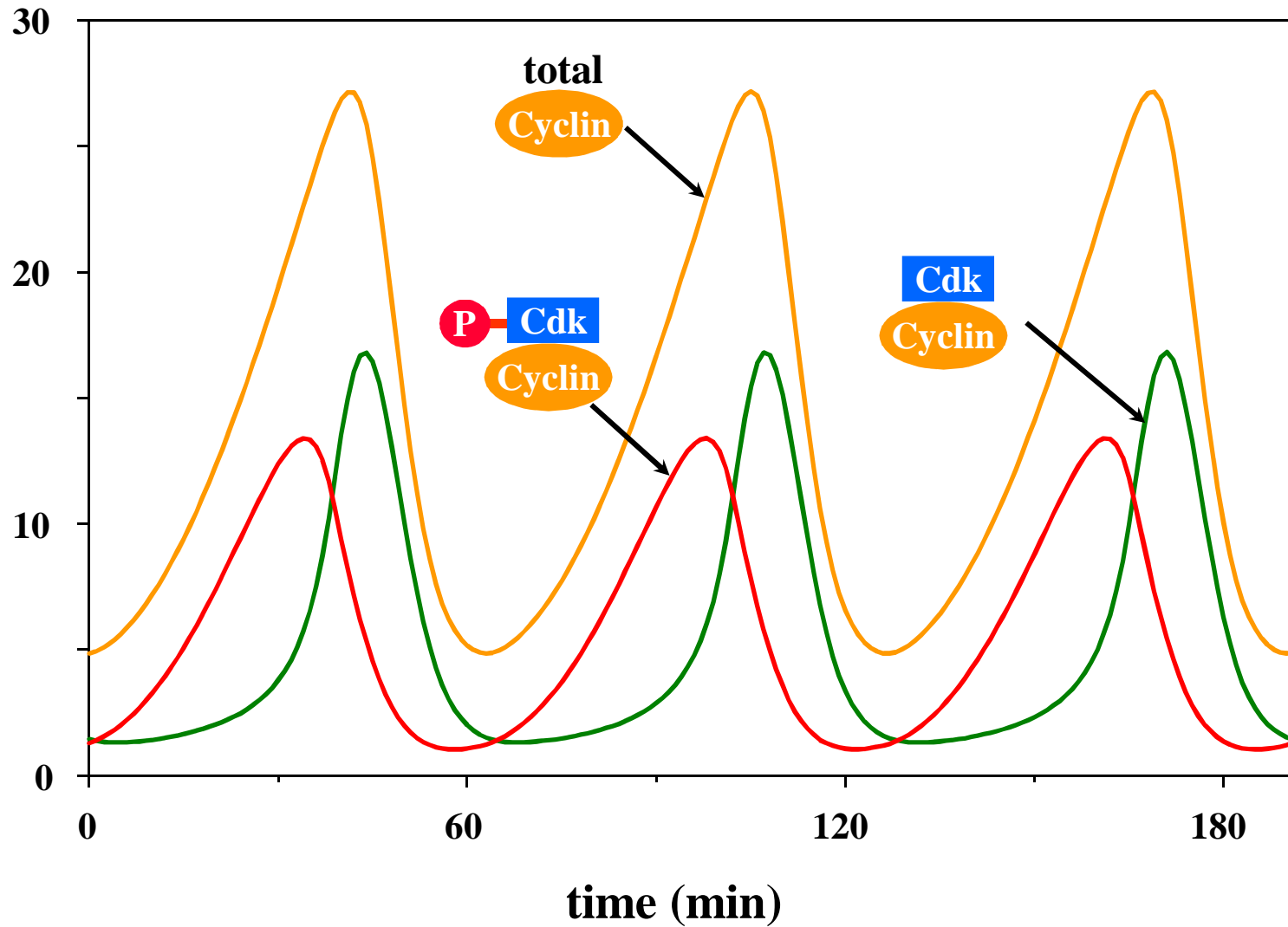
$$k_{25} = V_{25}'(\text{total Cdc25} - [\text{Cdc25P}]) + V_{25}''[\text{Cdc25P}]$$

$$k_{wee} = V_{wee}'[\text{Wee1P}] + V_{wee}''(\text{total Wee1} - [\text{Wee1P}])$$

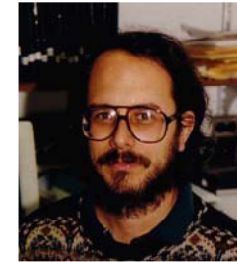
$$k_2 = V_2'(\text{total APC} - [\text{APC}^*]) + V_2''[\text{APC}^*]$$



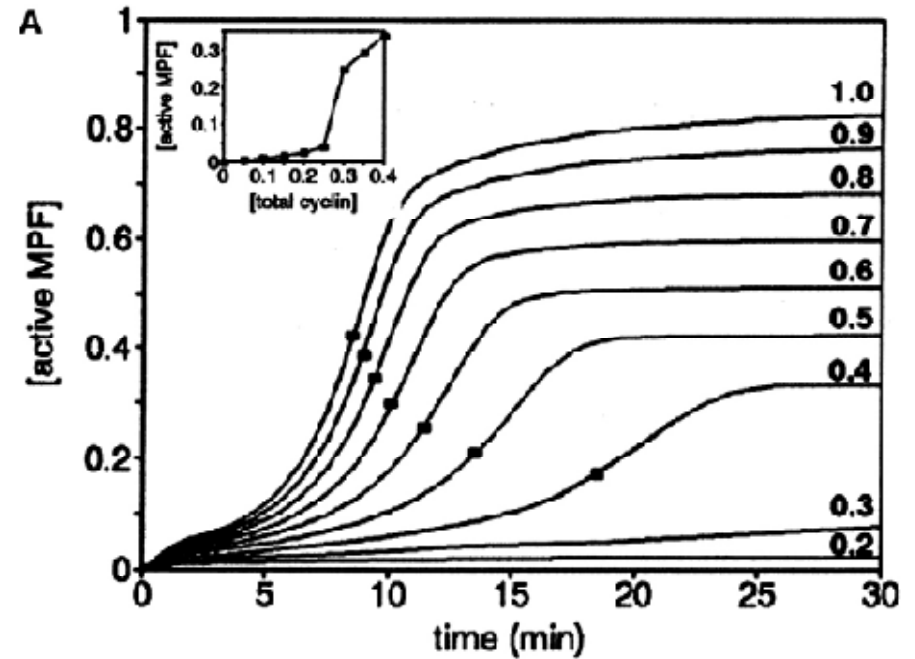
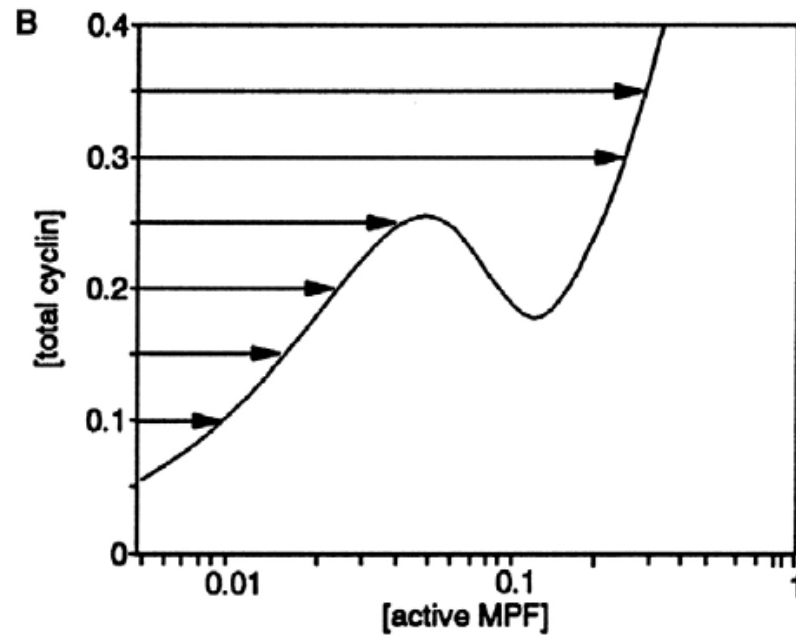
## Mathematical simulations generate oscillations in the model of *Xenopus* extracts



# Cyclin threshold to induce mitosis



Marc Solomon



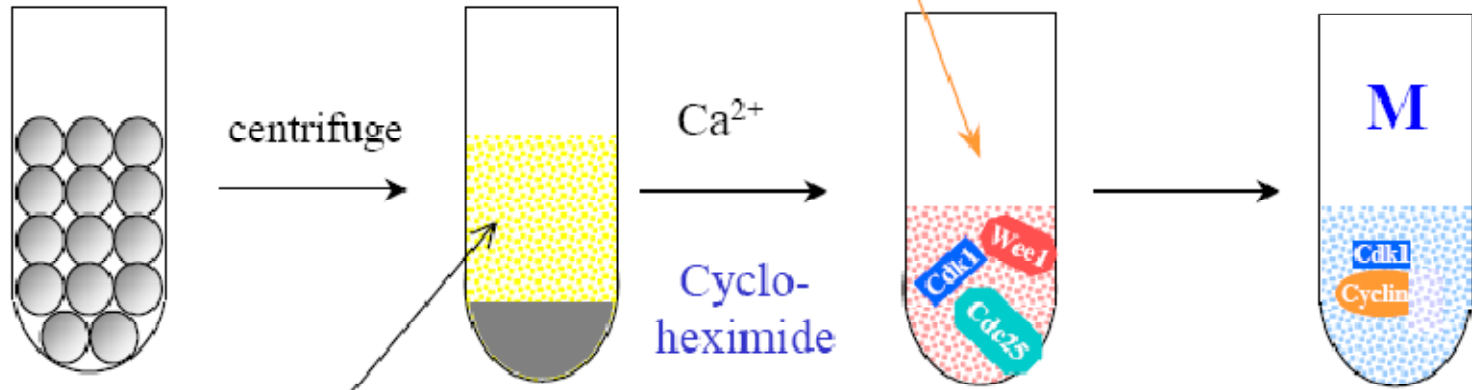
Solomon, M. J., Glotzer, M., Lee, T. H., Phillippe, M. & Kirschner, M. W. (1990) *Cell* 63, 1013–1024.



# Solomon's protocol for cyclin-induced activation of CDK

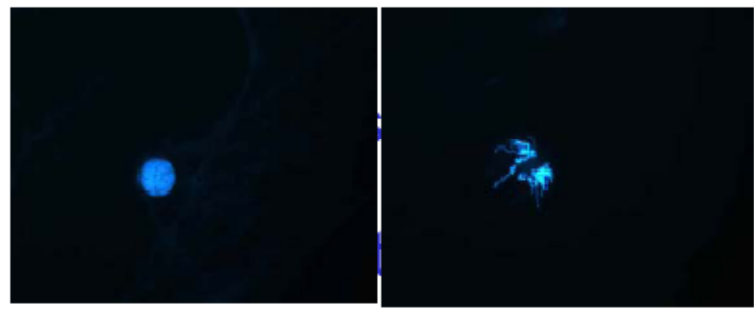
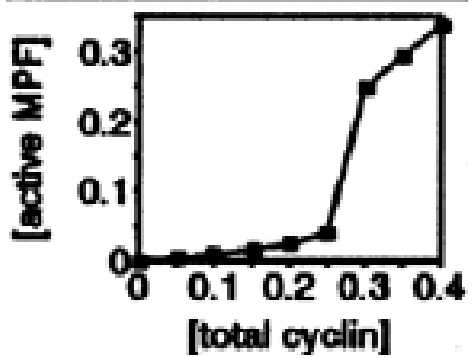
Solomon et al. (1990)  
Cell 63:1013.

Cyclin



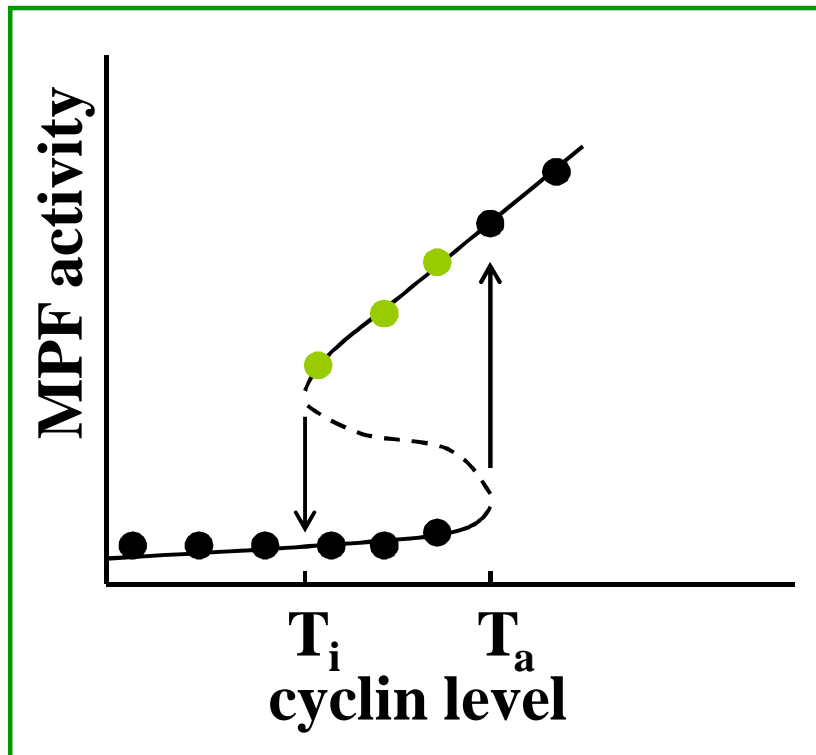
cytoplasmic extract

pellet

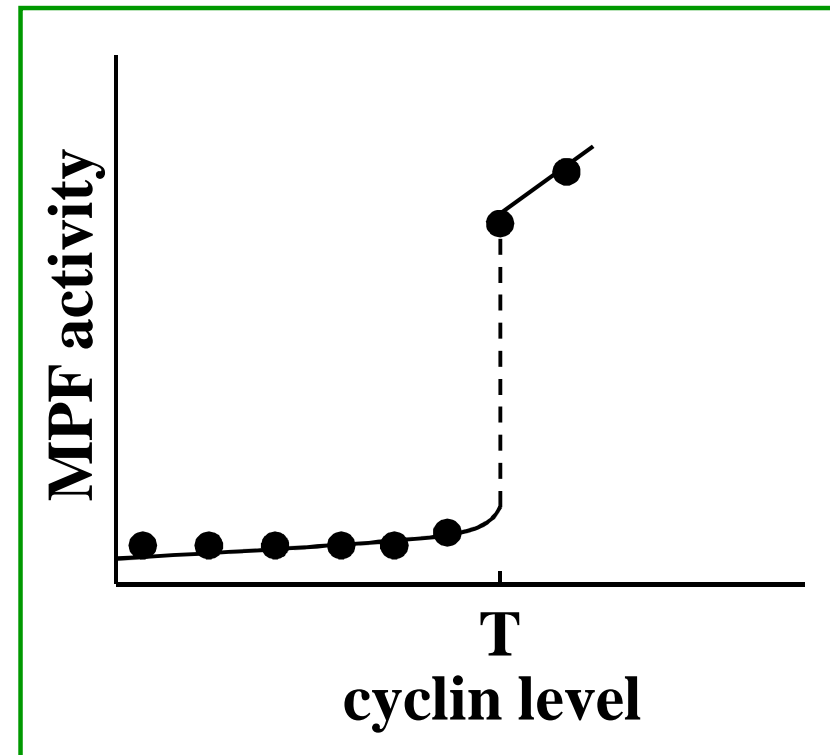


**Prediction: The threshold concentration of cyclin B required to activate MPF is higher than the threshold concentration required to inactivate MPF.**

**hysteretic**



**non-hysteretic**

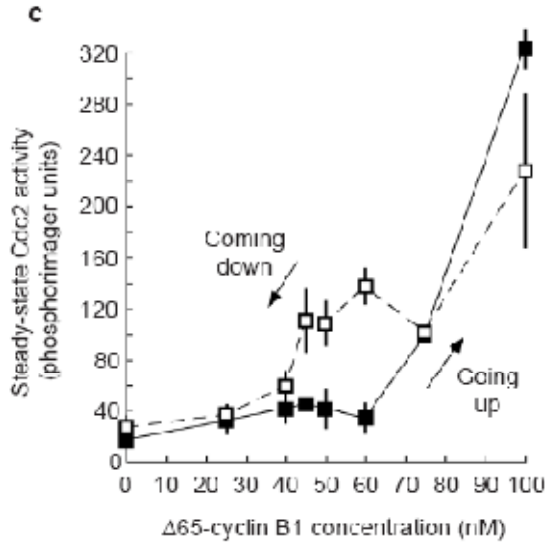


# Building a cell cycle oscillator: hysteresis and bistability in the activation of Cdc2

Joseph R. Pomerening\*, Eduardo D. Sontag† and James E. Ferrell Jr\*‡



Joe Pomerening



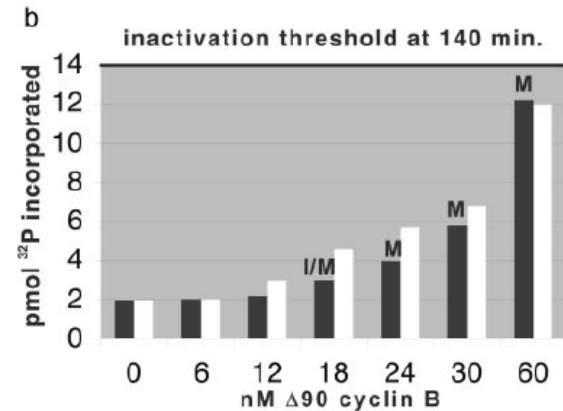
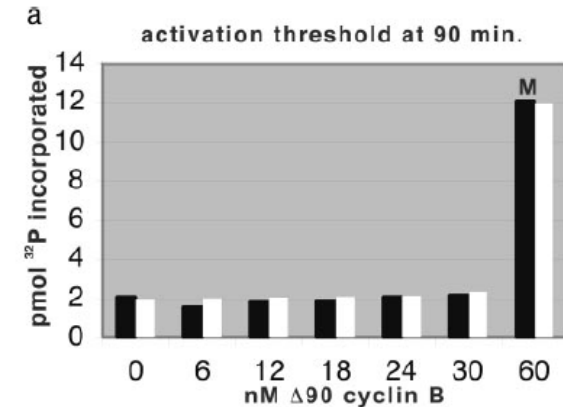
PNAS | February 4, 2003 | vol. 100 | no. 3 | 975-980

# Hysteresis drives cell-cycle transitions in *Xenopus laevis* egg extracts

Wei Sha\*, Jonathan Moore†, Katherine Chen\*, Antonio D. Lassaletta\*, Chung-Seon Yi\*, John J. Tyson\*, and Jill C. Sible\*\*



Jill Sible

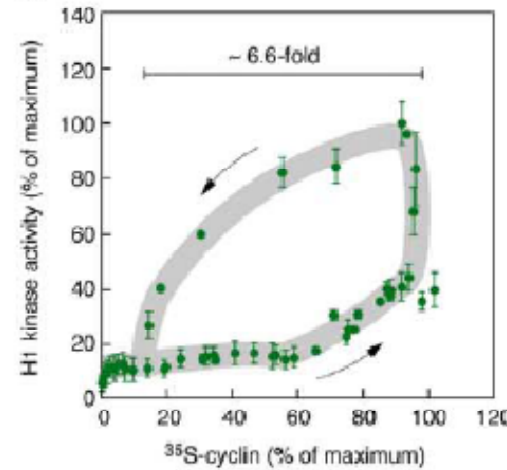
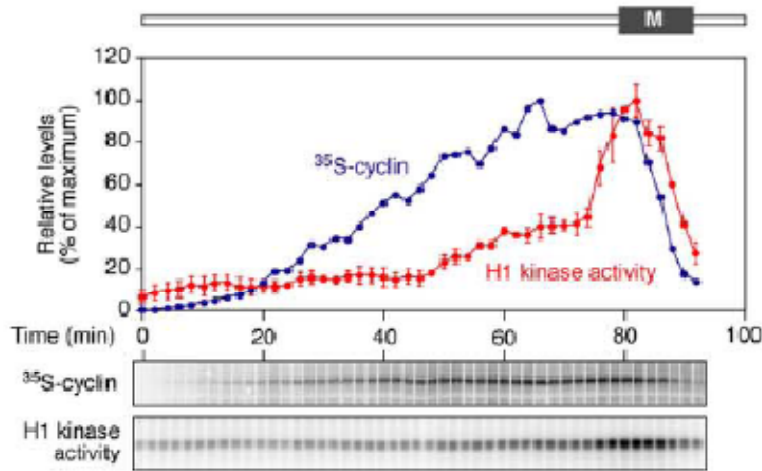


# Systems-Level Dissection of the Cell-Cycle Oscillator: Bypassing Positive Feedback Produces Damped Oscillations

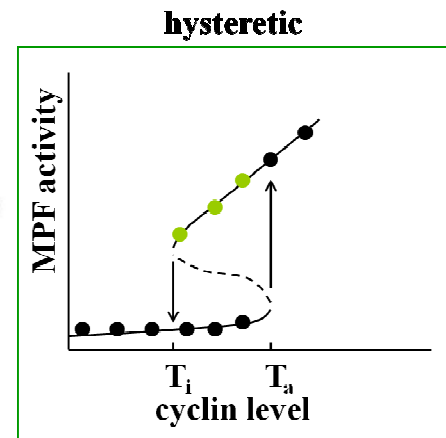
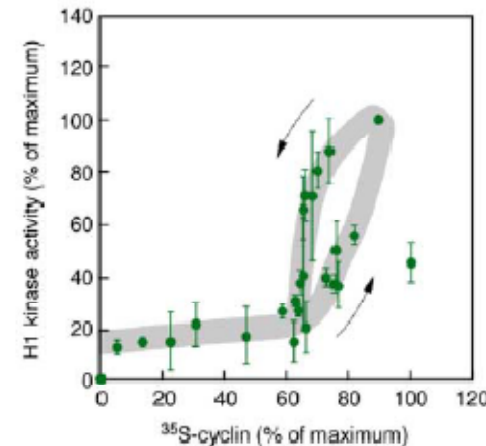
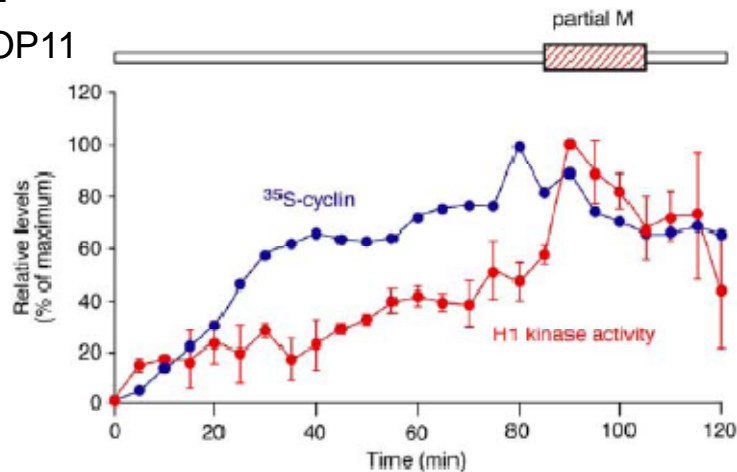
Joseph R. Pomerening,\* Sun Young Kim,  
and James E. Ferrell, Jr.



Joe Pomerening

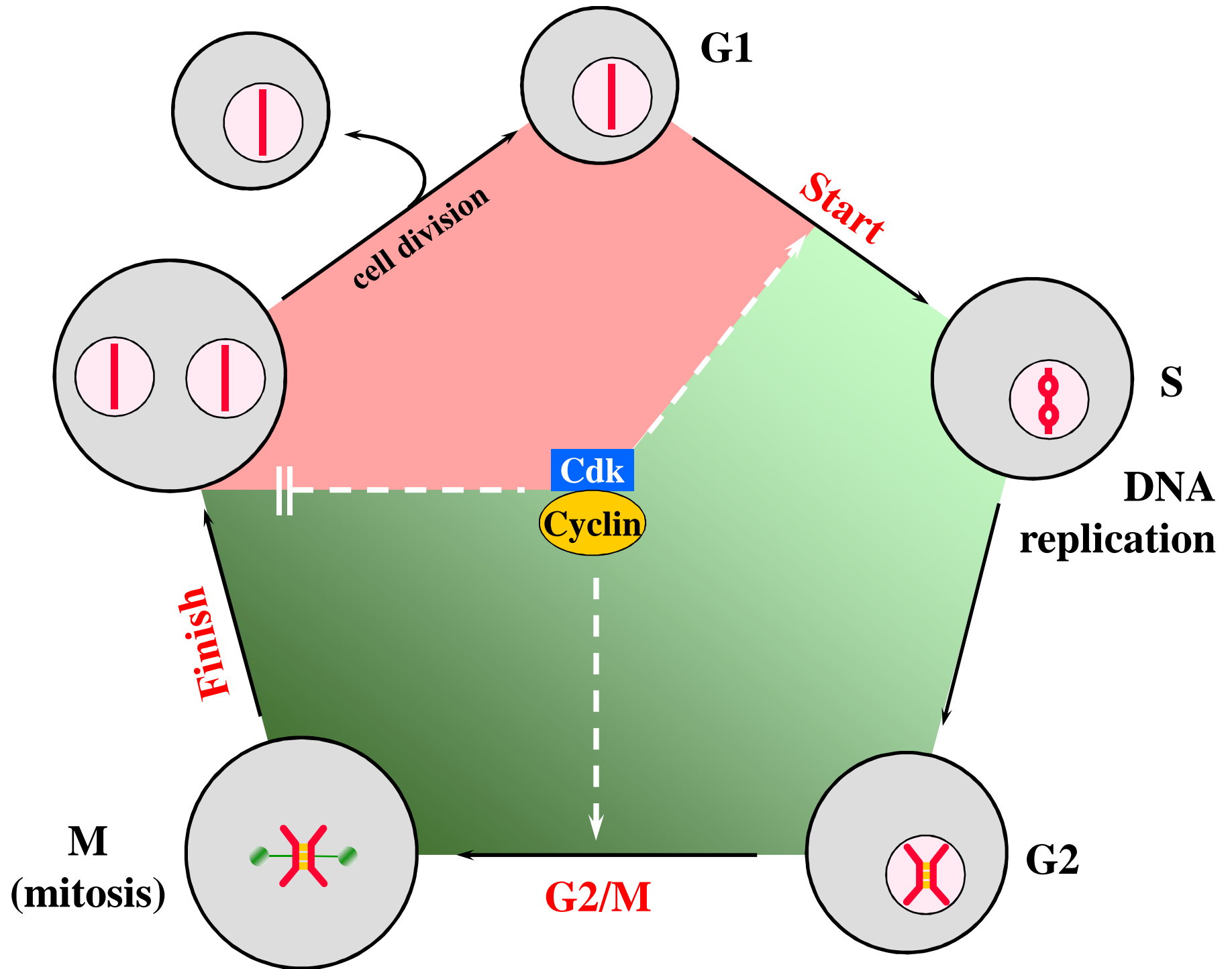


+Cdc2AF  
+Wee1-OP11



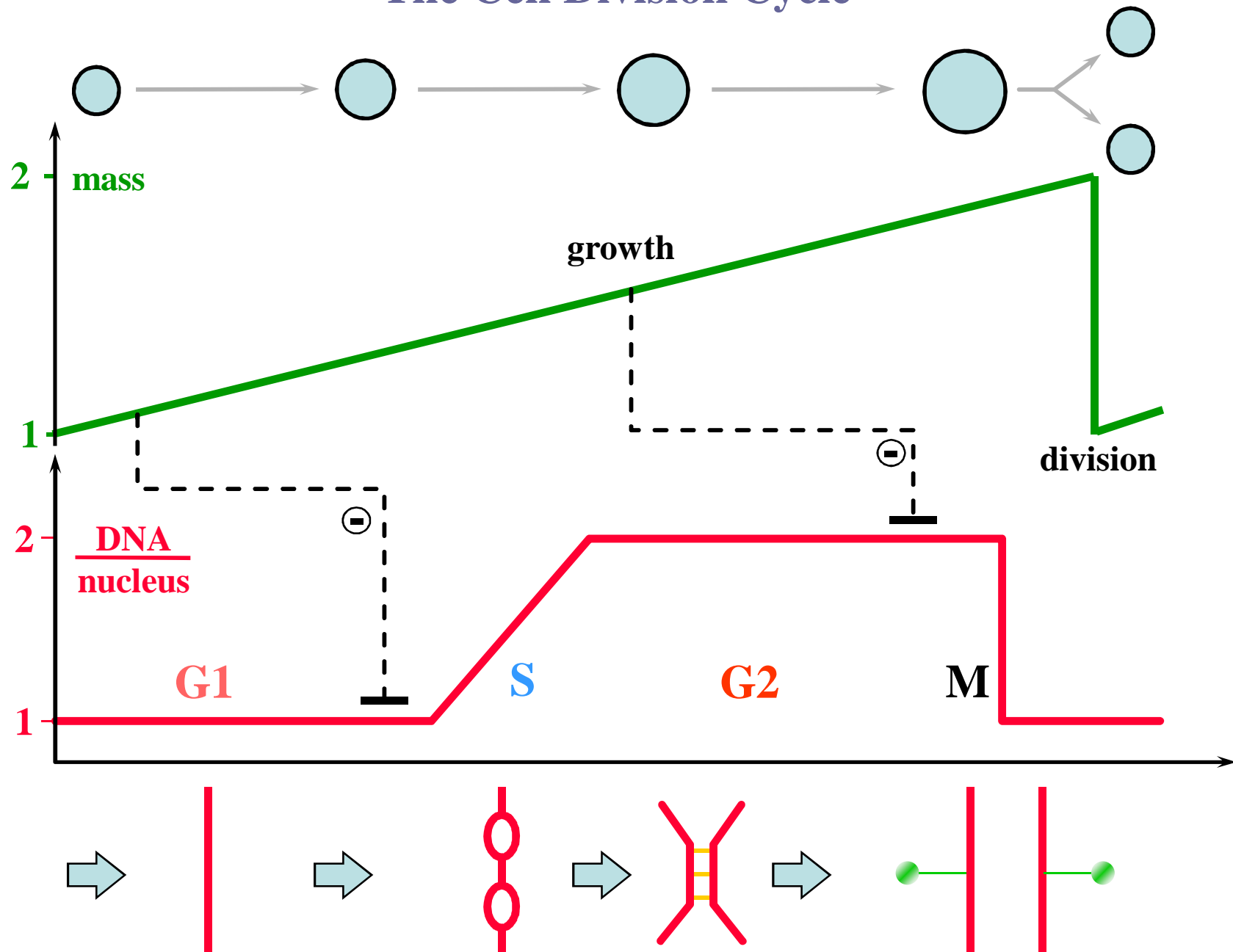
# **Modeling cell cycle regulation**

## **II. Somatic cycles**

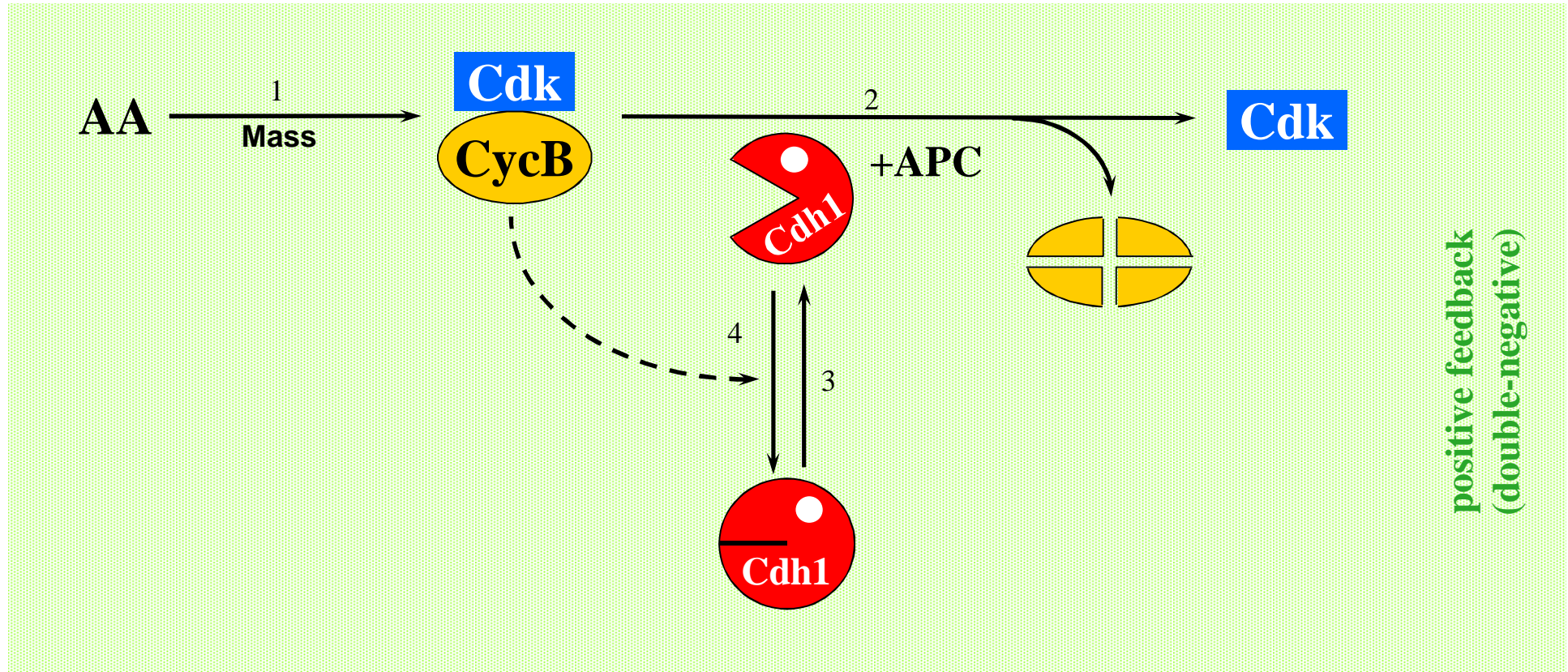




# The Cell Division Cycle



# Stabilizing the G1 phase

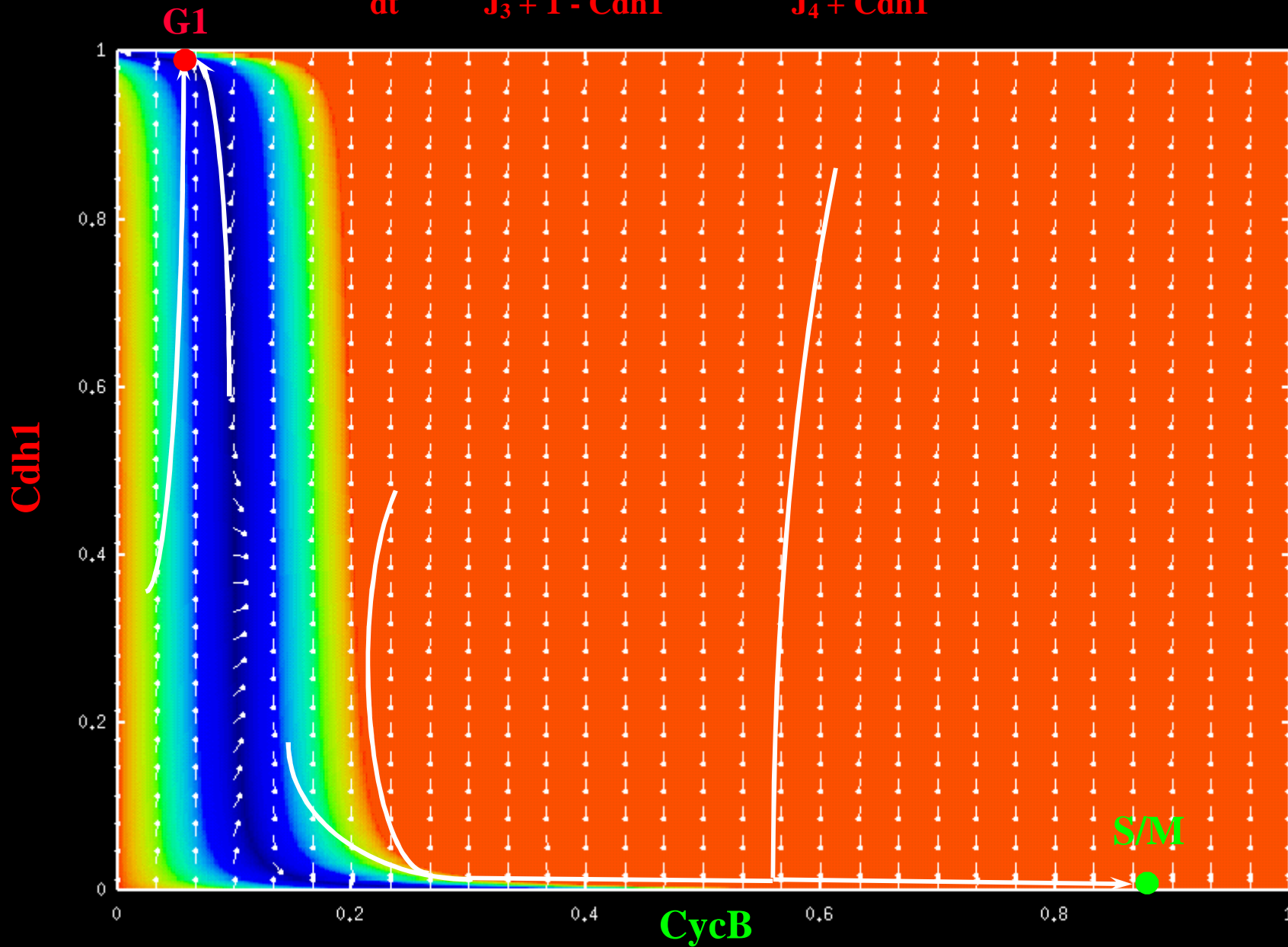


$$\frac{d \text{CycB}}{dt} = k_1 \cdot M - (k_2' + k_2'' \cdot \text{Cdh1}) \cdot \text{CycB}$$

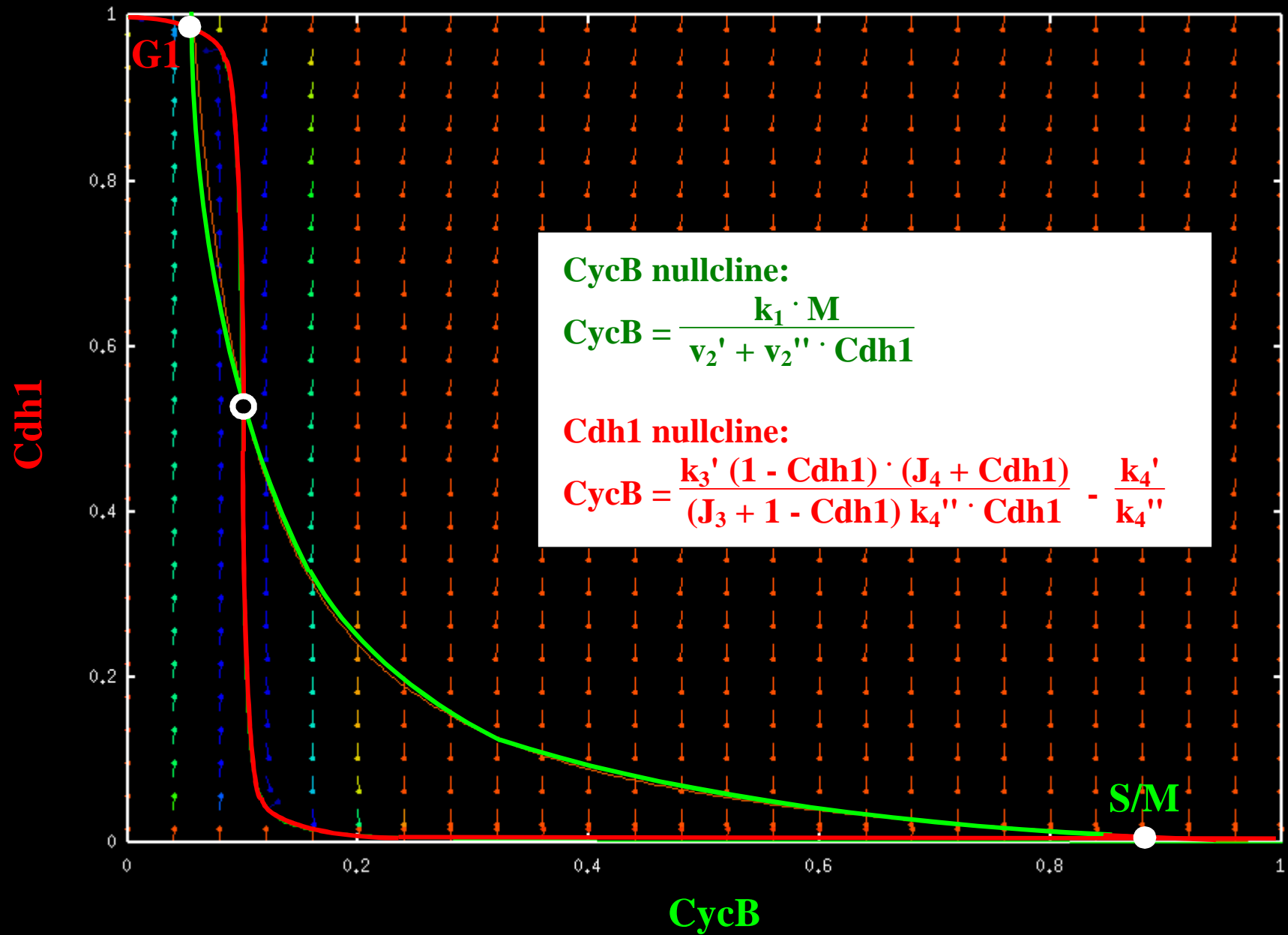
$$\frac{d \text{Cdh1}}{dt} = \frac{k_3' \cdot (1 - \text{Cdh1})}{J_3 + 1 - \text{Cdh1}} - \frac{(k_4' + k_4'' \cdot \text{CycB}) \text{Cdh1}}{J_4 + \text{Cdh1}}$$

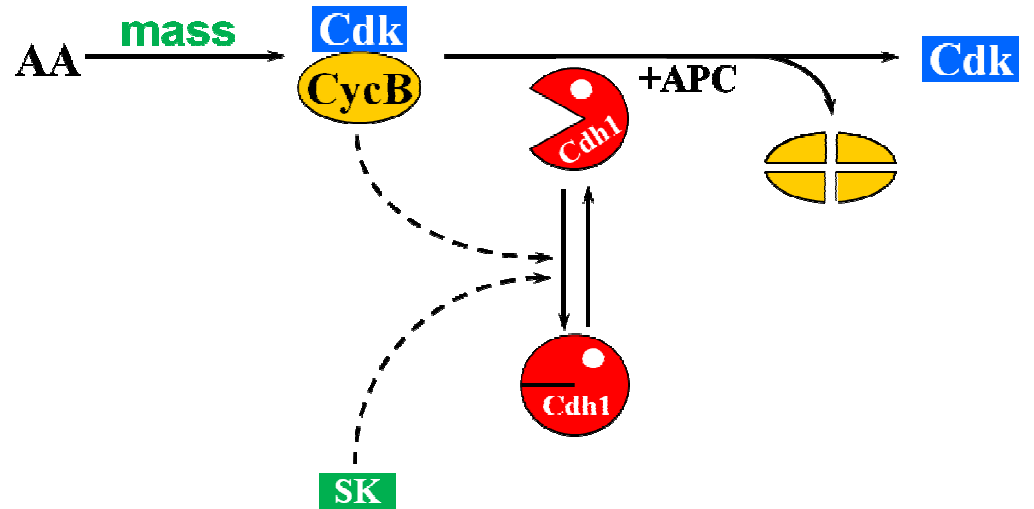
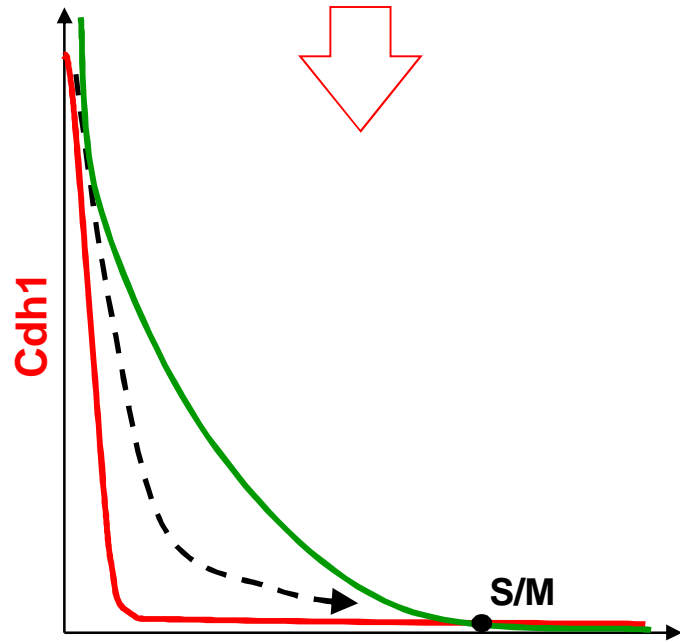
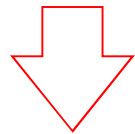
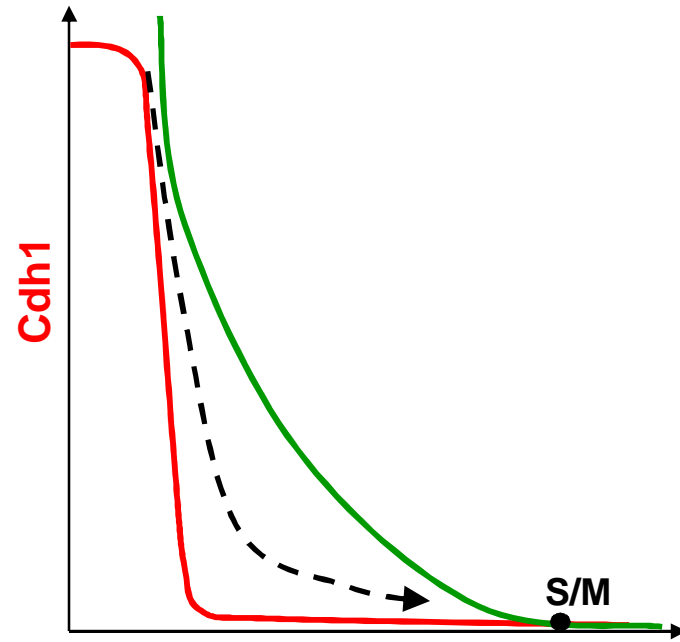
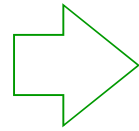
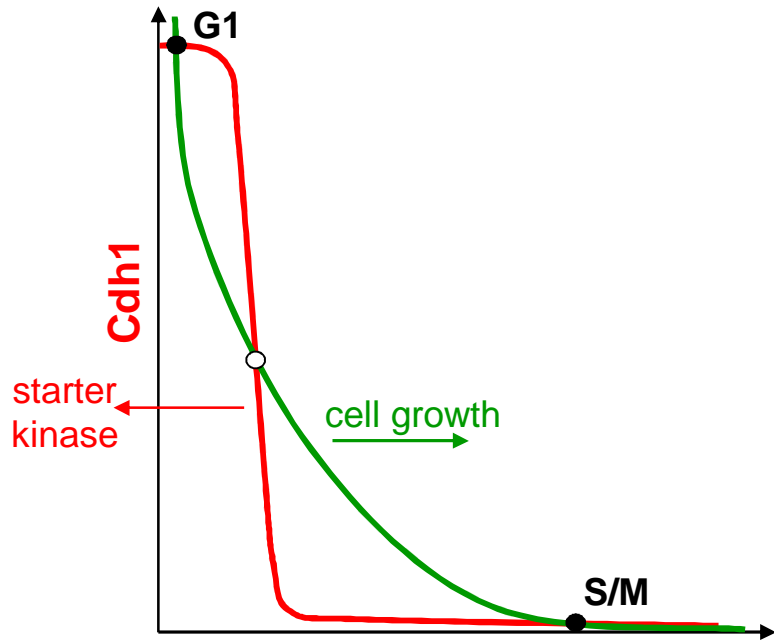
$$\frac{d \text{CycB}}{dt} = k_1 \cdot M - (k_2' + k_2'' \cdot \text{Cdh1}) \cdot \text{CycB}$$

$$\frac{d \text{Cdh1}}{dt} = \frac{k_3' \cdot (1 - \text{Cdh1})}{J_3 + 1 - \text{Cdh1}} - \frac{(k_4' + k_4'' \cdot \text{CycB}) \text{Cdh1}}{J_4 + \text{Cdh1}}$$

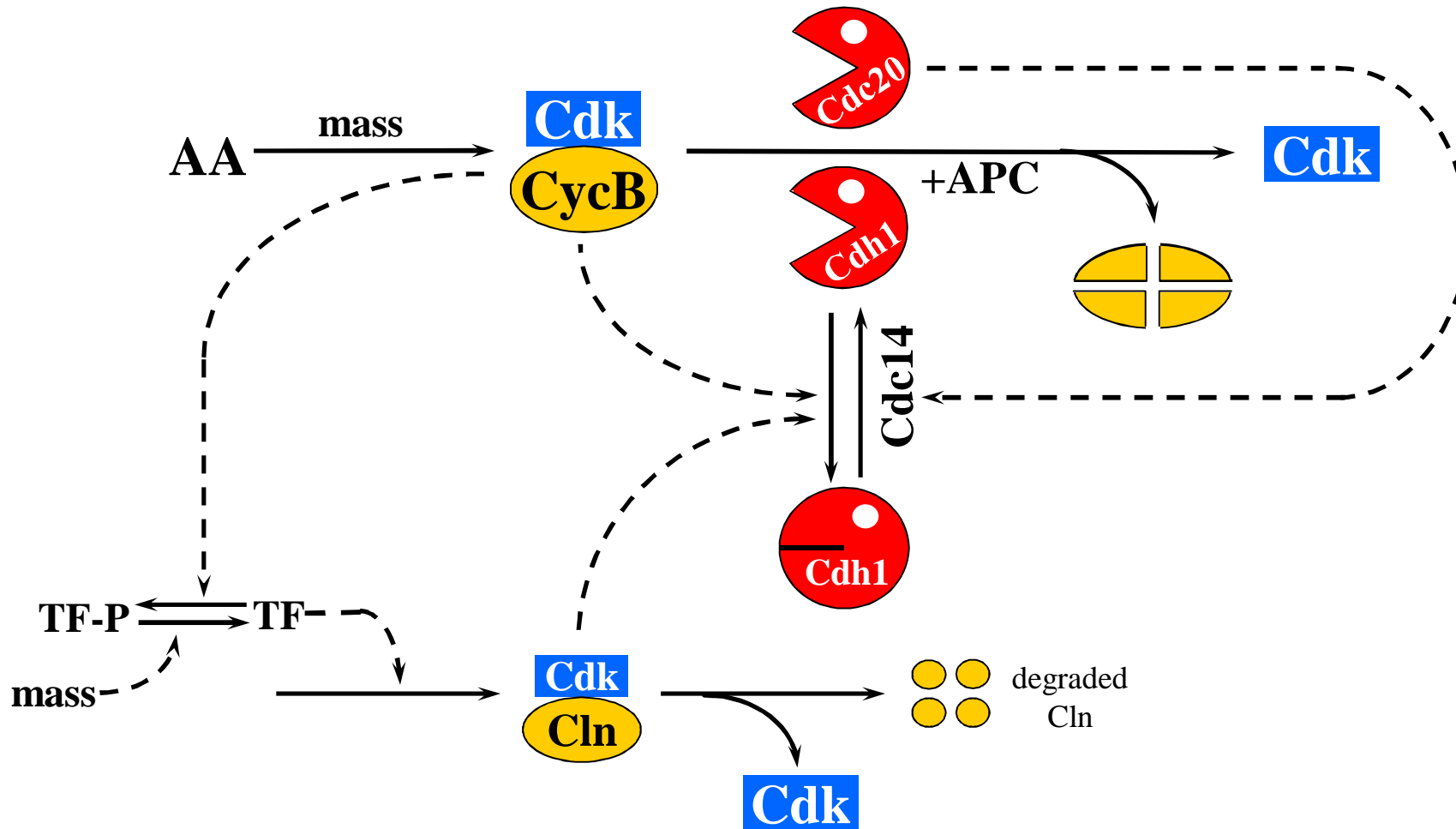


# Nullclines





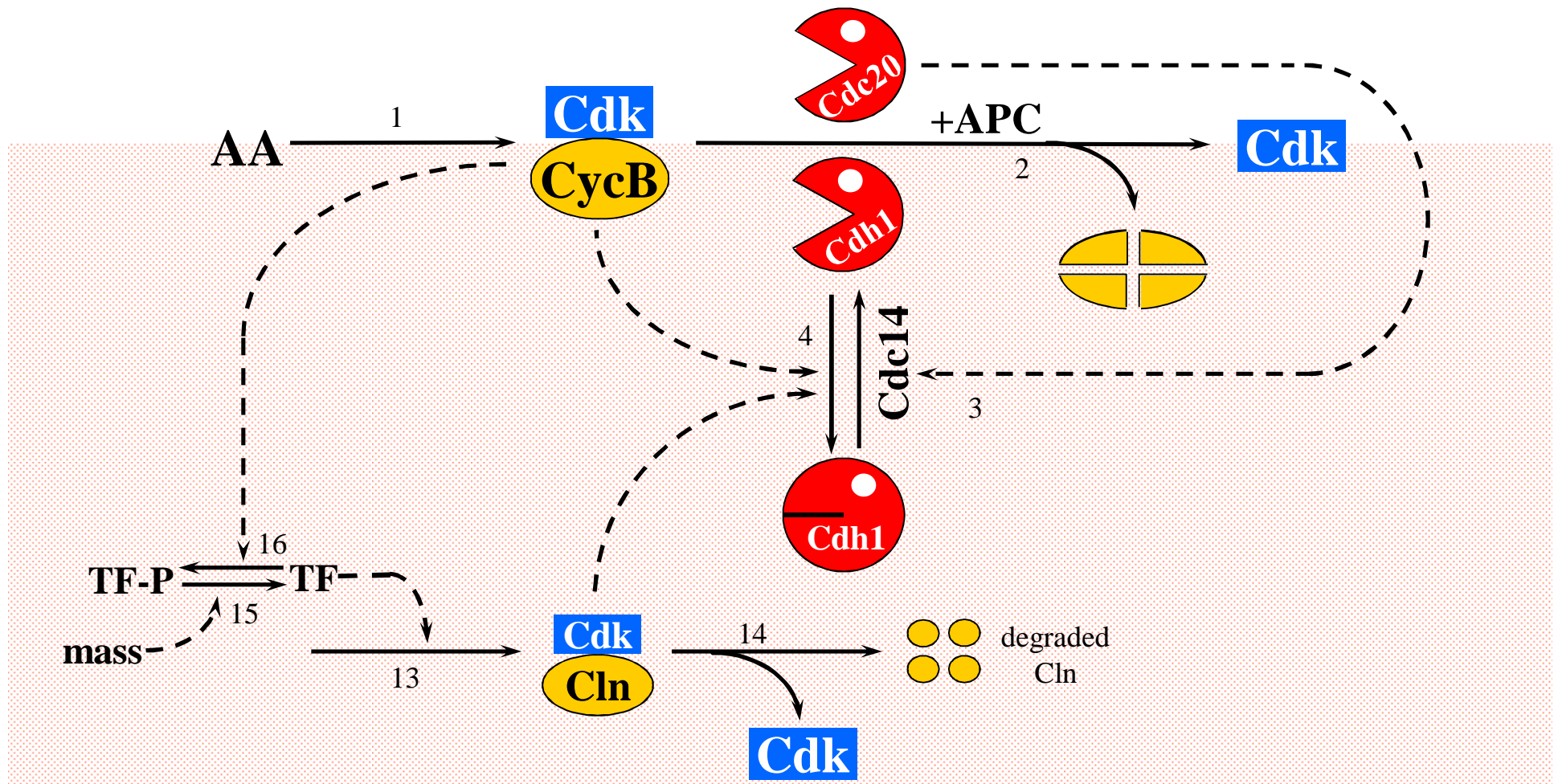
# Turning on/off the Cdk - Cdh1 switch

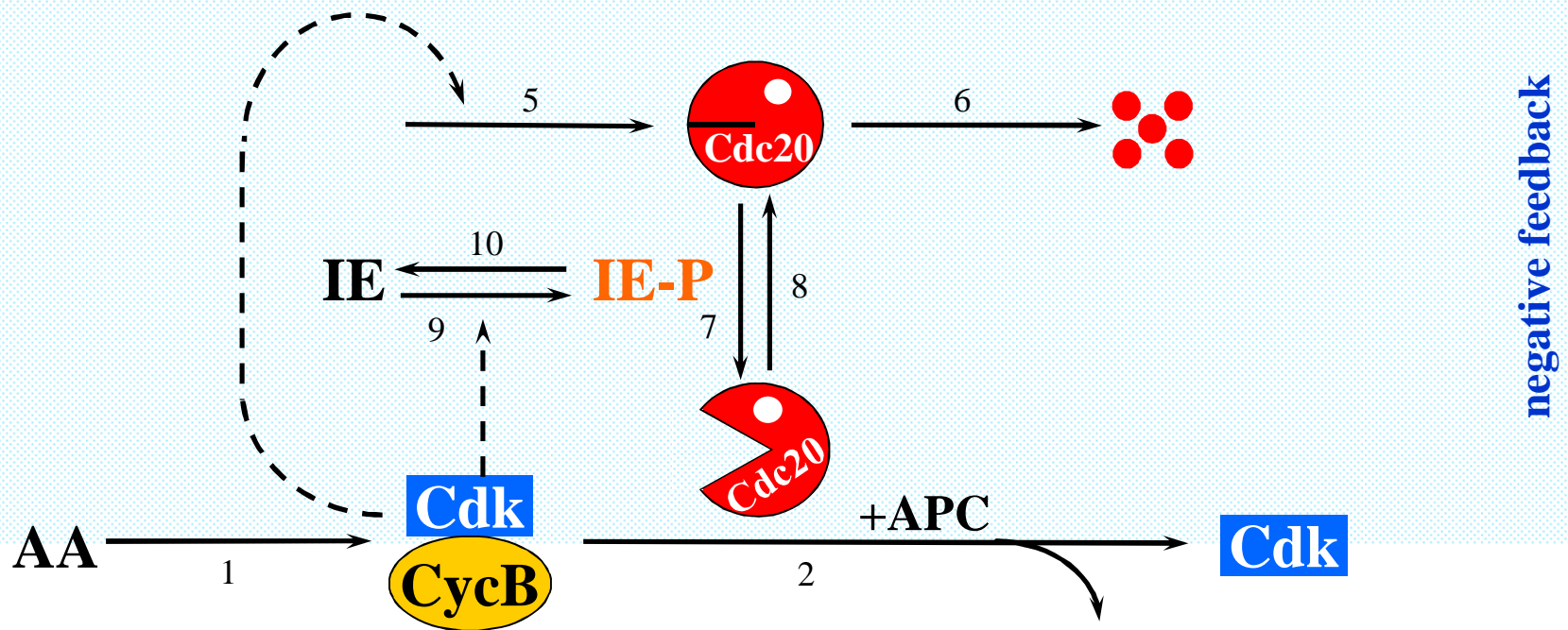


$$\frac{d \text{Cdh1}}{dt} = (k_3' + k_3'' \cdot \text{Cdc20}_A) \frac{1 - \text{Cdh1}}{J_3 + 1 - \text{Cdh1}} - (k_4' \cdot \text{SK} + k_4 \cdot \text{CycB}) \frac{\text{Cdh1}}{J_4 + \text{Cdh1}}$$

$$\frac{d \text{SK}}{dt} = k_{13}' + k_{13}'' \cdot \text{TF} - k_{14} \cdot \text{SK}$$

$$\frac{d \text{TF}}{dt} = k_{15}' \cdot \text{M} \frac{1 - \text{TF}}{J_{15} + 1 - \text{TF}} - (k_{16}' + k_{16}'' \cdot \text{CycB}) \frac{\text{TF}}{J_{16} + \text{TF}}$$





$$\frac{d \text{Cdc20}_T}{dt} = k_5' + \frac{k_5'' \cdot \text{CycB}^4}{J_5^4 + \text{CycB}^4} - k_6 \cdot \text{Cdc20}_T$$

$$\frac{d \text{Cdc20}_A}{dt} = k_7 \cdot \text{IE-P} \frac{\text{Cdc20}_T - \text{Cdc20}_A}{J_7 + \text{Cdc20}_T - \text{Cdc20}_A} - k_8 \cdot \frac{\text{Cdc20}_A}{J_8 + \text{Cdc20}_A} - k_6 \cdot \text{Cdc20}_A$$

$$\frac{d \text{IE-P}}{dt} = k_9 \cdot \text{CycB} \frac{1 - \text{IE-P}}{J_9 + 1 - \text{IE-P}} - k_{10} \cdot \frac{\text{IE-P}}{J_{10} + \text{IE-P}}$$



$$\frac{d\text{CycB}}{dt} = k_1 \cdot M - (k_2' + k_2'' \cdot \text{Cdh1} + k_2''' \cdot \text{Cdc20}_A) \text{CycB}$$

$$\frac{d \text{Cdh1}}{dt} = (k_3' + k_3'' \cdot \text{Cdc20}_A) \frac{1-\text{Cdh1}}{J_3+1-\text{Cdh1}} - (k_4' \cdot \text{SK} + k_4 \cdot \text{CycB}) \frac{\text{Cdh1}}{J_4 + \text{Cdh1}}$$

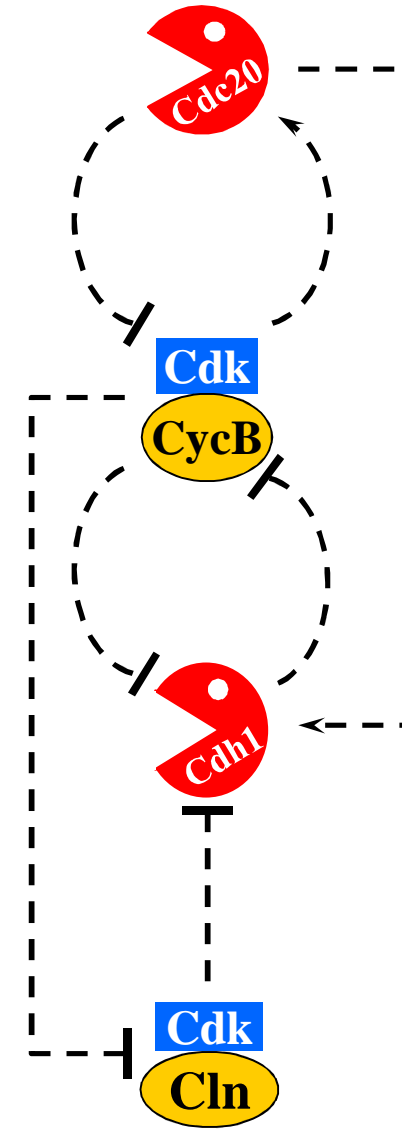
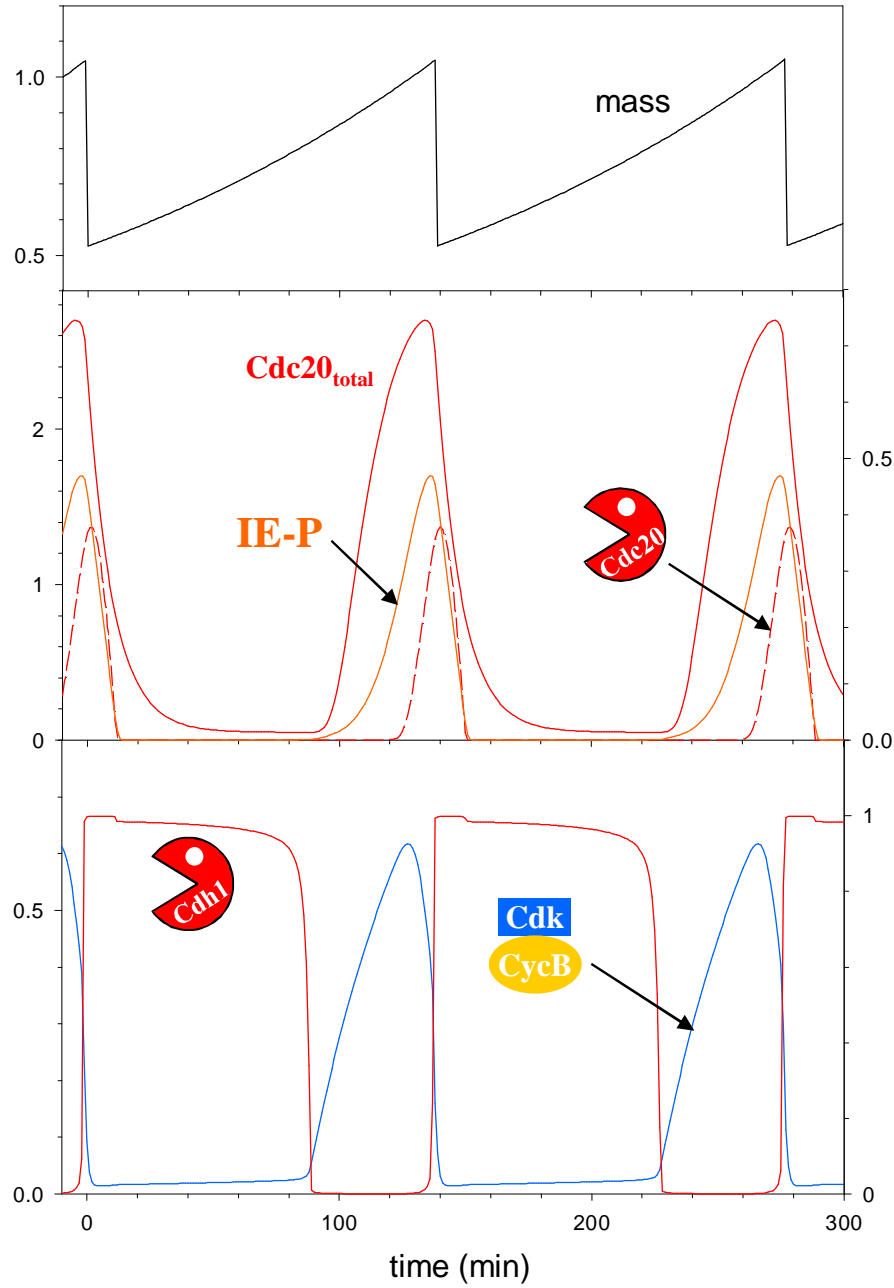
$$\frac{d \text{SK}}{dt} = k_{13}' + k_{13}'' \cdot \text{TF} - k_{14} \cdot \text{SK}$$

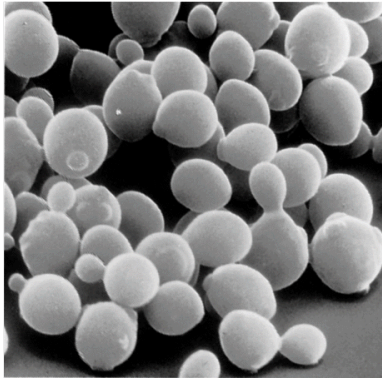
$$\frac{d \text{TF}}{dt} = k_{15}' \cdot M \frac{1 - \text{TF}}{J_{15} + 1 - \text{TF}} - (k_{16}' + k_{16}'' \cdot \text{CycB}) \frac{\text{TF}}{J_{16} + \text{TF}}$$

$$\frac{d \text{Cdc20}_T}{dt} = k_5' + \frac{k_5'' \cdot \text{CycB}^4}{J_5^4 + \text{CycB}^4} - k_6 \cdot \text{Cdc20}_T$$

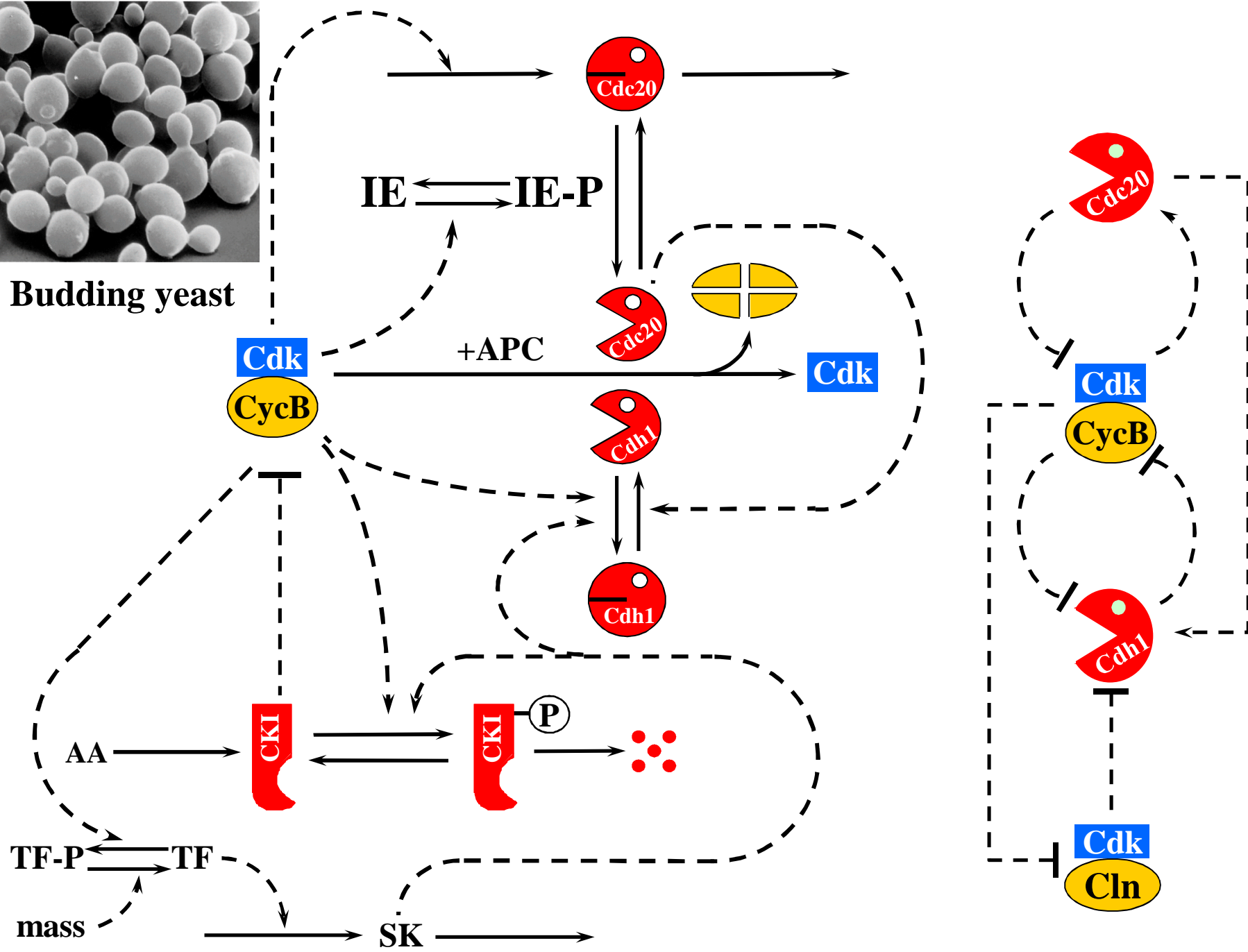
$$\frac{d\text{Cdc20}_A}{dt} = k_7 \cdot \text{IE-P} \frac{\text{Cdc20}_T - \text{Cdc20}_A}{J_7 + \text{Cdc20}_T - \text{Cdc20}_A} - k_8 \cdot \frac{\text{Cdc20}_A}{J_8 + \text{Cdc20}_A} - k_6 \cdot \text{Cdc20}_A$$

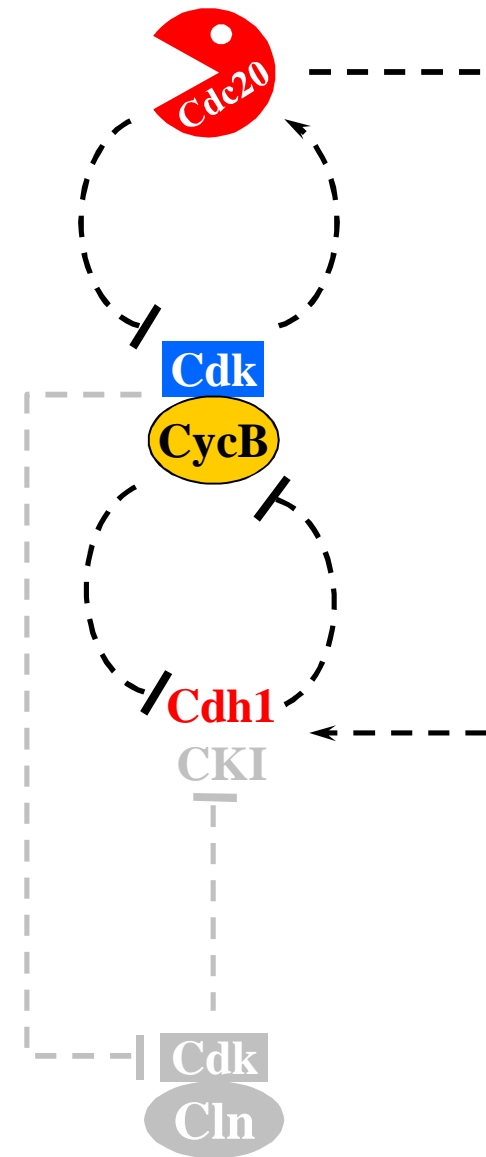
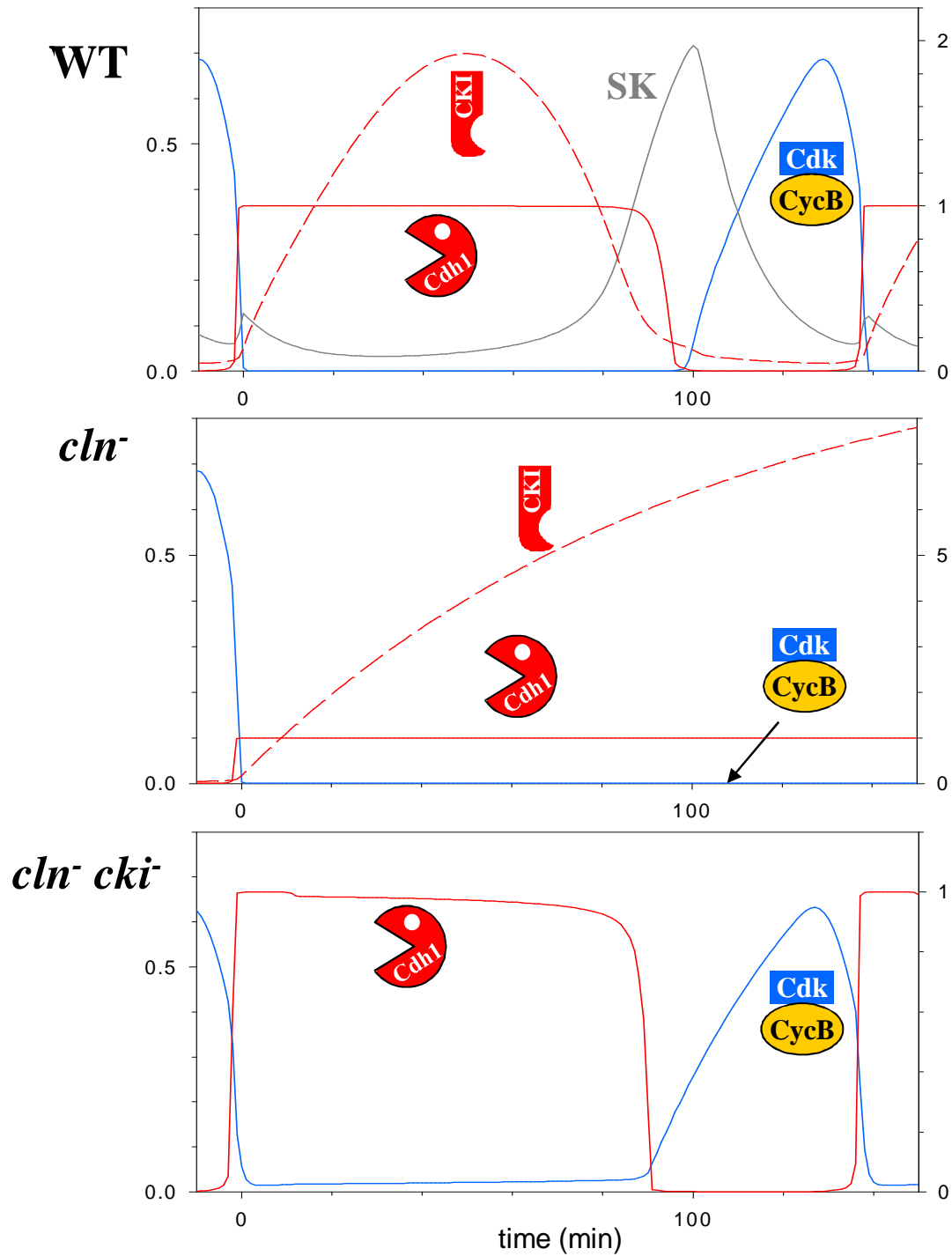
$$\frac{d \text{IE-P}}{dt} = k_9 \cdot \text{CycB} \frac{1 - \text{IE-P}}{J_9 + 1 - \text{IE-P}} - k_{10} \cdot \frac{\text{IE-P}}{J_{10} + \text{IE-P}}$$





Budding yeast





# What makes us believe that our set of equations and parameters are close to life?

Test the model! Compare the results to life:

What can be easy to measure?

- Length of different cell cycle phases
- Cell size at different transitions

Not only wild type, but many mutants!

One wild type parameter set. To simulate mutants use this as a basal parameter set.

Molecular Biology of the Cell  
Vol. 15, 3841–3862, August 2004

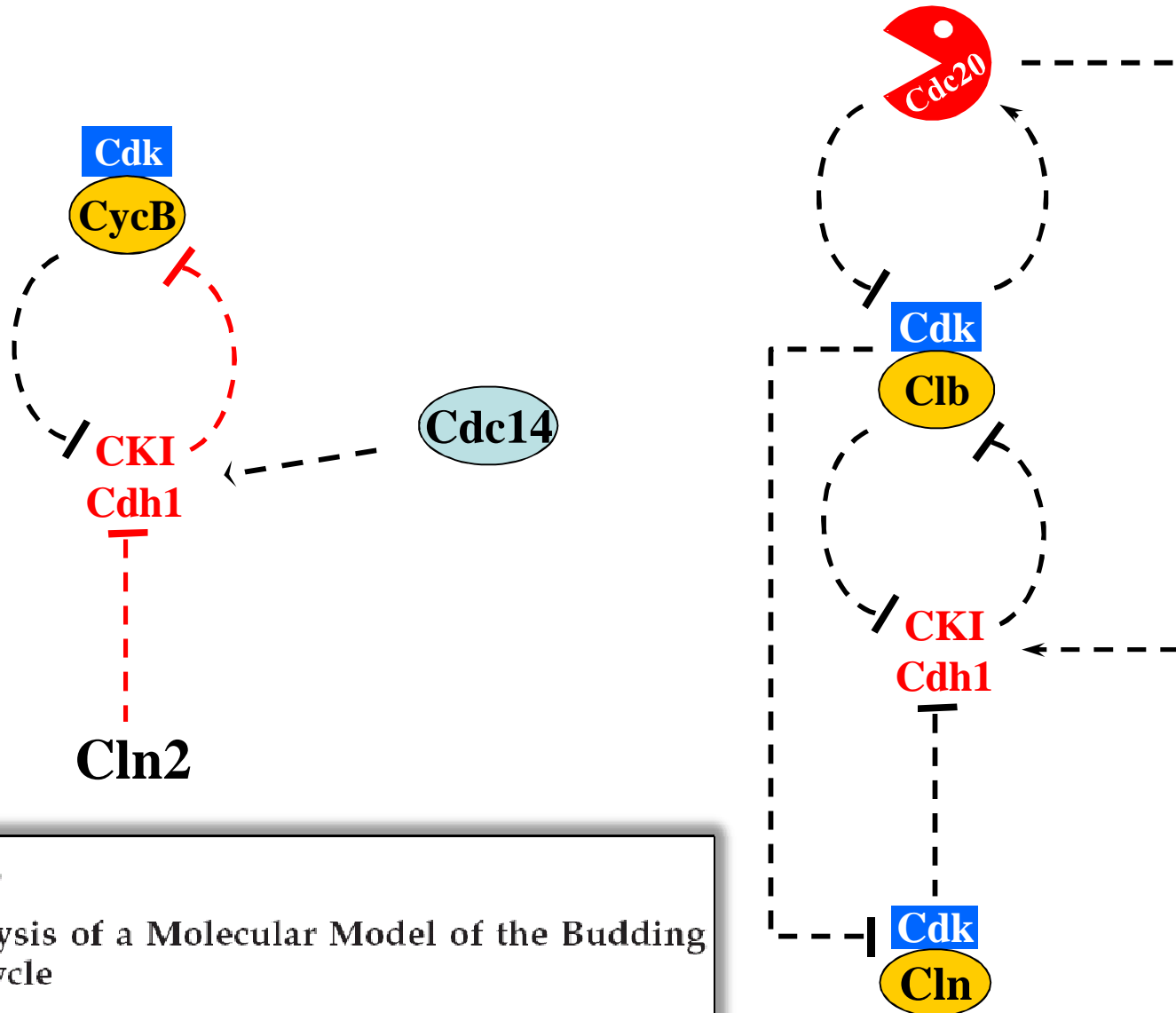
## Integrative Analysis of Cell Cycle Control in Budding Yeast<sup>D</sup>

Katherine C. Chen,<sup>\*†</sup> Laurence Calzone,<sup>\*</sup> Attila Csikasz-Nagy,<sup>‡</sup>  
Frederick R. Cross,<sup>§</sup> Bela Novak,<sup>‡</sup> and John J. Tyson<sup>\*†</sup>

Strain	$T_{G1}$ (min)	Changed parameter	Comments (Experimental results in boldface type)
44 (C)	84		CT 146 min (time of occurrence of event) Richardson, 1989, inviable
50	25	$D_{s3} = 0$ $k_{s,n2}^* = 20$	Schwob, 1993, Fig. 2, SBF activated soon after galactose induction
70	25	$D_{s3} = 0$ $k_{s,n2}^* = 0.1$	Cross, 1991, Fig. 4, <i>GAL-CLN1</i> induces <i>cln2</i>
52	98	$D_{s3} = k_{s,n2}^* = 0$ $k_{s,n2}^* = 0.012$ $k_{s,n5}^* = 0.04$	Epstein, 1992, viable
85	88	$D_{s3} = k_{s,n2}^* = 0$ $k_{s,n5}^* = 0.1$	Schwob, 1993, Fig. 6, viable
7		<i>GAL-CLB5</i> <i>cln1 cln2 cln3</i> 0.71 2.57 [5.49] [6.64] [7.38] <i>GAL-CLB2</i>	Amon, 1994, Fig. 8, G1 arrest We consider cells arrested in G1 if mass at DNA replication exceeds 5
8		<i>cln1 cln2 cln3</i> 1.30 2.12 1.50 2.24 3.01 24 <i>sic1</i> (85') (24') (94') (146')	Tyers, 1996, Fig. 2, viable but large
9		<i>cln1 cln2 cln3</i> 0.71 2.48 4.62 5.03 No mit <i>hct1</i>	Schwab, 1997, Fig. 3, M phase arrest
10		<i>cln1 cln2 cln3</i> 0.71 2.48 4.16 4.60 No mit <i>apc-ts</i>	Irniger, 1997, Fig. 2, M phase arrest

Current Budding yeast model 131 mutants  
Current Fission yeast model 59 mutants

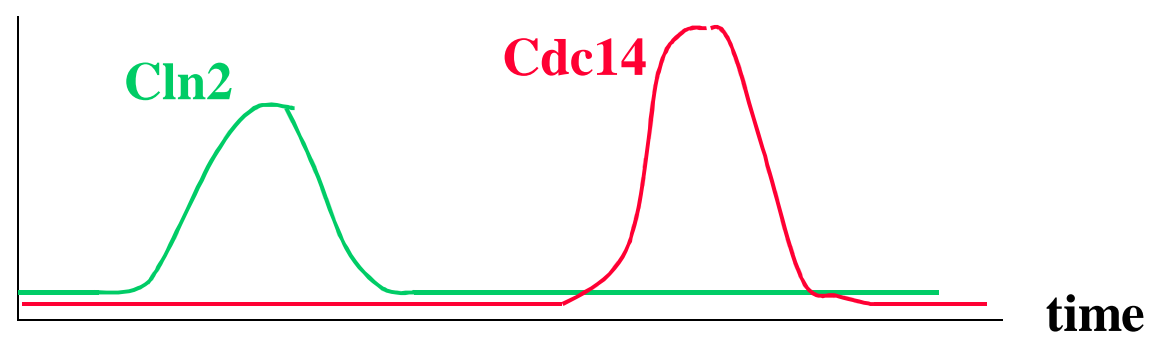
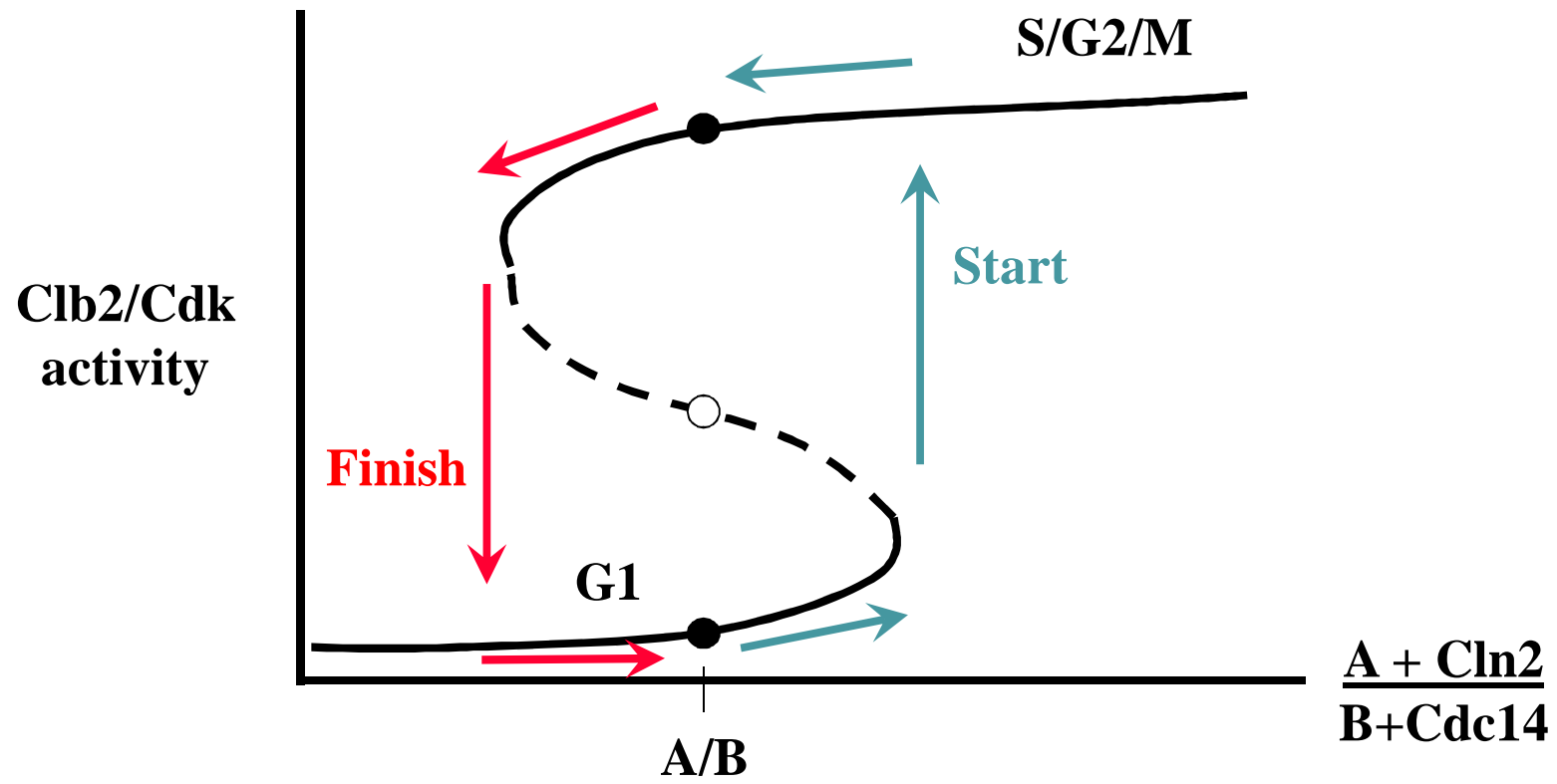
# Hysteresis in budding yeast cell cycle?

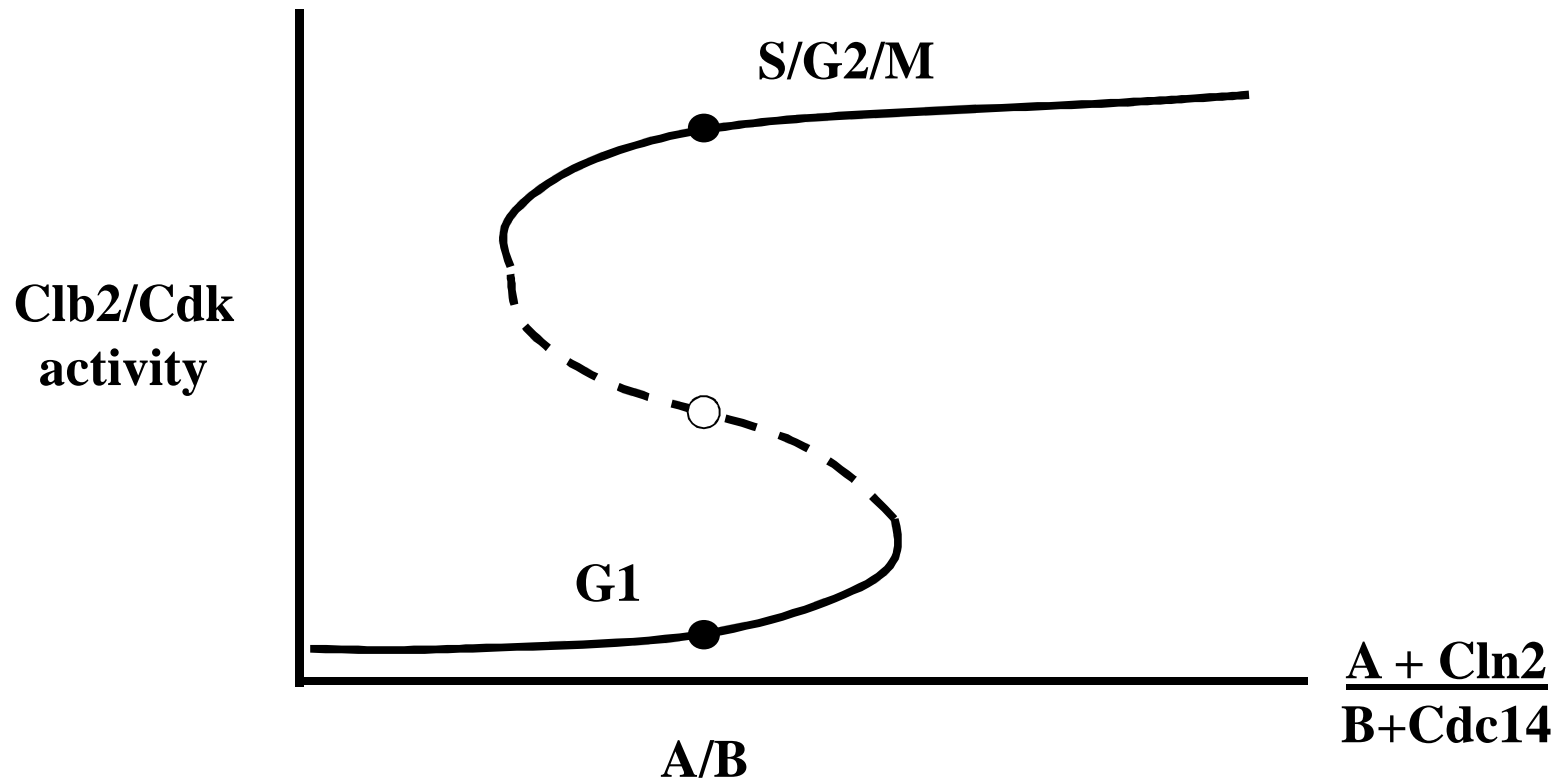


Molecular Biology of the Cell  
Vol. 11, 369-391, January 2000

## Kinetic Analysis of a Molecular Model of the Budding Yeast Cell Cycle

Katherine C. Chen,\* Attila Csikasz-Nagy,<sup>†</sup> Bela Gyorfy,<sup>†</sup> John Val,\*  
Bela Novak,<sup>†</sup> and John J. Tyson\*<sup>‡</sup>





“Neutral” 



# Protocol to demonstrate hysteresis



Fred Cross

Genotype:  $cln1\Delta cln2\Delta cln3\Delta GAL-CLN3 cdc14^{ts}$

Knockout all  
the G1cyclins

Turn on *CLN3*  
with galactose;  
turn off with  
glucose

Temperature-  
sensitive allele of  
*CDC14*: on at 23°C,  
off at 37°C.

**“Neutral” conditions: glucose at 37°C (no Cln’s, no Cdc14)**

Molecular Biology of the Cell  
Vol. 13, 52-70, January 2002

## Testing a Mathematical Model of the Yeast Cell Cycle

Frederick R. Cross,\* Vincent Archambault, Mary Miller, and  
Martha Klovstad

Start with all cells in G1

23°C

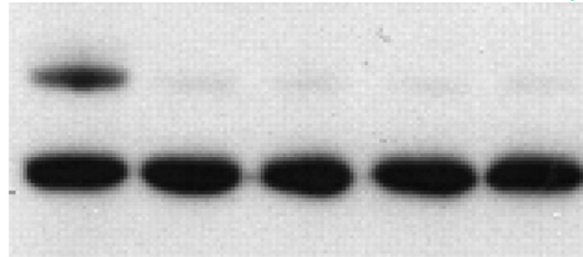
R+G

R = raffinose  
G = galactose

G R 0 0.5 1.0 hr

Make some Cln

Clb2p  
\*



Standard for protein loading

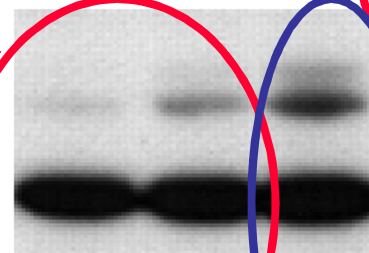
45 88 88 91 46 % Unbudded

G1 cells

Shift to neutral

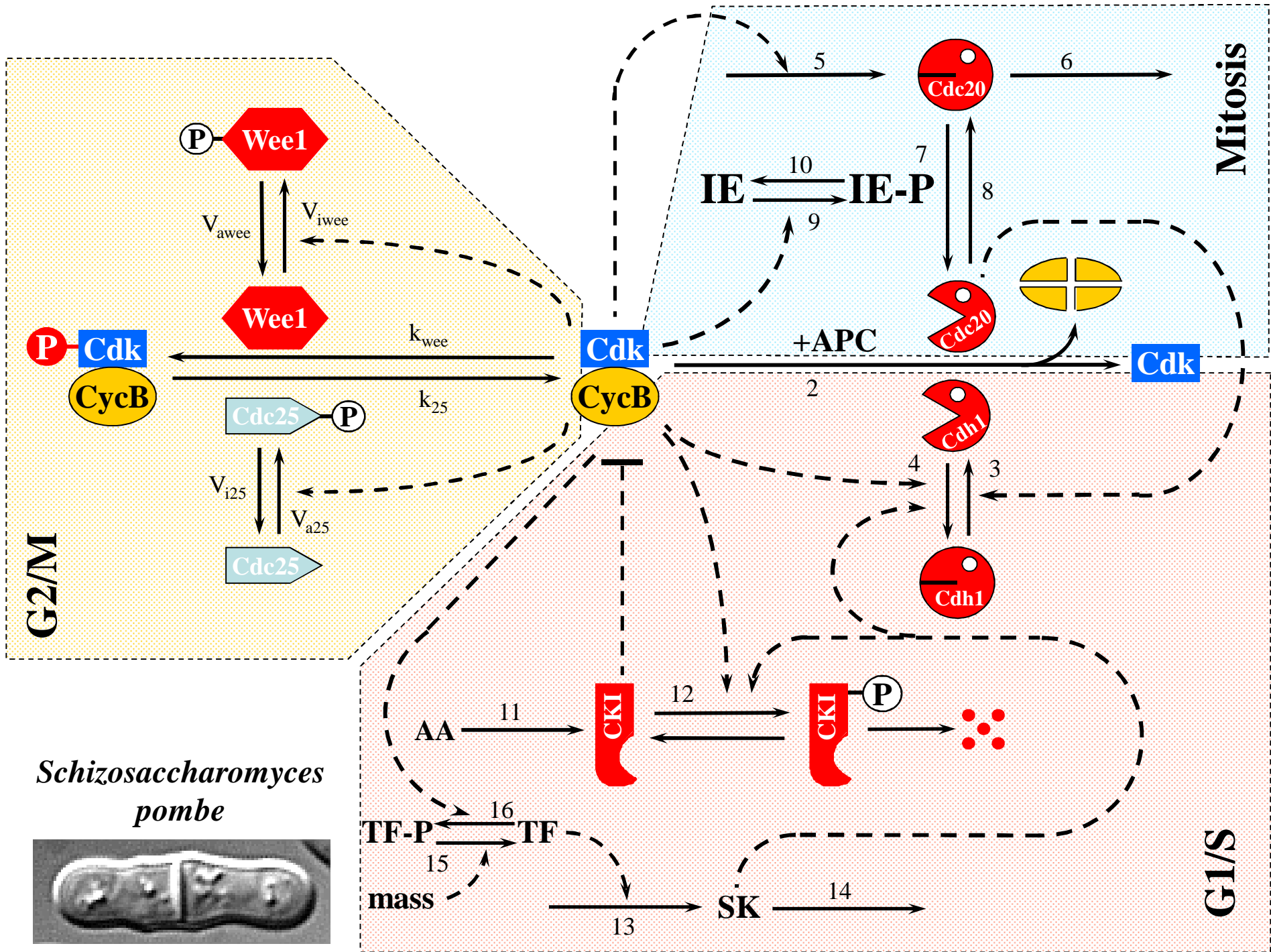
+ 2.5 hr  
Glucose  
37°C

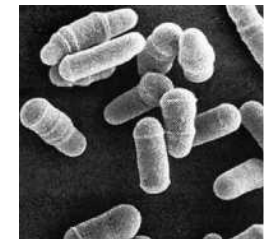
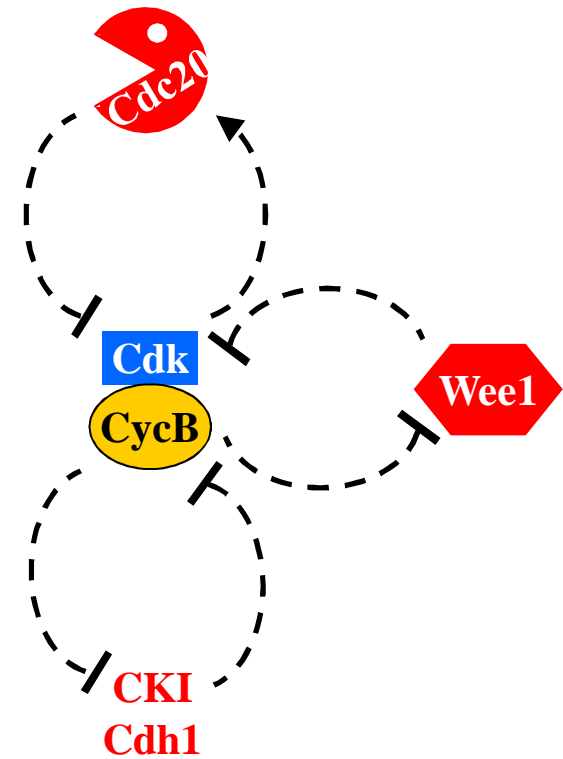
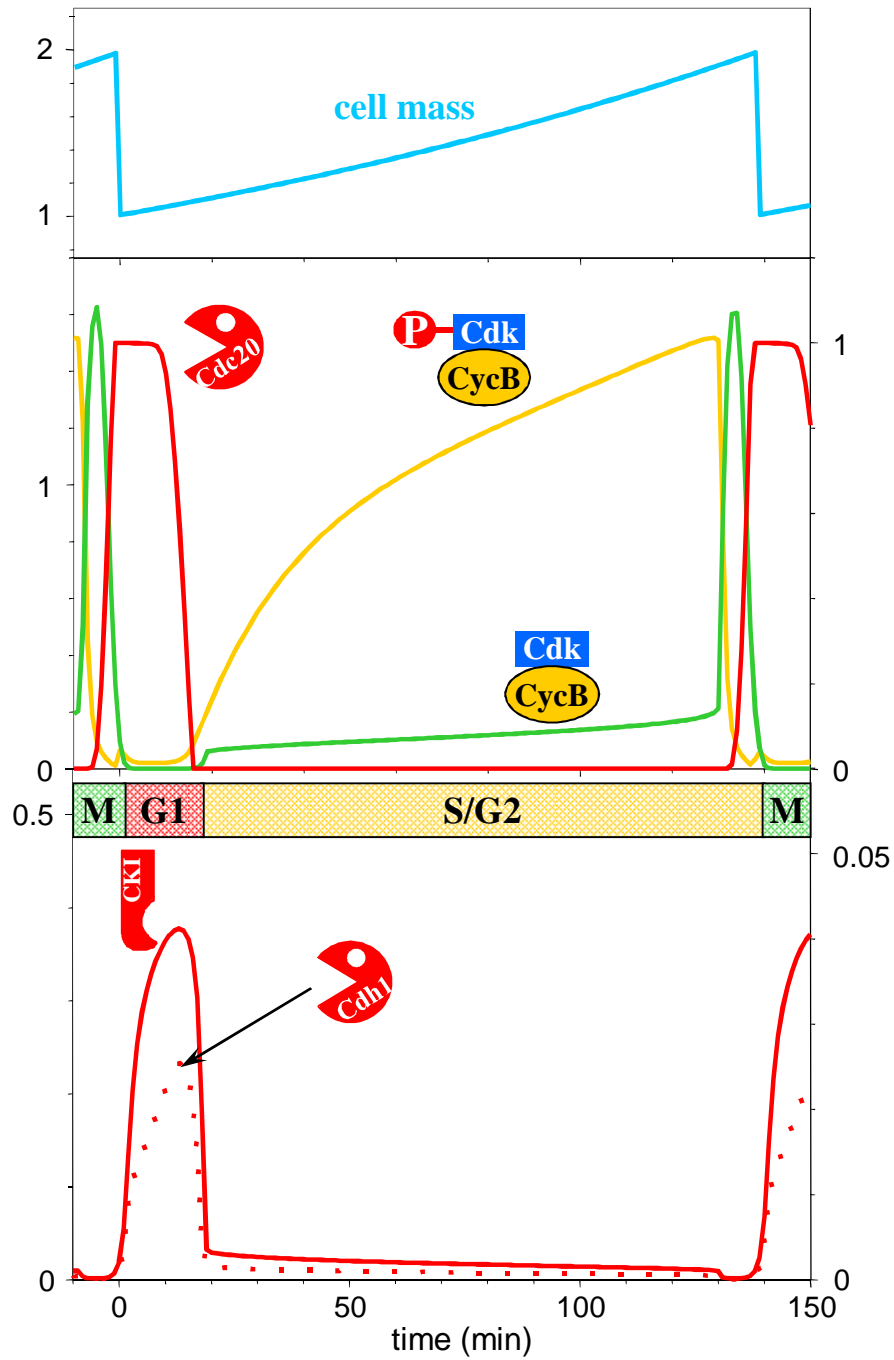
Clb2p  
\*



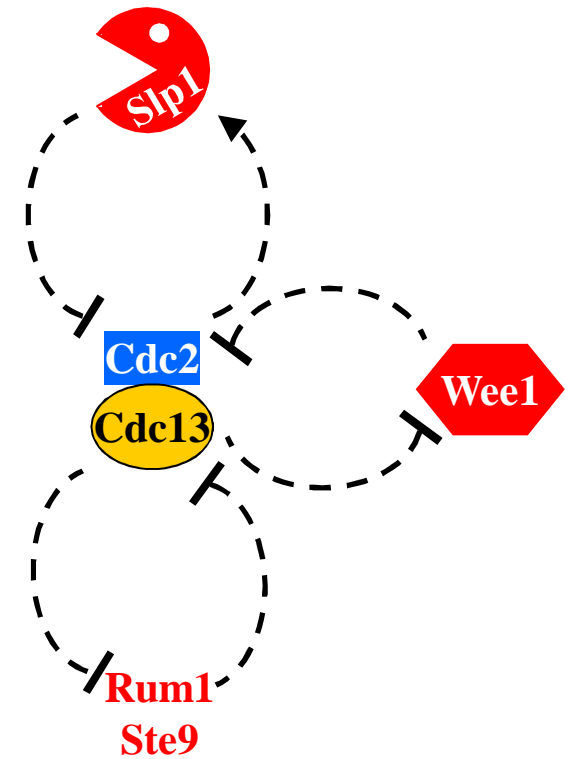
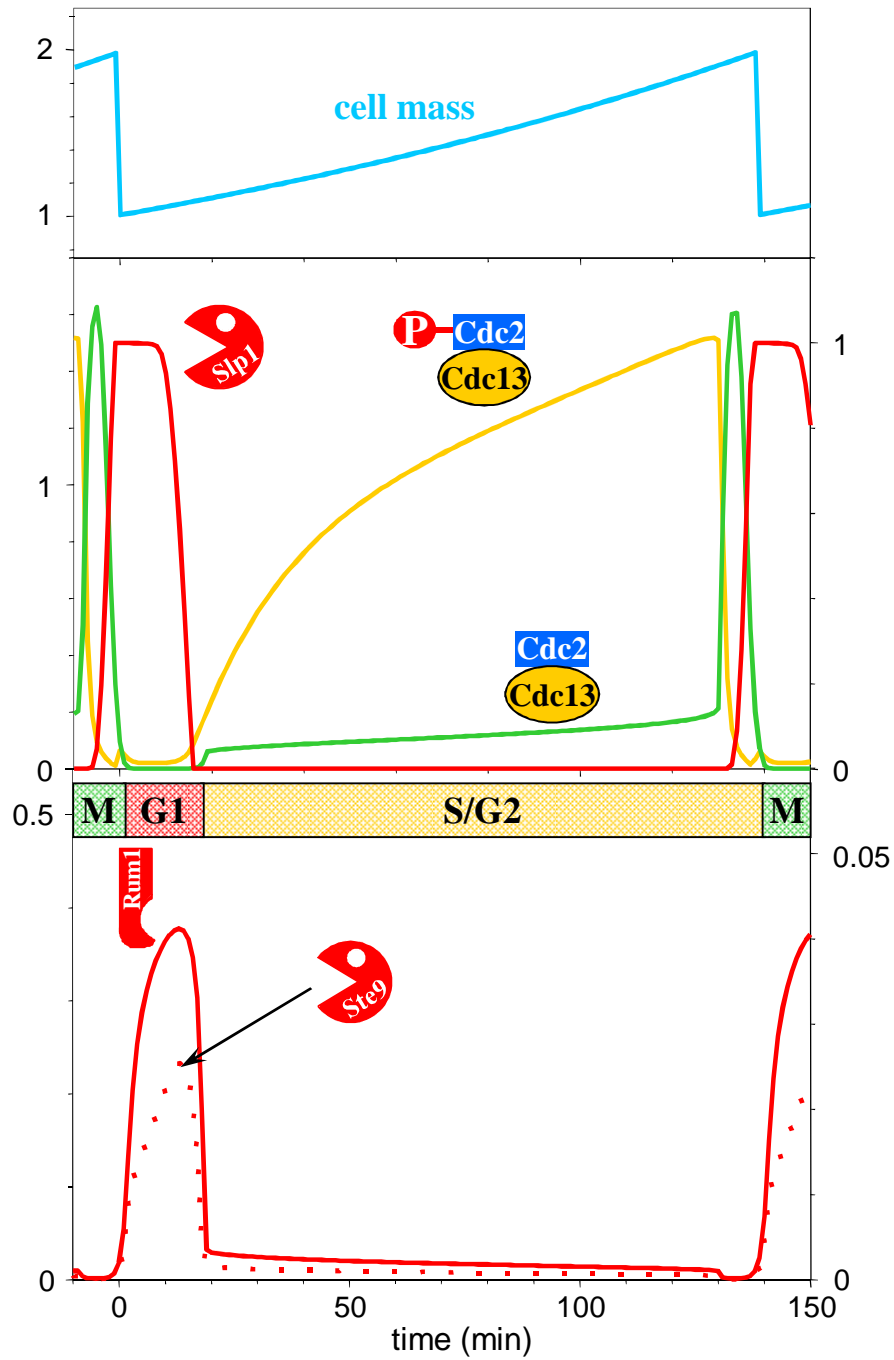
S/G2/M cells

85 86 26 % Unbudded





**Fission yeast**



**Fission yeast**

# Bifurcation diagram for cell cycle regulation (*Signal – response curve*)

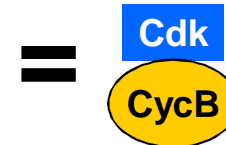
Bifurcation parameter (signal):

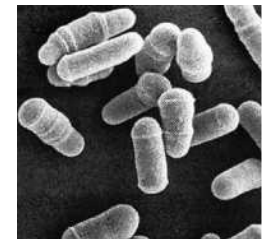
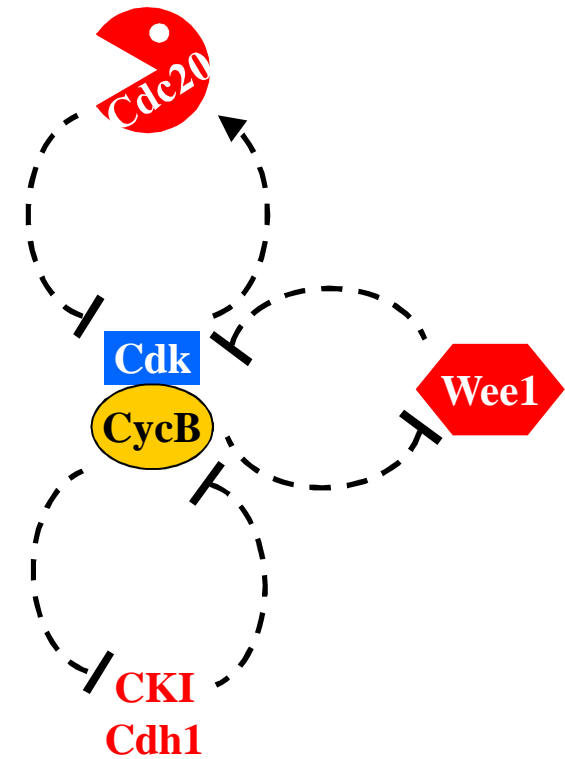
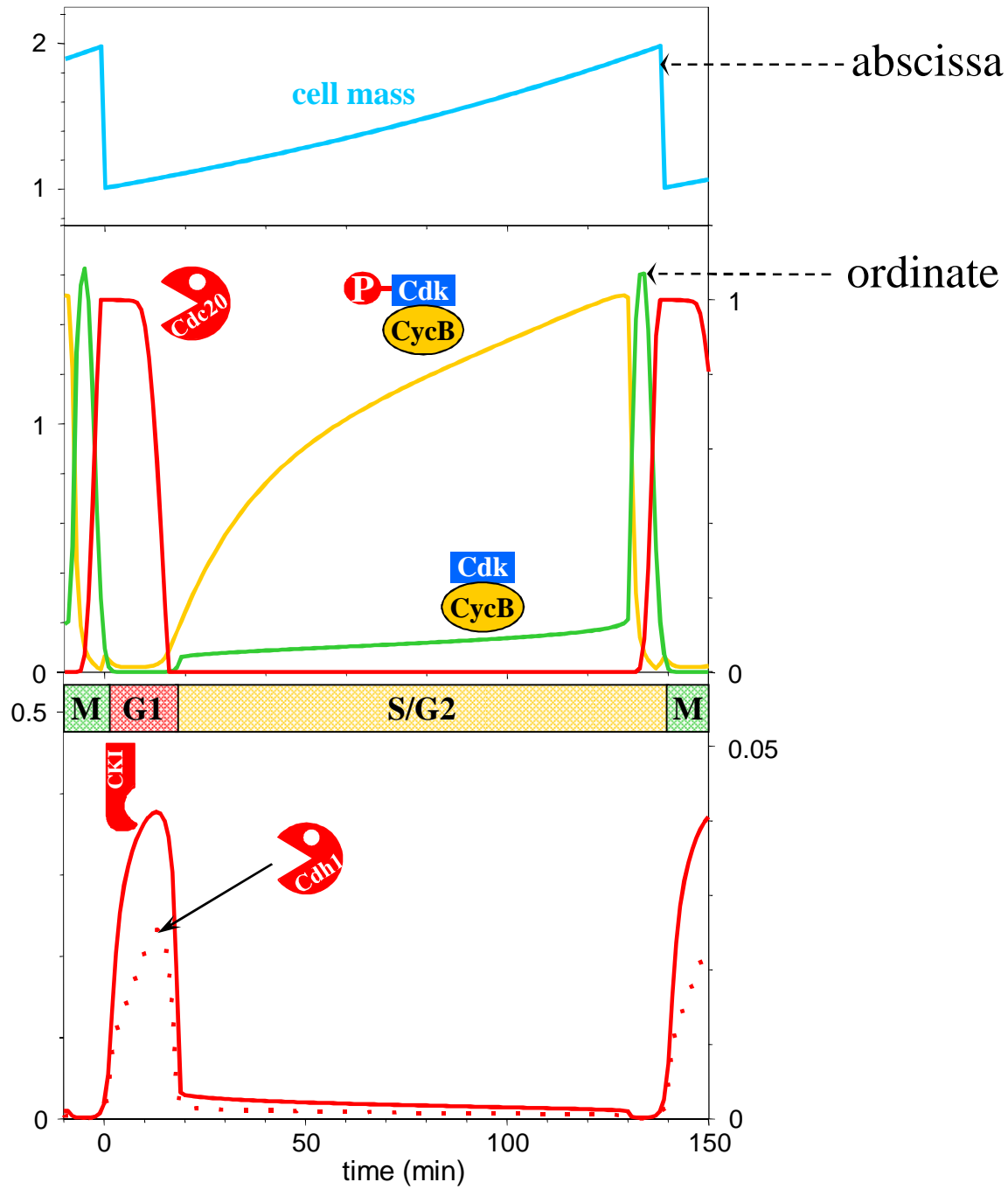
cell mass

(reports environmental conditions)

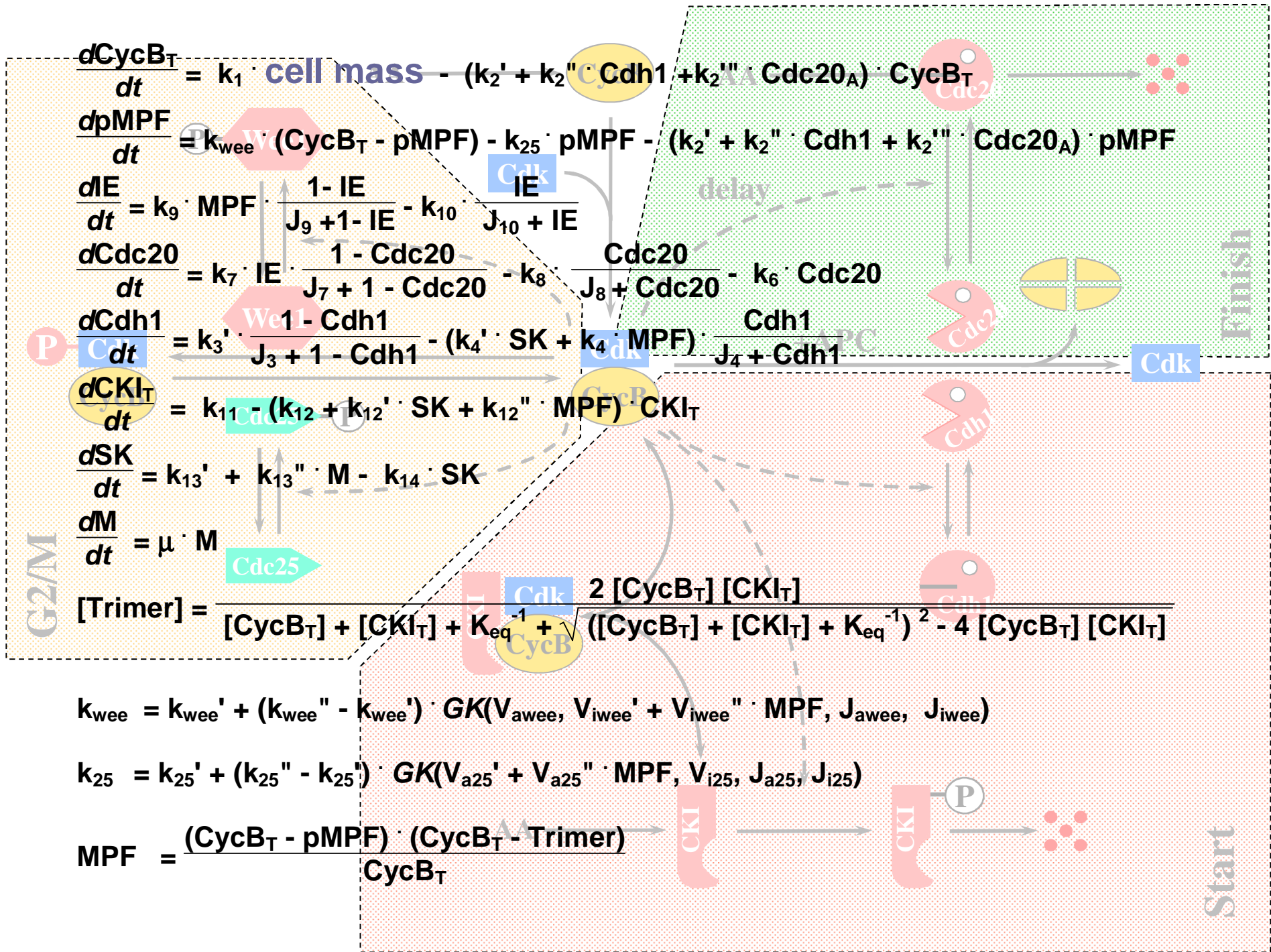
State variable (response):

main controller of the system

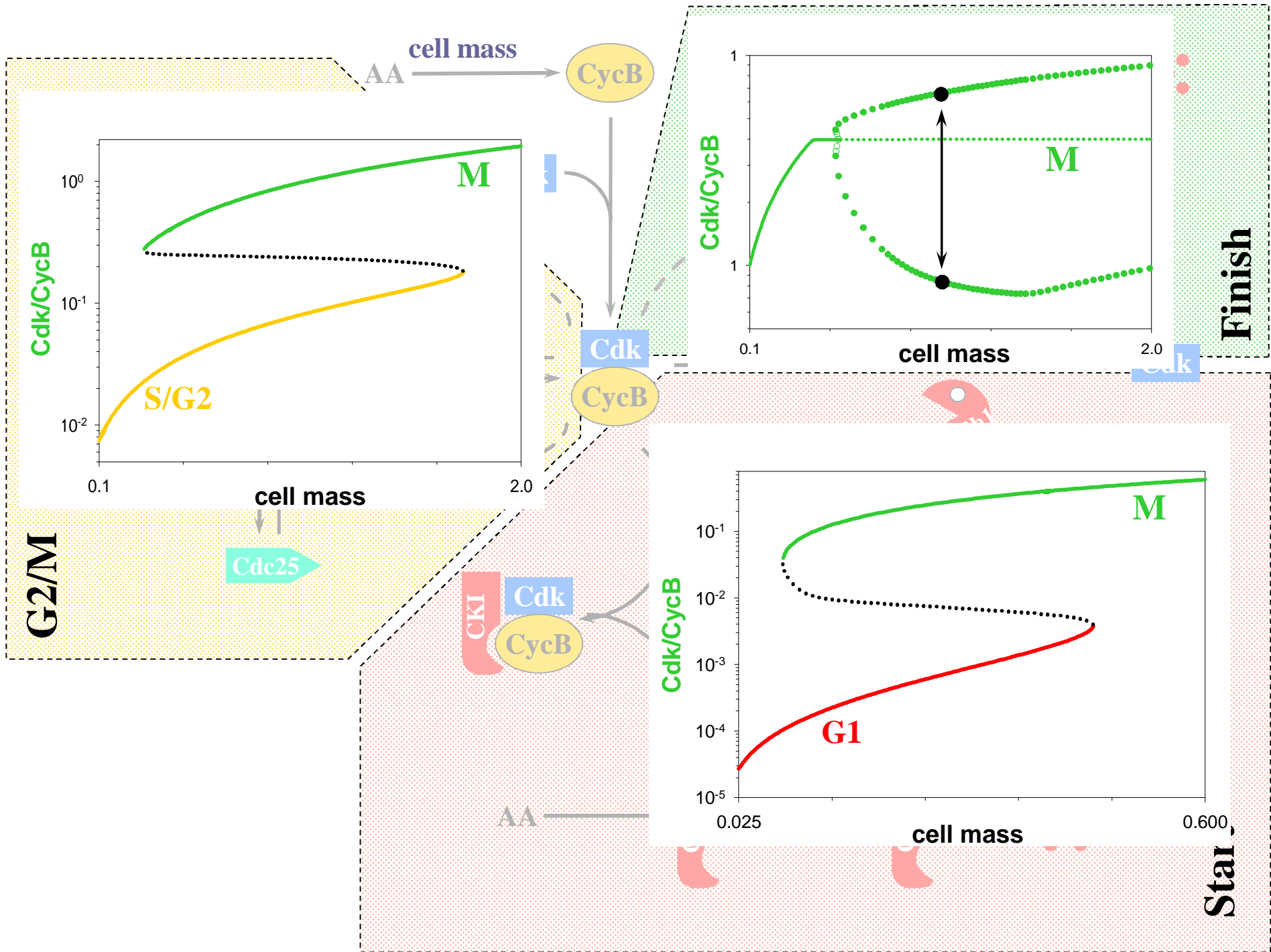


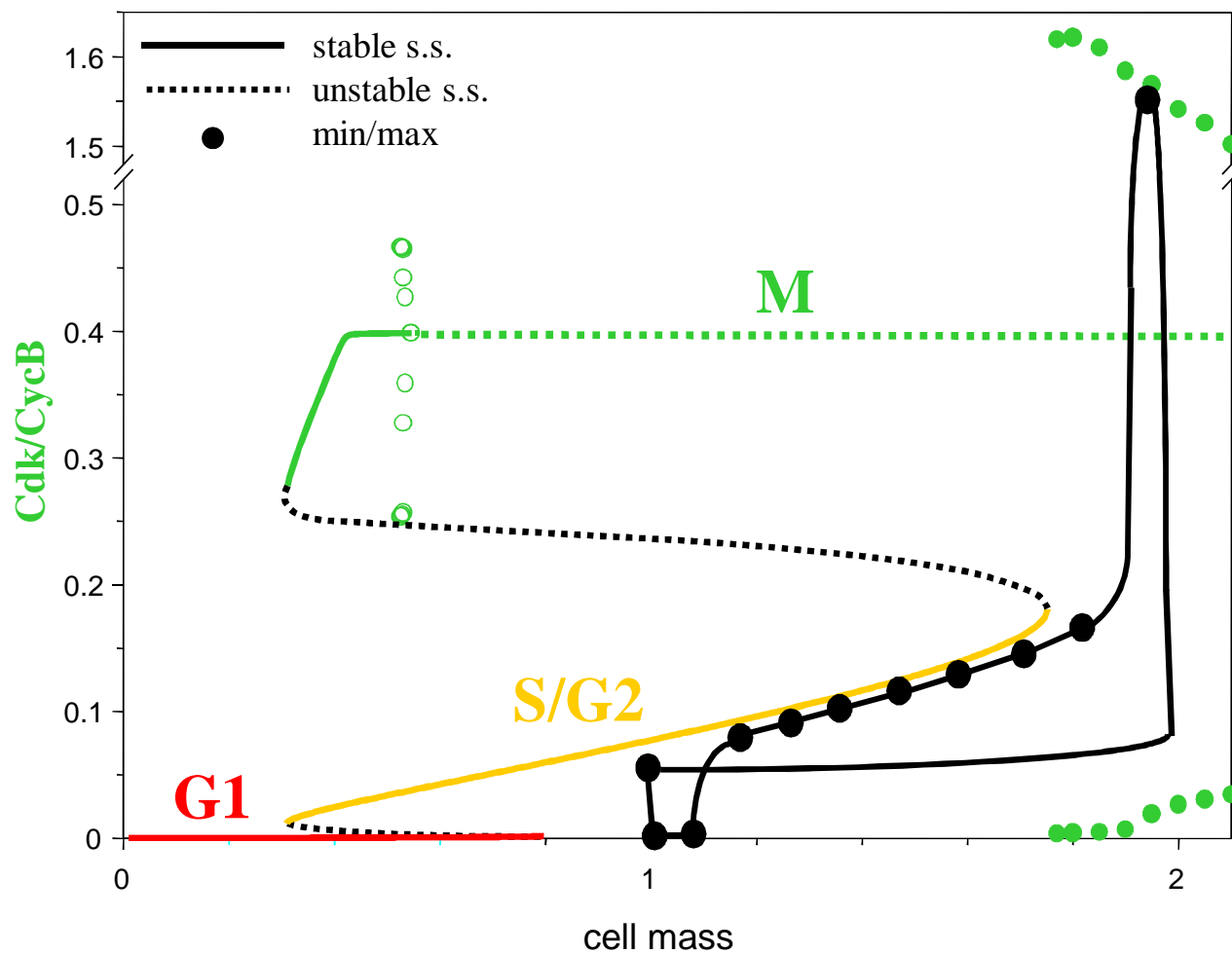
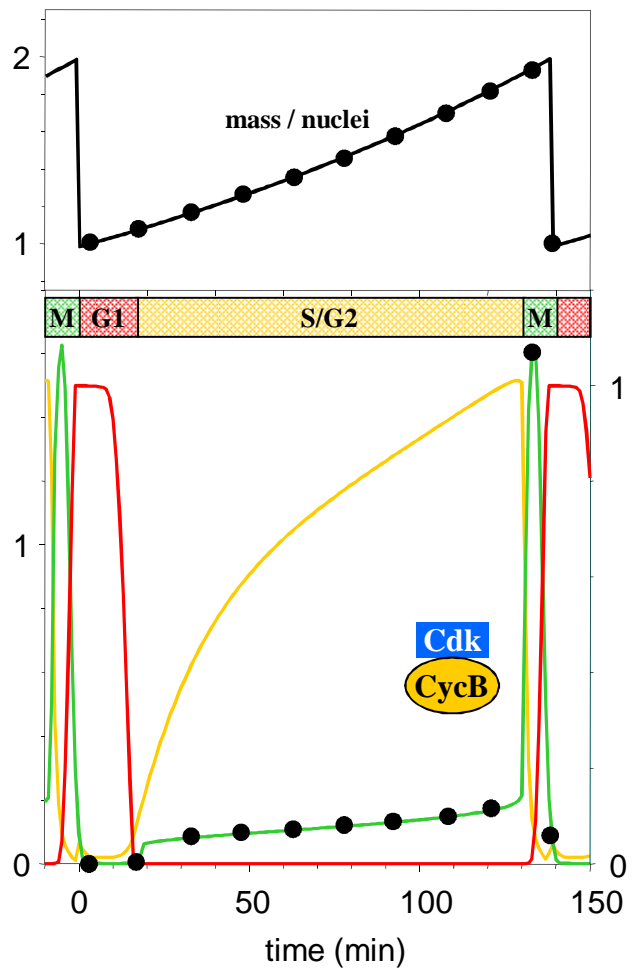


Fission yeast

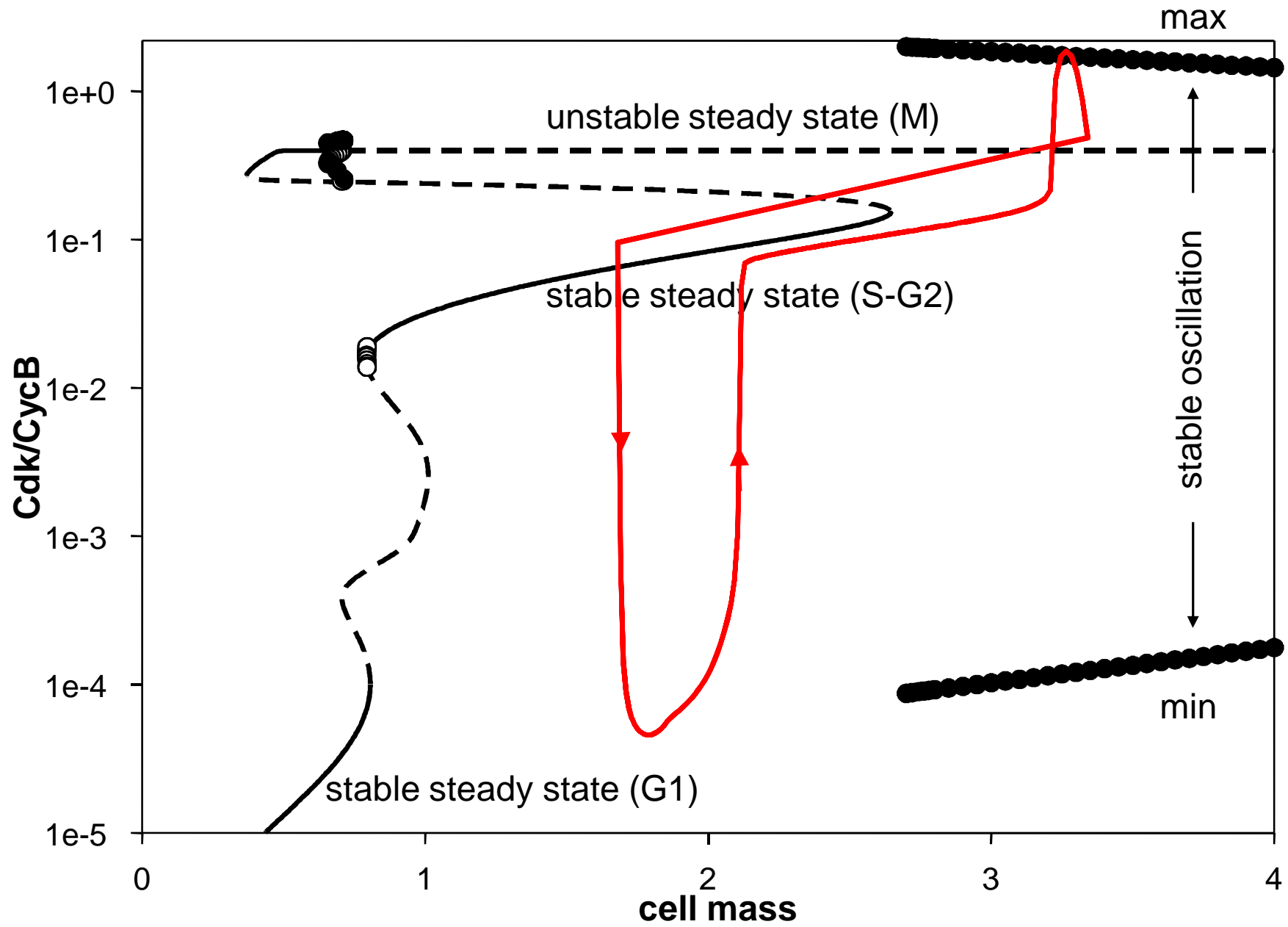




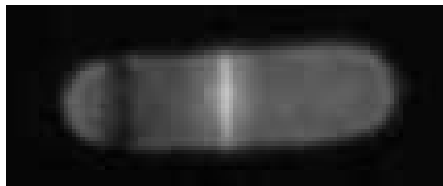




# Fission yeast – wild type

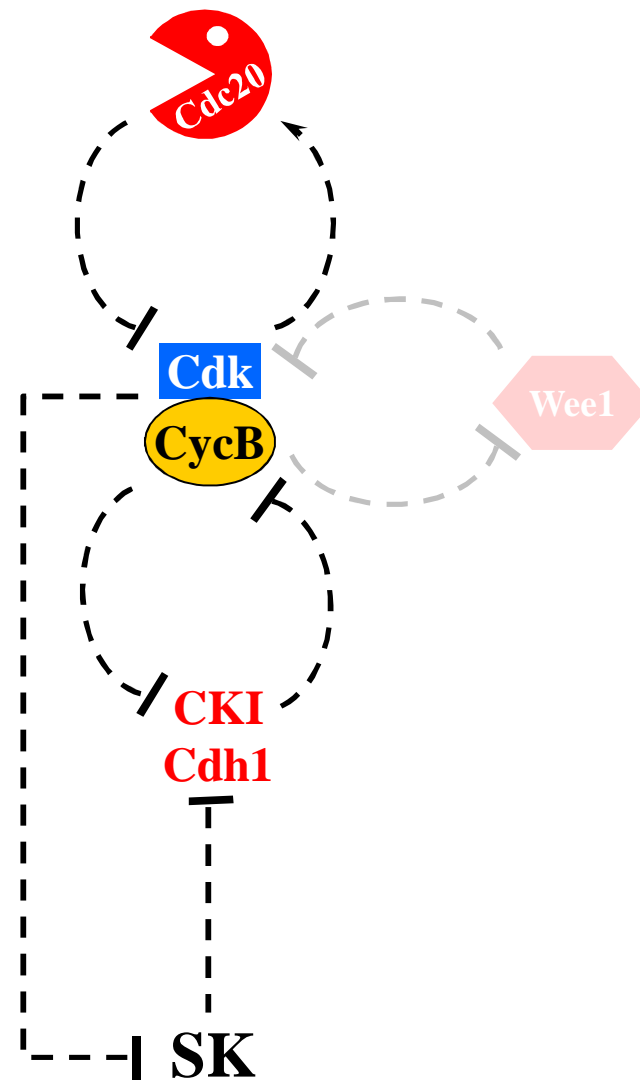
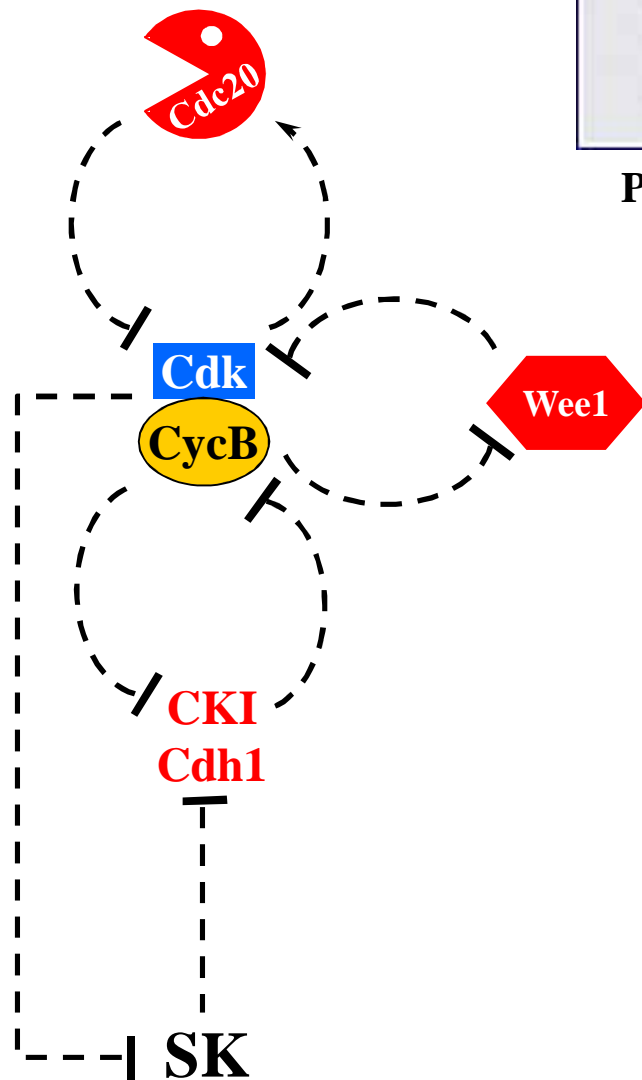


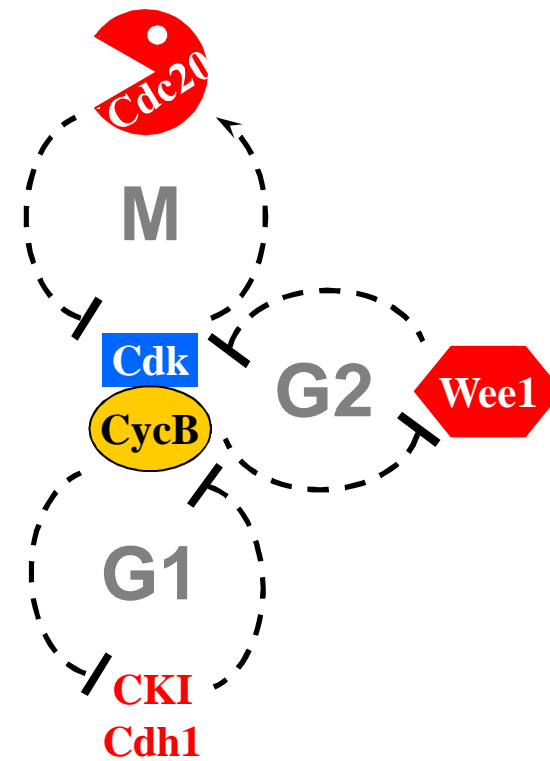
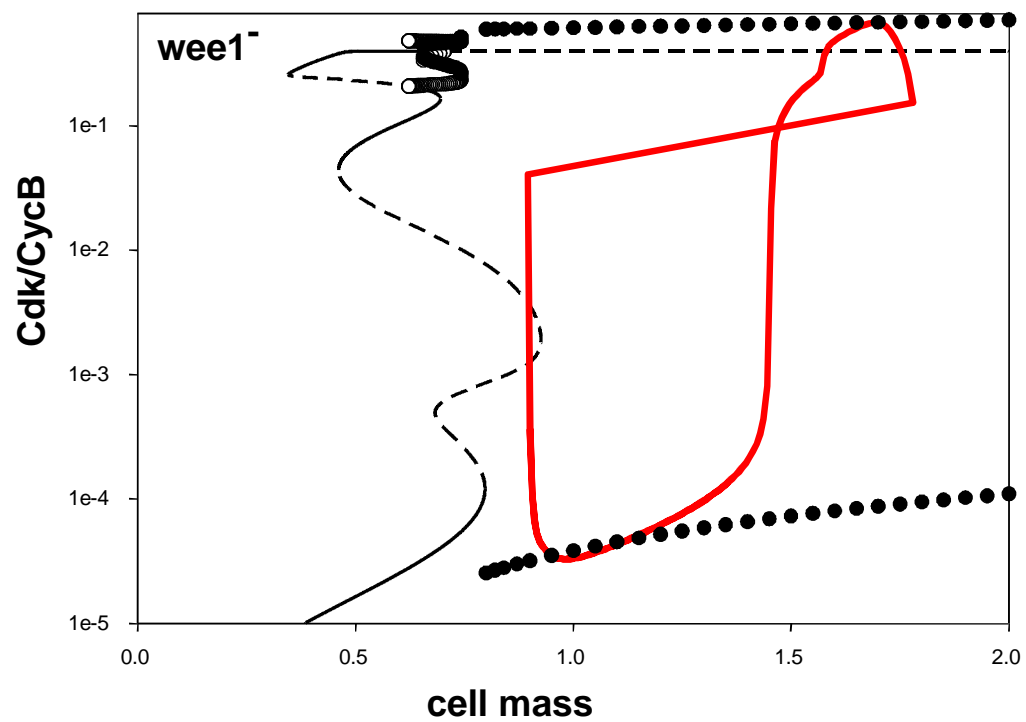
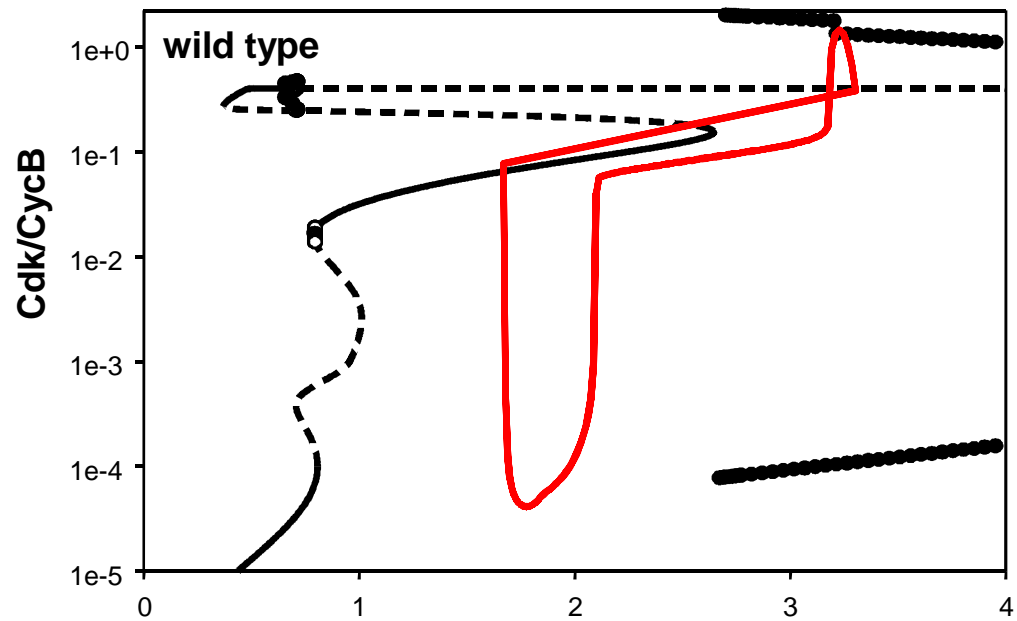
Wild type



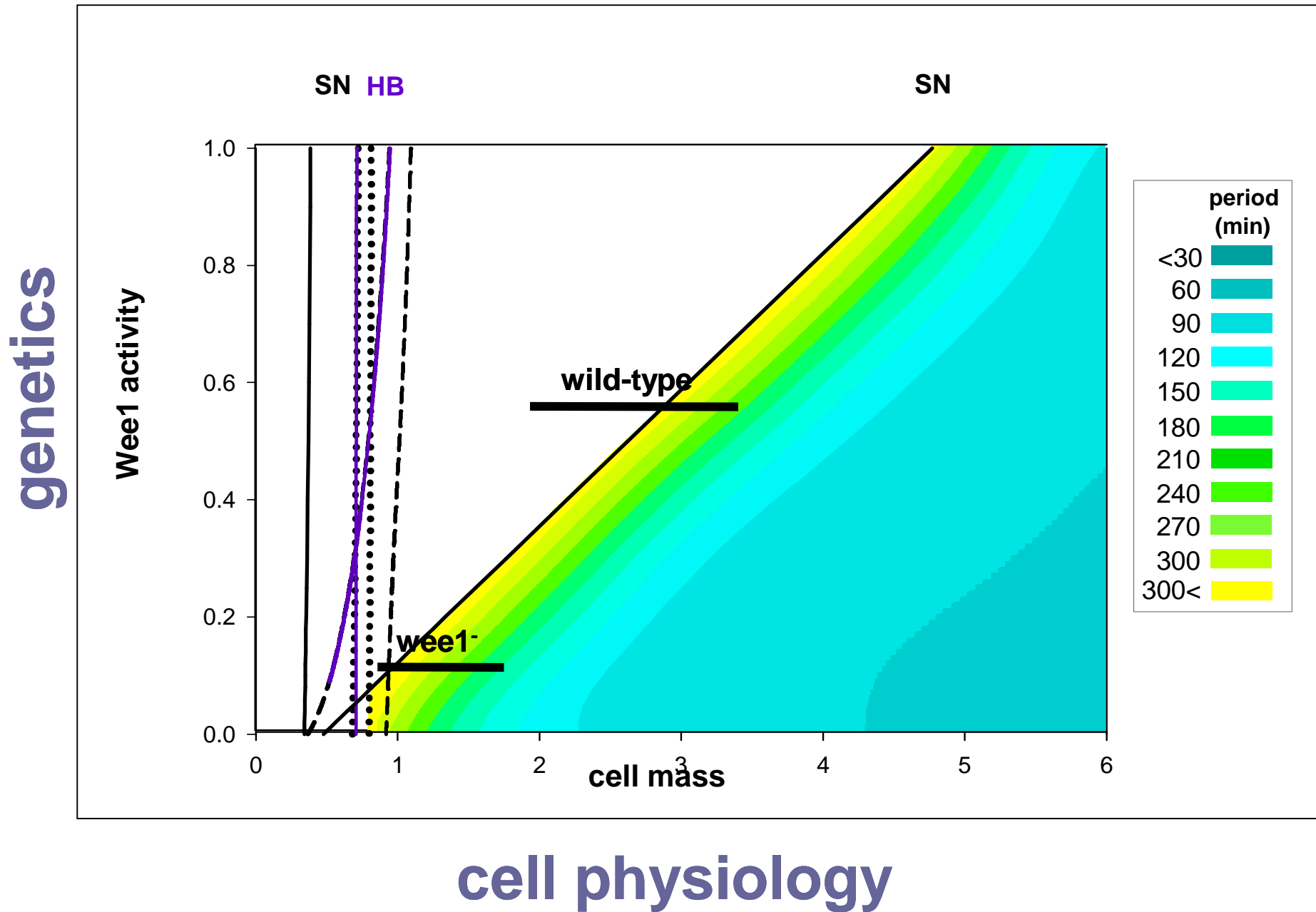
Paul Nurse

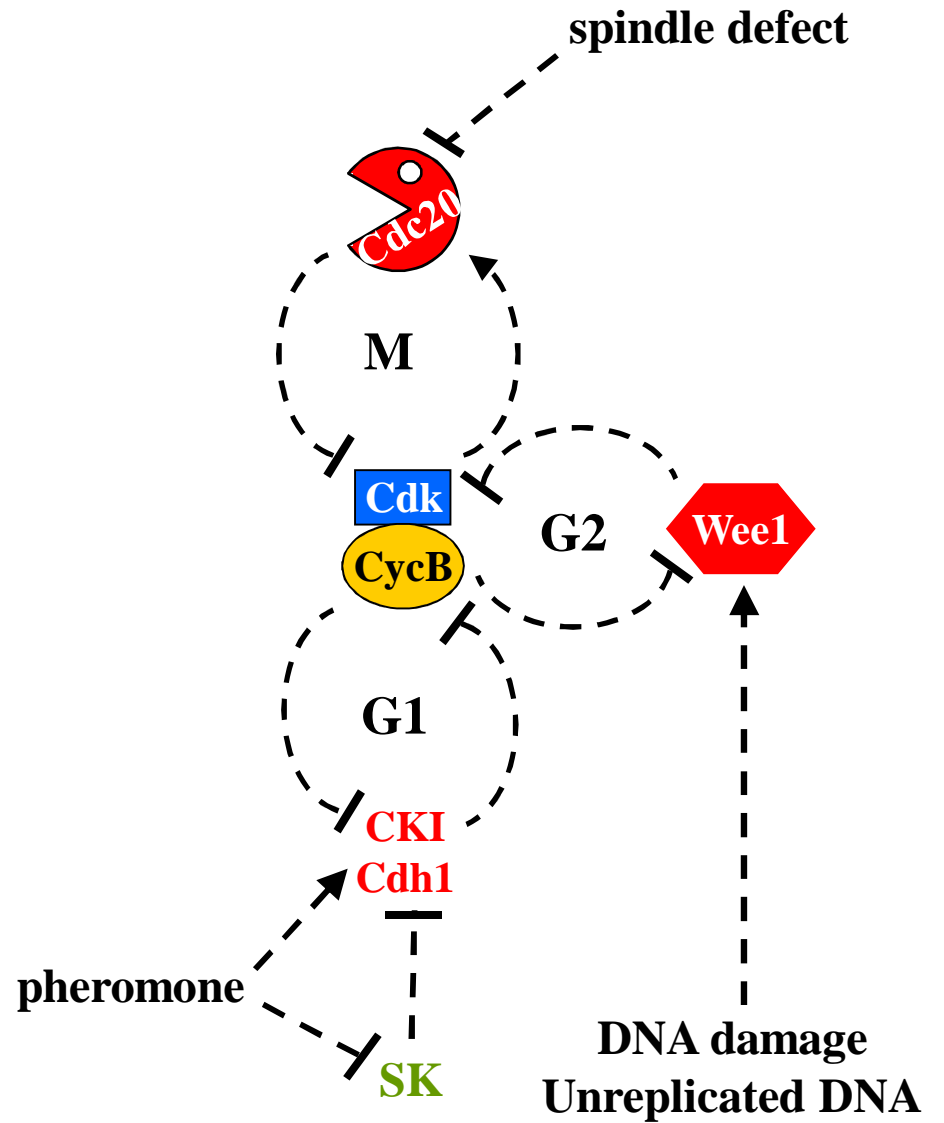
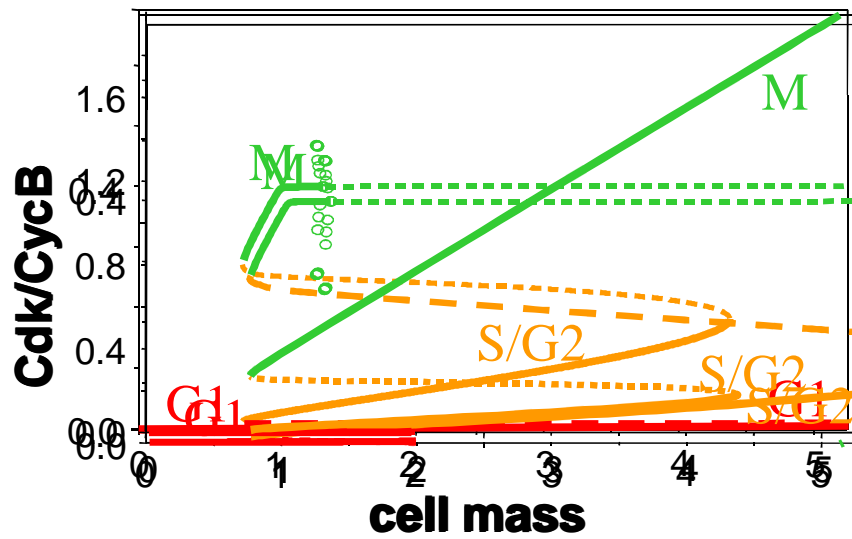
*wee1<sup>-</sup>* mutant





# Two dimensional bifurcation diagram





BioEssays 24:1095-1109, © 2002

## The dynamics of cell cycle regulation

John J. Tyson,<sup>1\*</sup> Attila Csikasz-Nagy,<sup>2,3</sup> and Bela Novak<sup>2</sup>



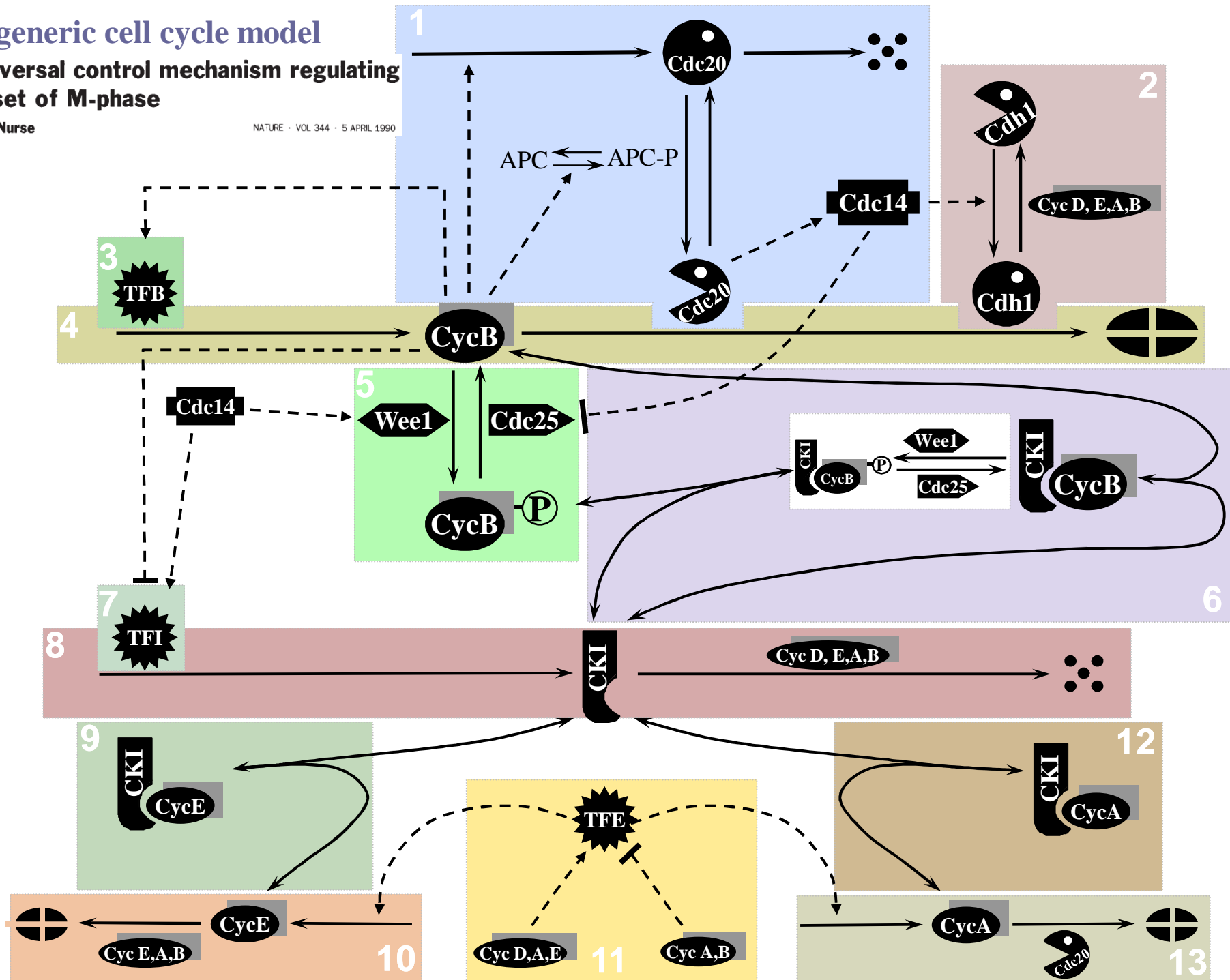


# generic cell cycle model

## Universal control mechanism regulating onset of M-phase

Paul Nurse

NATURE · VOL 344 · 5 APRIL 1990



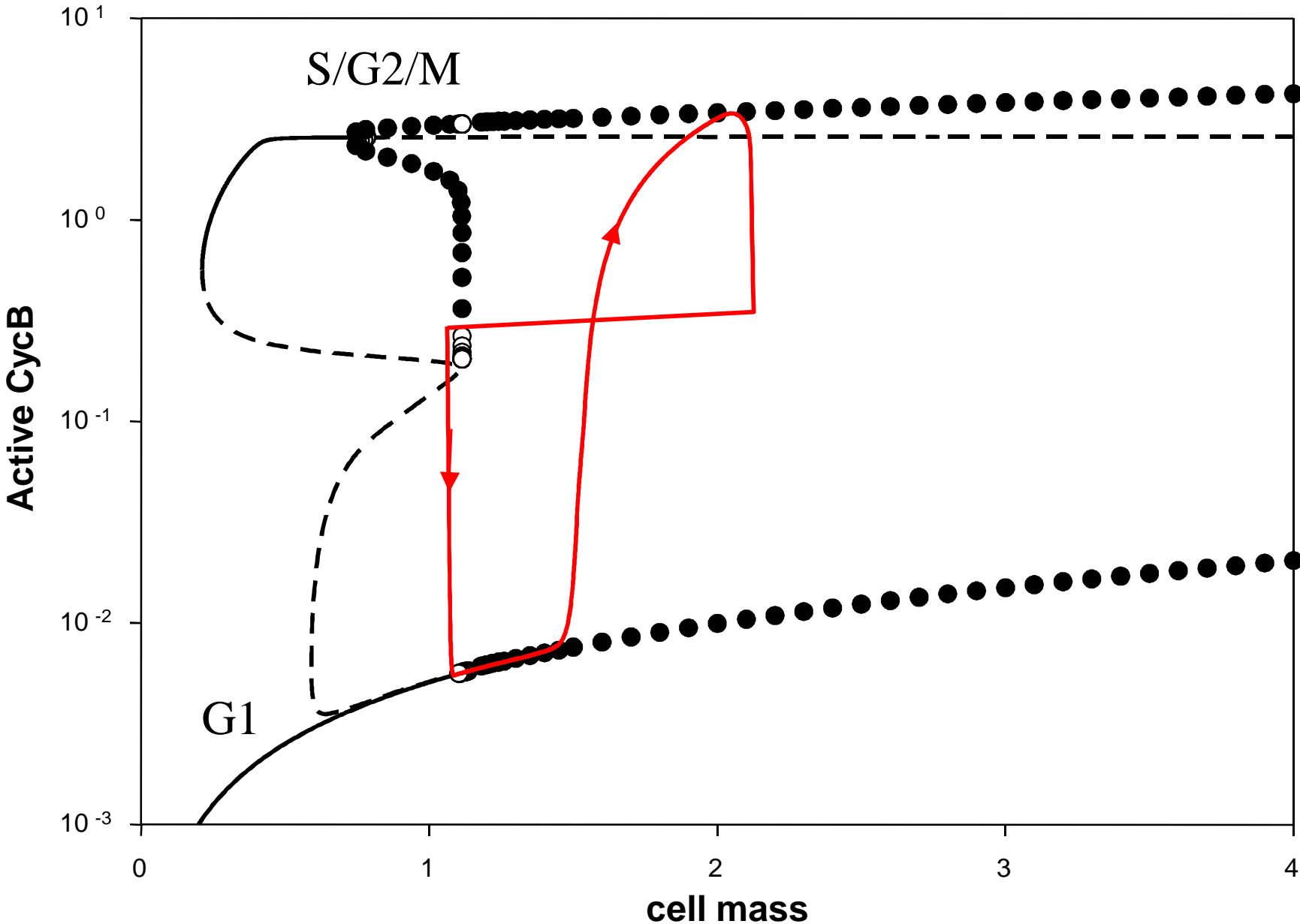
<b>This model</b>	<b>Budding yeast</b>	<b>Fission yeast</b>	<b>Mammalian cells</b>
CycB	Clb2	Cdc13	CycB
CycA	Clb5	Cig2	CycA
CycE	Cln2	-	CycE
CycD	Cln3	Puc1	CycD
CKI	Sic1	Rum1	Kip1
Cdh1	Cdh1	Ste9	Cdh1
Wee1	Swe1	Wee1	Wee1
Cdc25	Mih1	Cdc25	Cdc25c
Cdc20	Cdc20	Slp1	Cdc20
Cdc14	Cdc14	Clp1/Flp1	Cdc14
TFB	Mcm1	-	Mcm
TFE	Swi4/Swi6	Cdc10	E2F
TFI	Swi5	-	-
IE	APC	APC	APC

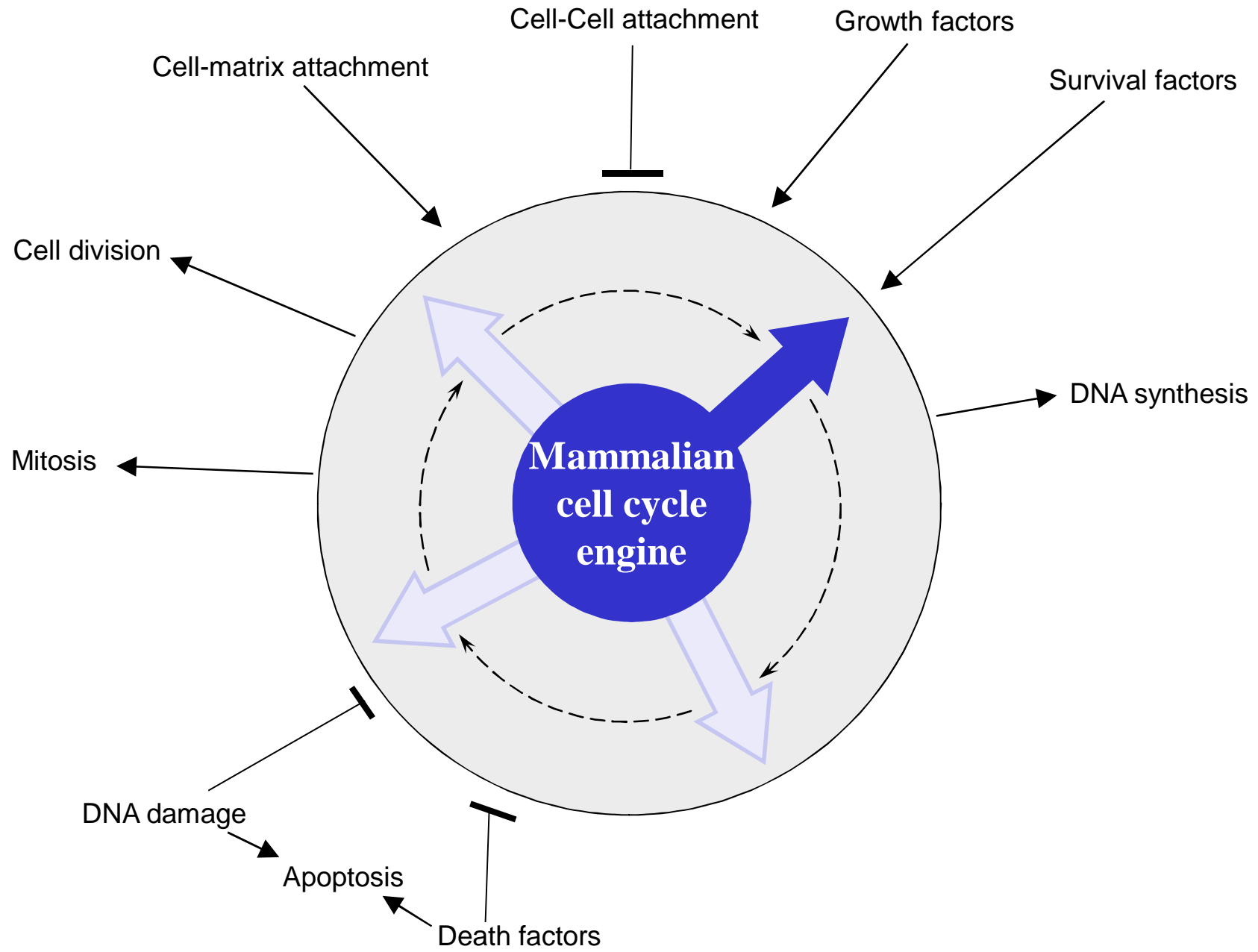
Biophysical Journal Volume 90 June 2006 4361–4379

## **Analysis of a Generic Model of Eukaryotic Cell-Cycle Regulation**

Attila Csikász-Nagy,<sup>\*†</sup> Dorjsuren Battogtokh,<sup>\*</sup> Katherine C. Chen,<sup>\*</sup> Béla Novák,<sup>†</sup> and John J. Tyson<sup>\*</sup>

*Mammalian cells*



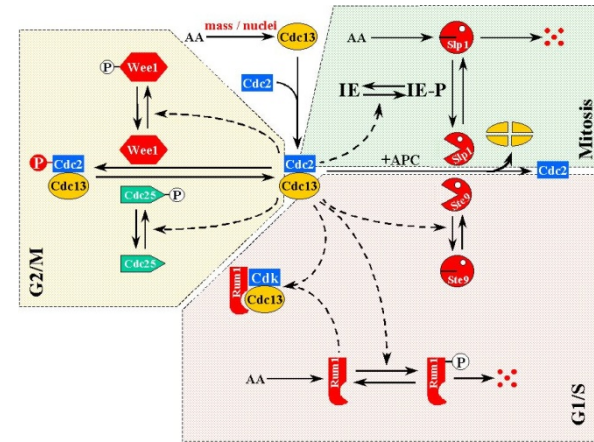


# The Dynamical Perspective

## Molecular Mechanism



## Physiological Properties



# The Dynamical Perspective

**Molecular Mechanism**



**Kinetic Equations**



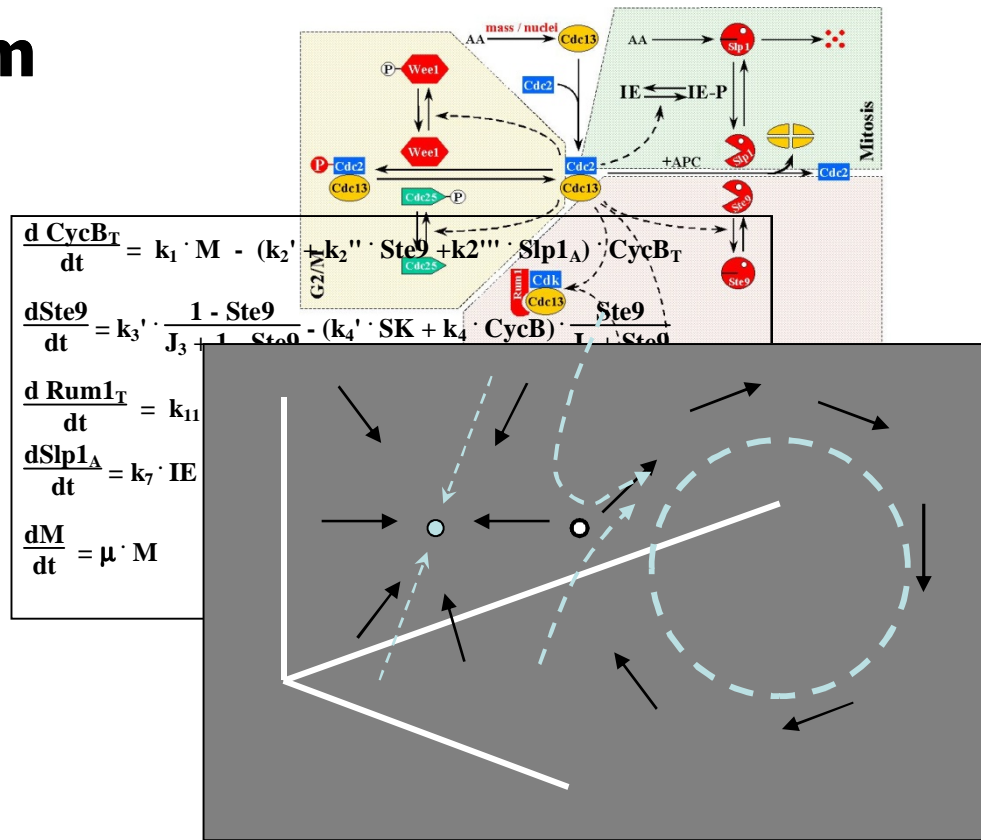
**Vector Field**



**Stable Attractors**



**Physiological Properties**



# References:

NATURE REVIEWS | MOLECULAR CELL BIOLOGY | DECEMBER 2001

## NETWORK DYNAMICS AND CELL PHYSIOLOGY

*John J. Tyson\*, Kathy Chen\* and Bela Novak†*

BioEssays 24:1095–1109, © 2002

## The dynamics of cell cycle regulation

**John J. Tyson,<sup>1\*</sup> Attila Csikasz-Nagy,<sup>2,3</sup> and Bela Novak<sup>2</sup>**

## Reverse Engineering Models of Cell Cycle Regulation

Attila Csikász-Nagy,\* Béla Novák and John J. Tyson

Adv Exp Med Biol. 2008;641:88-97.

BRIEFINGS IN BIOINFORMATICS. page 1 of 11

doi:10.1093/bib/bbp005

## Computational systems biology of the cell cycle

*Attila Csikász-Nagy*

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Kathy Chen



Andrea Ciliberto



Ákos Sveiczler