

Thin Beauville p -groups and Beauville surfaces

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Abstract

A *Beauville surface* of unmixed type is a compact complex surface which is the quotient of the product of two algebraic curves by a special action of a finite group. The groups that can appear in such a construction are called *Beauville groups*. It is easy to give a purely group theoretical description of Beauville groups. Given a group G and two elements $x, y \in G$, let $\Sigma(x, y)$ be the union of all conjugates of $\langle x \rangle$, $\langle y \rangle$ and $\langle xy \rangle$. Then G is called a Beauville group if the following two conditions hold:

- (1) G is a 2-generator group.
- (2) G has two sets of generators $\{x_1, y_1\}$ and $\{x_2, y_2\}$ such that $\Sigma(x_1, y_1) \cap \Sigma(x_2, y_2) = 1$.

In this case, $\{x_1, y_1\}$ and $\{x_2, y_2\}$ are said to form a *Beauville structure* for G .

Beauville groups have attracted much interest in recent years. One of the important results due to Catanese is that an abelian finite p -group has a Beauville structure if and only if it is isomorphic to $C_{p^n} \times C_{p^n}$, where $p \geq 5$ and $n \geq 1$.

In this talk, we first study the existence of Beauville structures in metabelian thin p -groups.

Next we fully determine the Beauville groups within two large families of p -groups of maximal class, namely metabelian groups and groups with a maximal subgroup of class at most 2.

Then we give an application of our methods to geometry. One Beauville group can have many different Beauville structures, which in turn can give rise to many non-isomorphic Beauville surfaces. We determine the number of non-isomorphic Beauville surfaces coming from a Beauville group of maximal class which is either metabelian or contains a maximal subgroup of class at most 2.

This is joint work with Gustavo A. Fernández-Alcober from the University of the Basque Country (Spain), Norberto Gavioli and Carlo M. Scoppola, from the University of LAquila (Italy).