TRENTO, A.A. 2019/20
MATHEMATICS FOR DATA SCIENCE/BIOSTATISTICS
EXERCISE SHEET # 1

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 1.1. Discuss the solutions x of the equation $ax = b$, where a, b are real numbers.

(SUGGERIMENTO: You should say under which conditions on a, b the equation has solutions, and then how many; and say under which conditions on a, b the equation has no solutions.)

Exercise 1.2. Discuss the solutions x, y of the equation

$$ax + by = c. \tag{1}$$

In detail,

(1) Discuss the case $a = b = 0$.
(2) Show how to find the solutions when $a \neq 0$.
(3) Show how to find the solutions when $b \neq 0$.
(4) Show that if a, b are both non-zero, and if x_0, y_0 is a solution of (1), then the solutions of (1) are exactly of the pairs x, y of the form

$$\begin{cases} x = -bt + x_0 \\ y = at + y_0 \end{cases}$$

where t is an arbitrary real number. (SUGGERIMENTO: We are going to discuss this in the lectures, but try and do it yourself.)

Exercise 1.3. Find the solution(s) of

$$\begin{cases} 2x + 3y = 5 \\ 7x - 4y = 8 \end{cases}$$

with the elimination methods.

Then try the substitution mathod(s):

(1) first solve the first equation, writing x in terms of y. Substitute this value of x in the second equation to get an equation in y alone;
(2) then solve the second equation, writing y in terms of x. Substitute this value of y in the second equation to get an equation in x alone.

Verify that all methods give the same results.

Exercise 1.4. Show that the system

$$\begin{cases} 2x + 3y = 5 \\ 6x + 9y = 15 \end{cases}$$

has infinite solutions, and spell them out.

Exercise 1.5. Show that the system

$$\begin{cases} 2x + 3y = 5 \\ 6x + 9y = -1 \end{cases}$$

has no solutions.