TIKHONOV THEOREM

Let $z = z(t)$ *and* $y = y(t)$ *denote* n- *and* m-*dimensional vectors respectively, and the (autonomous) system be*

$$\mu \frac{dz}{dt} = F(z, y, \mu), \qquad z(t_0) = z_0, \tag{2.7a}$$

$$\frac{dy}{dt} = f(z, y, \mu), \qquad y(t_0) = y_0. \tag{2.7b}$$

Let $z = \phi(y)$ *be a root of the equation* $F(z, y, o) = 0$ *defined in some closed and bounded domain* $D \subset R^m$. *Consider the degenerate system*

$$\frac{dy}{dt} = f(\phi(y), y, 0), \qquad y(t_0) = y_0, \tag{2.8}$$

and denote by $\bar{y}(t)$ *its solution. If:*

(a) $z = \phi(y)$ *is an isolated root in D, positively stable with respect to the adjoined system*

$$\frac{dz}{d\sigma} = F(z, y, 0), \qquad \sigma = \frac{t}{\mu}, \tag{2.9}$$

uniformly in $y \in D$;

(b) *the initial point* (z_0, y_0) *lies within the domain of influence of* $\phi(y)$;

(c) *the solution* $\bar{y} = \bar{y}(t)$ *of* (2.8) *belongs to D for all* $t \in [t_0, T]$,

then the solution $(z(t, \mu), y(t, \mu))$ *of the* **overall system** (2.7) *tends to the degenerate solution* $(\bar{z}(t) = \phi(\bar{y}(t)), \bar{y}(t))$ *as* $\mu \to 0$, *in the sense that, for any* $T_0 \in (t_0, T)$,

$$\lim_{\mu \to 0} z(t, \mu) = \bar{z}(t) \tag{2.10a}$$

for $t_0 < t \leqslant T_0 < T$, *and*

$$\lim_{\mu \to 0} y(t, \mu) = \bar{y}(t) \tag{2.10b}$$

for $t_0 \leqslant t \leqslant T_0 < T$.