

TRENTO, A.A. 2019/20
MATHEMATICS FOR DATA SCIENCE/BIOSTATISTICS
EXERCISE SHEET # 13

Important! In solving the exercises

- explain what you are doing,
- explain why you are doing what you are doing, and
- spell out all intermediate steps.

Exercise The sample mean \bar{X} and the sample covariance C of a set of 2-dimensional samples are

$$\bar{X} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}, \quad C = \begin{bmatrix} 17 & -6 \\ -6 & 8 \end{bmatrix}.$$

- (1) Find the principal directions of C .
- (2) Find the principal components of

$$v = \begin{bmatrix} 40 \\ 30 \end{bmatrix}$$

Exercise 13.1. The sample mean \bar{X} and the sample covariance C of a set of 3-dimensional samples are

$$\bar{X} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}, \quad C = \begin{bmatrix} \frac{11}{6} & \frac{5}{6} & -\frac{5}{3} \\ \frac{5}{6} & \frac{11}{6} & -\frac{5}{3} \\ -\frac{5}{3} & -\frac{5}{3} & \frac{13}{3} \end{bmatrix}.$$

The characteristic polynomial of C is $p(\lambda) = -(\lambda - 1)^2 \cdot (\lambda - 6)$.

- (1) Find the principal directions of C .
- (2) Find the principal components of

$$v = \begin{bmatrix} 40 \\ 30 \\ 20 \end{bmatrix}$$

Exercise 13.2. The sample mean \bar{X} and the sample covariance C of a set of 3-dimensional samples are

$$\bar{X} = \begin{bmatrix} 20 \\ 20 \\ 50 \end{bmatrix}, \quad C = \begin{bmatrix} 14 & 4 & 2 \\ 4 & 14 & -2 \\ 2 & -2 & 17 \end{bmatrix}.$$

The characteristic polynomial of C is $p(\lambda) = -(\lambda - 18)^2 \cdot (\lambda - 9)$.

- (1) Find the principal directions of C .

(2) Find the principal components of

$$v = \begin{bmatrix} 40 \\ 30 \\ 20 \end{bmatrix}$$