Mathematical biology

A/A 2009-10

Program

- Classic models in population ecology
  - Growth of single populations:
    - Models of Malthus and Verhulst;
    - Generalized logistic model;
    - Allee effect ;
    - Models for the harvesting of a renewable resource;
    - Idea of modelling through a birth-and-death stochastic process.
  - Prey-predator models:
    - Volterra model
    - Prey with logistic growth
    - Functional response: Rosenzweig-MacArthur model; periodic solutions and Hopf bifurcation.
    - Construction of Holling functional response through prey handling time.
    - Effect of fishing (or harvesting): Volterra's principle.
  - The chemostat:
    - Modelling growth of a species in the chemostat;
    - Reduction to a single equation.
  - Competition among species:
    - Volterra's classic model: competitive exclusion;
    - Lotka-Volterra models: possible behaviours;
    - Definition of cooperative and competitive systems; proof of convergence in 2 dimensions;
    - Competition for 1 non-renewable resource;
    - Competition in the chemostat.
  - Models with several species and trophic levels:
    - Introduction to some interesting cases: 2 predators and 1 prey; 1 predatore and 2 preys; 3 trophic levels; 3 species in non-transitive competition.
  - Models of discrete-time population growth:
    - Logistic and Ricker model: equilibria, periodic solutions, chaos;
    - Models with age classes; linear case and possible extensions.
- Other areas in mathematical biology
  - Epidemic models:
    - SIS and SIR models with closed or open population;
    - Definition of R_0 and relation with the initial growth rate r of an epidemic.
  - Trasmissione of neural impulse:
    - biophysical background; Hodgkin-Huxley model;
    - FitzHugh-Nagumo model and an euristic analysis.
  - Regulation of cellular cycle
    - biological background;
    - Novak-Tyson model.
  - Spatial diffusion
    - Modelling through reaction-diffusion equations;
    - Travelling-wave solutions: examples of logistic and Nagumo equation with diffusion;
    - Turing instability in reaction-diffusion systems.

- Mathematical theory
    - Main ideas in the qualitative theory for ordinary differential equations:
        - Linearized stability; stable and unstable manifold; ω-limit sets, Liapunov functions and applications.
        - Poincaré-Bendixson theory. Bendixson and Dulac criteria.
        - Ideas from bifurcation theory.
    - Discrete maps:
        - Equilibria, periodic solutions, stability.