TRENTO, A.A. 2019/20
MATHEMATICS FOR DATA SCIENCE/BIOSTATISTICS
EXERCISE SHEET # 8

Important! In solving the exercises
- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise 8.1. Find the determinant of each of the following matrices. If a matrix is invertible, find its inverse.

$$\begin{bmatrix}0\end{bmatrix}, \begin{bmatrix}2\end{bmatrix}, \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, \begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}, \begin{bmatrix}1 & 2 & 3\\ 1 & -1 & 0\\ 2 & -2 & 1\end{bmatrix}, \begin{bmatrix}1 & 2 & 3\\ 1 & -1 & 0\\ 2 & 1 & 3\end{bmatrix},$$

$$\begin{bmatrix}1 & 2 & 3 & 4\\ 1 & -1 & 0 & 1\\ 2 & 1 & 3 & -1\\ 2 & 2 & -1 & 0\end{bmatrix}, \begin{bmatrix}1 & 2 & 3 & 4\\ 1 & -1 & 0 & 1\\ 2 & 1 & 3 & -1\\ 0 & 0 & 0 & 6\end{bmatrix}, \begin{bmatrix}1 & 2 & 3 & 4 & 1\\ 1 & -1 & 0 & 1 & -1\\ 2 & 1 & 3 & -1 & 0\\ -1 & 0 & 4 & 6 & 2\\ 0 & 0 & 0 & 6 & 2\end{bmatrix}.$$

Exercise 8.2. Prove that the following are equivalent for a square matrix A:

(1) A is invertible, and
(2) the only solution of the system $AX = 0$ is $X = 0$.

Exercise 8.3. Let A, B be two $n \times n$ matrices. Prove that if $AB = I_n$, then $BA = I_n$.

Here

$$I_n = \begin{bmatrix}1 & 0 & \dots & 0\\ 0 & 1 & \dots & 0\\ & & \ddots & 0\\ 0 & 0 & \dots & 1\end{bmatrix}$$

is the $n \times n$identity matrix.

(Hint: Show that if x is a solution of $Bx = O$, then $x = O$.)