TRENTO, A.A. 2019/20
MATHEMATICS FOR DATA SCIENCE/BIOSTATISTICS
EXERCISE SHEET # 2

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 2.1. Find a parametric equation of the line $x + 4y = 7$.

Exercise 2.2. Find a parametric equation of the line $4y = 7$.

Exercise 2.3. Find a Cartesian equation of the line $(x, y) = (3, 4) + t(1, 5)$.

Exercise 2.4. Find a Cartesian equation of the line $(x, y) = (3, 4) + t(5, 0)$.

Exercise 2.5. Prove that the set $\mathbb{R}^n$ together with the operations of sum of n-tuples and multiplication by scalars is a vector space.

Exercise 2.6. Let m and n be positive integers. Prove that the set V of real matrices $m \times n$ together with the sum of matrices and multiplication by scalars is a vector space.

Exercise 2.7. Consider the matrices

$$a = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} -1 & -4 & 1 \\ 2 & -1 & 0 \end{bmatrix}, \quad c = \begin{bmatrix} 2 & 2 & 7 \\ 3 & -1 & -4 \end{bmatrix}.$$

compute the matrices

$$a + b, \quad a + c, \quad b + c.$$

Exercise 2.8. Consider the matrices

$$a_1 = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -1 & -4 \\ 2 & -1 \\ 1 & 0 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 2 & 2 \\ 3 & -1 \end{bmatrix}.$$

Indicate which of the nine products $a_i \cdot a_j$ are allowable, and compute them.

Exercise 2.9. Consider the following matrices

$$b_1 = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, b_2 = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 5 \end{bmatrix}, b_3 = \begin{bmatrix} -1 & -2 \\ 3 & 5 \\ 1 & 1 \end{bmatrix}, b_4 = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}.$$

Indicate which of the sixteen products $b_i \cdot b_j$ are allowable, and compute them.

Exercise 2.10. Consider the matrices

$$a = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Compute $a \cdot b$ and $b \cdot a$.

Deduce the following

- when multiplying matrices a and b, it might well happen that $a \cdot b \neq b \cdot a$ even when both products are allowable, and
- the product of two non-zero matrices (meaning that not all if their entries are zero) might well be the zero matrix.