Lecture 6 — Strongly correlated photons

Quantum field theory corresponding to photon driven-dissipative QPE

$$H = \int dz \, \Big[\hat{E}^\dagger(r) \Big[\omega_0 - \frac{\hbar \nabla^2}{2m} + \delta\omega(r_\parallel) \Big] \hat{E}^\dagger(r) +$$

$$+ \frac{g_{NL}}{2} \hat{E}^\dagger(r) \hat{E}^\dagger(r) \hat{E}(r) \hat{E}(r) +$$

$$+ K E_{inc}(r,t) \hat{E}^\dagger(r) + K^* E^*_{inc}(r,t) \hat{E}(r) \Big] +$$

$$+ \text{loss terms } (+ \text{ amplification})$$

Restrict to coherent drive

External potential $\delta\omega(r) = \delta\omega_0 (\cos k_\ell x + \cos k_\ell y)$

lattice in both directions

↳ approx to 1 state per site

⇒ generalized Bose-Hubbard Hamiltonian

$$H_{BH} = \sum_{\langle ij \rangle} - J \hat{a}_i^\dagger \hat{a}_j + \sum_i \Big[\frac{U}{2} \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i + \omega_0 \hat{a}_i^\dagger \hat{a}_i +$$

$$+ F_i(t) \hat{a}_i^\dagger + F_i^* \hat{a}_i^\dagger \Big] + \text{losses}$$

- J = tunneling (or hopping) amplitude
- U = charging energy
- F_i = driving
- $\hat{a}_i$ = destruction operator on wf of site i

Isolated system $H = \sum_i \omega_i a_i^\dagger a_i + \frac{U}{2} a_i^\dagger a_i^\dagger a_i a_i +$

$- J \sum_{\langle i,j \rangle} a_i^\dagger a_j$

Restrict to commensurate occupation $N = M$ (N: photons, M: sites)

* for $\frac{U}{J} \ll 1$: ground state $|\psi_S\rangle = \frac{1}{\sqrt{N!}} \left(\sum_i \frac{1}{\sqrt{N}} \hat{a}_i^\dagger \right)^N |vac\rangle$

"superfluid state", extended coherence in space

$g^{(1)}(r,r') = 1$

Energy $= N \left[-J \cdot c + \frac{U}{2} \right]$ (c = # of neighboring sites)

* for $\frac{U}{J} \gg 1$ ground state $|\psi_M\rangle = \prod_i \hat{a}_i^\dagger |vac\rangle$

"Mott-insulator" state, fixed occupation,

no coherence $g^{(1)}(r,r') = \delta_{r,r'}$

Energy $= 0$

↳ Crossover for $\frac{U}{J} = \frac{U}{J}\Big|_{crit.}$

Experiment with atoms: Greiner et al. Nature 415, 39 ('02)

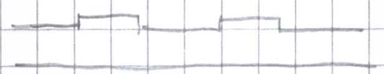
Proposals with photons: Angelakis et al. PRA 76, 031805 ('07)

Greentree et al. Nat. Phys. 2, 856 (06)

Hartmann et al. Nat. Phys. 2, 849 (06)

Most promising physical realisations :

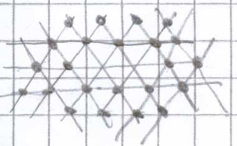
a) - planar microcavities + lateral patterning



nonlinearity → QW exciton or
Q. dot (pb. localization of dot
+ inhomog. broadening)

(analogously → micropillars)

b) - photonic crystal cavity :



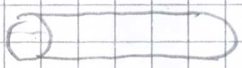
missing holes in periodic array → defect state
with sub-λ localization, high Q

nonlinearity → QW or Q dots
↳ exp. observed antibunching

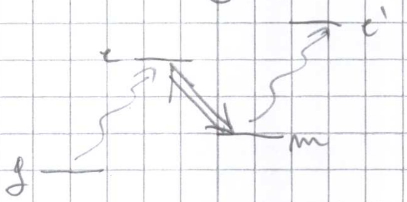
⟹ ab) losses forbid thermalization / adiabatic sweep in g.s.
↳ dissipation/driving to be included

c) - hollow fibers + atoms (inside or at surface)

single-mode propagation → 1D.



nonlinearity from atoms, e.g. EIT



pb. not yet high enough optical thickness

d) superconducting circuits

→ interruptions define weakly coupled cavity modes.

coplanar waveguide in μ-wave region

nonlinearity : Josephson junction circuit → effective 2-level atom
(more in Haroche's lectures)

advantages $\frac{\Omega}{\gamma} \gtrsim 100$ (while ~ 5 for semiconductors)

disadvantages $T < 1K$ (hard cryogenics)
no single photon detector
has to use linear amplifiers to measure field amplitudes
↳ antibunching observed

Tonks - Girardeau gas in 1D (lattice $N \ll M$ or continuum 1D)

$U/J = \infty$ impenetrable photons "fermionized"

$\psi_B(x_1 \ldots x_N) = 0$ whenever $x_i = x_j$, fully symmetric wrt exchange

Girardeau J. Math. Phys 1, 516 ('60)

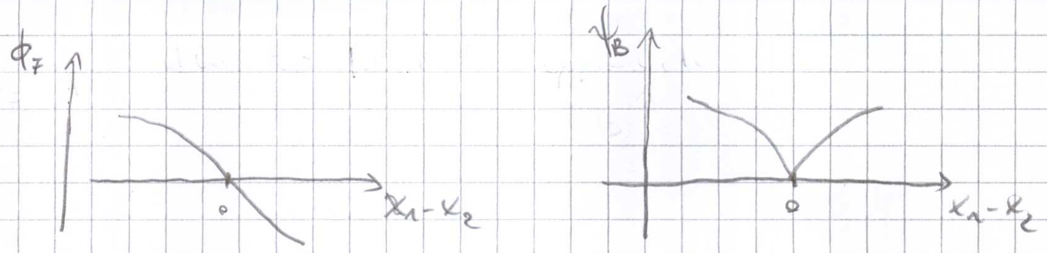
$\psi_B(x_1 \ldots x_N) = \varepsilon_{x_1 \ldots x_N} \cdot \phi_F(x_1 \ldots x_N)$ for each eigenstate

↳ non-interacting fermions

impenetrable bosons

→ parity of ordering to $x_1 < x_2 < \ldots < x_N$)

ground state $\longrightarrow$ Fermi sphere in ϕ_F



when two particles approach:

$$\begin{cases} \phi_F \sim x_1 - x_2 \\ \phi_B \sim |x_1 - x_2| \end{cases}$$

To observe TG states in presence of dissipation:

$\rightarrow$ each eigenstate gives peak in transmission spectrum

$\rightarrow$ structure of eigenstate deduced from coherence functions of secondary emission

PRL 103, 033601 ('09)

New directions $\longrightarrow$ more complex many-body states
(theory) interplay of amplification + strong interactions
effect of artificial gauge field $J \rightarrow J e^{i\phi_{ij}}$
Laughlin states, fractional statistics ..

(experiment) demonstrate a strongly correlated photon gas.