Robust optimality of linear saturated control in uncertain linear network flows

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Abstract—We propose a novel approach that, given a linear saturated feedback control policy, asks for the objective function that makes robust optimal such a policy. The approach is specialized to a linear network flow system with unknown but bounded demand and poliptic bounds on controlled flows. All results are derived via the Hamilton-Jacobi-Isaacs and viscosity theory.

Keywords: Optimal control, Robust optimization, Inventory control, Viscosity solutions.

I. INTRODUCTION

Consider the problem of driving a continuous time state $z(t) \in \mathbb{R}^m$ within a target set $T = \{\xi \in \mathbb{R}^m : |\xi| \leq \epsilon\}$ in a finite time $T \geq 0$ with $\epsilon \geq 0$ a-priori chosen and keeping the state within $T$ from time $T$ on. Such a problem is shortly referred to as the $\epsilon$-stabilizability problem. Viscosity solutions results are derived via the Hamilton-Jacobi-Isaacs and viscosity theory. Consider the problem of driving a continuous time state $z(t) \in \mathbb{R}^m$ within a target set $T = \{\xi \in \mathbb{R}^m : |\xi| \leq \epsilon\}$ in a finite time $T \geq 0$ with $\epsilon \geq 0$ a-priori chosen and keeping the state within $T$ from time $T$ on. Such a problem is shortly referred to as the $\epsilon$-stabilizability problem. Viscosity solutions results are derived via the Hamilton-Jacobi-Isaacs and viscosity theory.

In a previous work [2], it has been shown that under certain conditions on the matrix $D$ (recalled below), the following (linear) saturated control policy drives the state $z$ within $T$:

$$ u(t) = \text{sat}_{u^{-},u^{+}}(-kz(t)) = \begin{cases} \text{sat}_{u^{-},u^{+}}(-kz_1(t)), & \ldots, \text{sat}_{u^{-},u^{+}}(-kz_n(t)) \end{cases} \in \mathbb{R}^n, $$

with $k > 0$ and where

$$ \text{sat}_{\alpha,\beta}(\xi_i) = \begin{cases} \beta, & \text{if } \xi_i > \beta, \\ \xi_i, & \text{if } \alpha \leq \xi_i \leq \beta, \\ \alpha, & \text{if } \xi_i < \alpha. \end{cases} $$

Then, we deduce that the saturated control policy returns an admissible solution for problem (1)-(3). In the light of this consideration, we focus on the following problem.

Problem 1: We wish to design the integrand $g^\sigma(.)$ of the objective function (1) in (4) such that the saturated control turns optimal for the min-max problem (1)-(3).

REFERENCES
