Addenda: how to create and observe a supercurrent

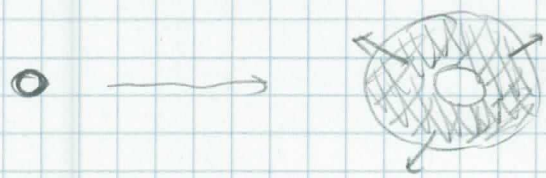
1) TOF - expansion after releasing the trap

* circulation = 0

→ able to fill the hole

* circulation ≠ 0

→ destructive interference in center



see also:

M. Cozzini, B. Jackson, S. Stringari, PRA 73, 013603 ('06)
C. Ryu, M. F. Andersen ... PRL 99, 260401 ('07)

2) dragging of sound waves

$$\omega_{\pm n} = \omega^0_{\pm n} \pm k \cdot v$$

↓ Doppler shift

create standing wave pattern along ring (4-poles modulation):

superposition of $k = \pm 2\left(\frac{2\pi}{L}\right)$

$$\delta\rho(x,t) = \sqrt{\frac{n}{L}}\,(U_k + V_k)\,\left(b_{+k}\, e^{ikx} + b^{\dagger}_{-k}\, e^{ikx} + b_{-k}\, e^{-ikx} + b^{\dagger}_{k}\, e^{-ikx}\right)$$

$$= \sqrt{\frac{n}{L}}\,(U_k + V_k)\,\left[(b_k + b^{\dagger}_{-k})\, e^{ikx} + (b_{-k} + b^{\dagger}_{k})\, e^{-ikx}\right]$$

at $t=0$: $\langle b_k \rangle = \langle b_{-k} \rangle = \beta \in \mathbb{R}$

$$\langle \delta\rho(x,t=0) \rangle = 2\sqrt{\frac{n}{L}}\,\beta\,\left(e^{ikx} + e^{-ikx}\right) = 4\beta\sqrt{\frac{n}{L}} \cdot \cos kx$$

at later t: $\langle b_k \rangle = \beta\, e^{-i(\omega_k^0 + k\cdot v)t}$, $\langle b_{-k} \rangle = \beta\, e^{-i(\omega_k^0 - k v)t}$

$$\langle \delta\rho(x,t) \rangle = 2\sqrt{\frac{n}{L}}\,\beta\,\Big[\left(e^{-i\omega_k^0 t} + e^{i\omega_k^0 t}\right) e^{-ikvt}\, e^{ikx} + \left(e^{-i\omega_k^0 t} + e^{i\omega_k^0 t}\right) e^{ikvt}\, e^{-ikx}\Big] =$$

$$= 4\sqrt{\frac{n}{L}}\,\beta \cdot \cos(\omega_k^0 t) \cdot \cos(k(x - vt))$$

→ precession of cos pattern

↳ oscillation at ω_k^0

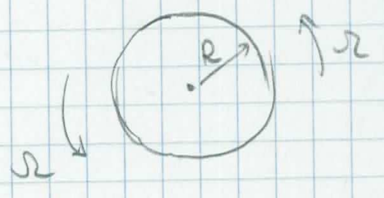
see also: P. C. Haljan, B.P. Anderson, I. Coddington and E. A. Cornell, PRL 86, 2922 (2001)

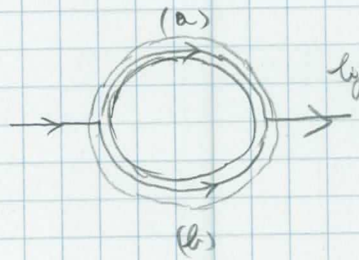
F. Chevy, K.W. Madison, J. Dalibard, PRL 85, 2223 (2000)

3) (speculative) : <u>slow light propagation along BEC</u>

BEC at rest in fixed stars reference frame

has velocity $v = \omega \cdot R$ in lab. frame (assumed to be rotating at Ω).





light path along BEC

(a) path : accumulates a phase $\Delta\phi = k\, L \cdot \frac{v}{v_g} = L \cdot \pi R \, \frac{\Omega R}{v_g} = \pi \frac{\Omega R^2}{v_g}$

(b) path : opposite phase $-\pi \frac{\Omega R^2}{v_g}$

$\Longrightarrow$ can be measured interferometrically

lower v_g $\Longrightarrow$ better sensitivity to rotations

<u>see also</u>:

P. Zimmer and M. Fleischhauer, PRL 92, 253201 (2004)
M. Artoni and IC, PRA 67, 011602 (2003)

To spin up the BEC from $k=0$:

* standard stirring ineffective up to $v \simeq c_s$
  (Landau criterion)
  - vortex state can be energetically favoured
  - no path to reach it without crossing a high mountain pass in energy

* thermal nucleation: lower T across T_{BEC} → BEC with random k

* stir thermal gas, then cool → BEC with finite k (+ fluctuations)
  ↳ and metastable!

* phase imprinting techniques :

  → 2 component BECs :
  M.R Matthews, B.P. Anderson ... PRL 83, 002498 ('99

  → Raman process using Laguerre-Gauss → imports OAM to atoms
  M F Andersen, C. Ryu, P. Cladé, ... PRL 97, 170406 ('06
  C. Ryu, M. F. Andersen ... PRL 99, 260401 ('07)

Physics even more interesting in harmonic traps : see Stringari and Pitaevskii book

- overcritical rotations
- nucleation of vortices from surface instabilities