Exercise 7 : Extinction vs. Absorption 22/3/17

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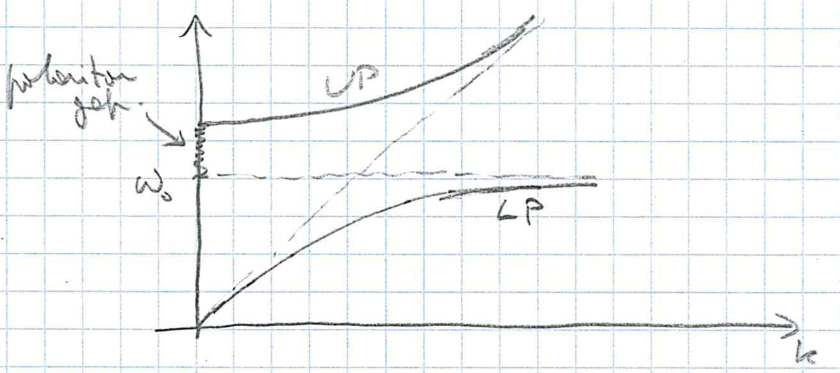


Lossless case $\gamma = 0^+$

1.

Within polariton gap $\varepsilon(\omega) < 0$.

$$c^2 k^2 = \varepsilon(\omega)\,\omega^2 \implies k \text{ purely imaginary}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \implies ik \times E = \frac{i\omega}{c} B$$

$$B = \frac{c}{\omega} k \times E.$$

$S = \frac{c}{4\pi} E \times B$. For oscillating fields $\bar{S} = \frac{c}{8\pi} \mathrm{Re}\left[E^* \times B\right]$

In our case: $\bar{S} = \frac{c}{8\pi} \mathrm{Re}\left[E^* \times \frac{c}{\omega}(k \times E)\right] =$

$$= \frac{c^2}{8\pi\omega} \mathrm{Re}\left[|E|^2 k - \cancel{(E^* \cdot k)} \cdot E\right] = 0$$

imaginary

0

Absorption : $\varepsilon = \varepsilon_r + i\varepsilon_i \implies k = \frac{\omega}{c}\sqrt{\varepsilon_r + i\varepsilon_i} = k_r + ik_i$

$$\bar{S} = \frac{c^2}{8\pi\omega} \mathrm{Re}\left[|E|^2 (k_r + ik_i)\right] \neq 0.$$

3. Within a band $\varepsilon(\omega) > 0$:

$$\bar{S} = \frac{c^2}{8\pi\omega}\left(\sqrt{\varepsilon(\omega)}\,\frac{\omega}{c}\cdot|E|^2\right) = \frac{c}{8\pi}\sqrt{\varepsilon}\cdot|E|^2$$

$$\bar{u} = \frac{1}{16\pi}\left[\frac{d(\omega\varepsilon)}{d\omega}|E|^2 + |B|^2\right] =$$

$$= \frac{1}{16\pi}\left[\frac{d(\omega\varepsilon)}{d\omega} + \frac{\cancel{c^2}}{\cancel{\omega^2}}\varepsilon\frac{\cancel{\omega^2}}{\cancel{c^2}}\right]|E|^2 = \frac{1}{16\pi}\left[\varepsilon + \frac{d(\omega\varepsilon)}{d\omega}\right]|E|^2$$

$$v_g = \frac{d\omega}{dk} = \left(\frac{dk}{d\omega}\right)^{-1} = \left(\frac{d}{d\omega}\left(\sqrt{\varepsilon}\,\frac{\omega}{c}\right)\right)^{-1} =$$

$$= \left(\frac{1}{2\sqrt{\varepsilon}}\varepsilon'\frac{\omega}{c} + \sqrt{\varepsilon}\,\frac{1}{c}\right)^{-1} = \left(\frac{1}{c\sqrt{\varepsilon}}\left(\varepsilon + \frac{1}{2}\varepsilon'\omega\right)\right)^{-1} =$$

$$= \left(\frac{1}{2c\sqrt{\varepsilon}}\left(2\varepsilon + \omega\varepsilon'\right)\right)^{-1} = \left(\frac{1}{2c\sqrt{\varepsilon}}\cdot\left(\varepsilon + \varepsilon + \omega\varepsilon'\right)\right)^{-1} =$$

$$= \left(\frac{1}{2c\sqrt{\varepsilon}}\left(\varepsilon + \frac{d}{d\omega}(\varepsilon\omega)\right)\right)^{-1}$$

So $$\bar{u}\cdot v_g = \frac{1}{16\pi}\cancel{\left[\varepsilon + \frac{d}{d\omega}(\varepsilon\omega)\right]}|E|^2\,\frac{1}{\frac{1}{2c\sqrt{\varepsilon}}\cancel{\left(\varepsilon + \frac{d}{d\omega}(\varepsilon\omega)\right)}} =$$

$$= \frac{1}{8\pi}c\sqrt{\varepsilon}\,|E|^2 = \bar{S}.$$

2. At interfaces : $E_{\parallel}$ continuous

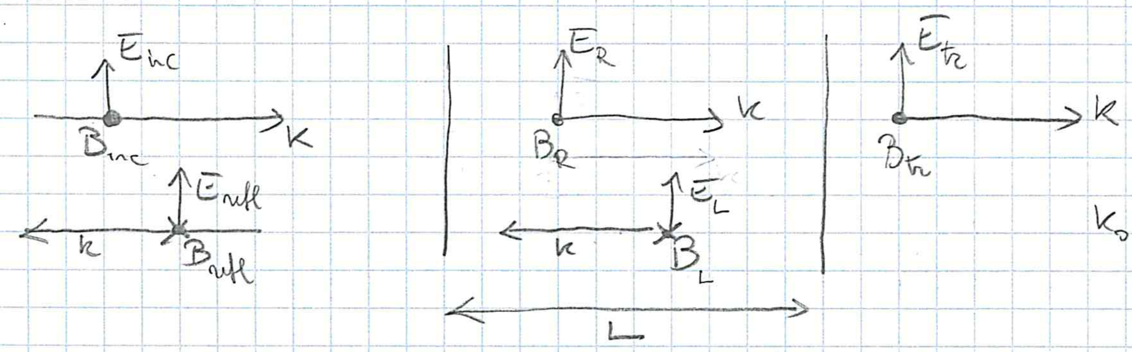
B continuous.

Within gap $k = \frac{\omega}{c}\sqrt{\varepsilon_0}$ purely imaginary

Expanding in components $B = \frac{c}{\omega} k\cdot E = \sqrt{\varepsilon_0}\, E = i\sqrt{|\varepsilon_0|}\, E$

(in medium)

$B = E$ (in vacuo)



$k_0 = +\frac{\omega}{c}\sqrt{|\varepsilon_0|}$ A

$E_{inc} + E_{refl} = E_R + E_L$

$B_{inc} - B_{refl} = B_R - B_L$

$E_{tr} = E_R e^{-k_0 L} + E_L e^{k_0 L}$

$B_{tr} = B_R e^{-k_0 L} - B_L e^{k_0 L}$

$B_{L,R} = i\sqrt{|\varepsilon_0|}\, E_{L,R}$

$B_{inc,\,refl,\,tr} = E_{inc,\,refl,\,tr}$

$$\begin{cases} iA\, E_{inc} + iA\, E_{refl} = iA\, E_R + iA\, E_L \\ E_{inc} - E_{refl} = iA\, E_R - iA\, E_L \end{cases}$$

$$\Longrightarrow (1+iA)E_{inc} - (1-iA)E_{refl} = 2iA\, E_R$$

$$\begin{cases} iA\, E_{tr} = iA\, E_R\, e^{-k_0 L} + iA\, E_L\, e^{k_0 L} \\ E_{tr} = iA\, E_R\, e^{-k_0 L} - iA\, E_L\, e^{k_0 L} \end{cases}$$

$$\Longrightarrow (1+iA)E_{tr} = 2iA\, e^{-k_0 L} E_R \Longrightarrow E_R = \frac{(1+iA)\, e^{k_0 L}}{2iA} E_{tr}$$

$$\begin{cases} E_{inc} + E_{refl} = E_R + E_L \\ e^{-k_0 L} E_{tr} = e^{-2k_0 L} E_R + E_L \end{cases}$$

$$\Longrightarrow E_{inc} + E_{refl} - e^{-k_0 L} E_{tr} = E_R \left(1 - e^{-2k_0 L}\right)$$

$$E_{inc} + E_{refl} - e^{-k_0 L} E_{tr} = \frac{(1+iA)\left(e^{k_0 L} - e^{-k_0 L}\right)}{2iA} E_{tr}$$

$$(1+iA) E_{inc} - (1-iA) E_{refl} = (1+iA)\, e^{k_0 L} E_{tr}$$

$$(1-iA) E_{inc} + (1-iA) E_{refl} = (1-iA)\, e^{-k_0 L} E_{tr} + \frac{(1+iA)(1-iA)}{2iA}\left(e^{k_0 L} - e^{-k_0 L}\right) E_{tr}$$

$$E_{inc} = E_{tr} \cdot \frac{1}{2}\left[(1-iA)\, e^{-k_0L} + (1-iA)\left(\frac{iA+1}{2iA}\right)\left(e^{k_0L} - e^{-k_0L}\right) + (1+iA)\, e^{k_0L}\right]$$

$$E_{inc} = E_{tr} \cdot \frac{1}{2}\left[2 \cdot ch\, k_0L + 2iA\, sh\, k_0L + \frac{1+A^2}{2iA}\, 2\, sh\, k_0L\right]$$

$$= E_{tr}\left[ch\, k_0L + sh\, k_0L\left(iA + \frac{1+A^2}{2iA}\right)\right]$$

$$E_{tr} = \frac{E_{inc}}{ch\, k_0L + sh\, k_0L \; \frac{-1-A^2}{2iA}} \overset{k_0L \gg 1}{\simeq} \frac{2E_{inc}}{e^{k_0L}\left(1 + \frac{1-A^2}{2iA}\right)}$$

$$= E_{inc}\, e^{-k_0L}\, \frac{4iA}{(1+iA)^2}$$

$\longrightarrow$ exponential suppression of transmission as in tunnel effect

5 bis

Physical interpretation : two successive transmissions, no interference

1) vac $\searrow$ medium

$$\begin{cases} B_{inc} - B_{refl} = B_{tr} \\ E_{inc} + E_{refl} = E_{tr} \end{cases}$$

$$E_{inc} - E_{refl} = iAE_{tr}$$

$$2E_{inc} = (1+iA)\,E_{tr}$$

$$E_{tr} = \frac{2}{1+iA}\,E_{inc}.$$

2) medium $\rightarrow$ vac

$$\begin{cases} E'_{inc} + E'_{refl} = E'_{tr} \\ iA\,E'_{inc} - iA\,E'_{refl} = E'_{tr} \end{cases}$$

$$iA\,E'_{inc} + iA\,E'_{refl} = iA\,E'_{tr}$$

$$2iA\,E'_{inc} = (1+iA)\,E'_{tr}$$

$$E'_{tr} = \frac{2iA}{1+iA}\,E'_{inc}$$

Propagation in medium : $E'_{inc} = e^{-k_0 L}\,E_{tr}$

$$\Rightarrow E'_{tr} = \frac{2iA}{1+iA}\,e^{-k_0 L}\,\frac{2}{1+iA}\,E_{inc} = \frac{4iA}{(1+iA)^2}\,e^{-k_0 L}\,E_{inc}$$

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With weak losses (1st perturbative order in γ):

$$\varepsilon(\omega) = 1 + \frac{4\pi n\, e^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega} \simeq 1 + \frac{4\pi n\, e^2/m}{\omega_0^2 - (\omega + i\gamma/2)^2} =$$

$$= \underbrace{\varepsilon_0(\omega + i\gamma/2)}_{\text{lossless } \varepsilon \text{ evaluated @ } \omega + i\gamma/2} = \varepsilon_0(\omega) + \frac{d\varepsilon_0}{d\omega}\, i\gamma/2 .$$

$$(k_r + i k_i)^2 = k^2 = \frac{\omega^2}{c^2}\varepsilon(\omega) \simeq \frac{\omega^2}{c^2}\left(\varepsilon_0(\omega) + i\frac{\gamma}{2}\frac{d\varepsilon_0}{d\omega}\right)$$

$$k_r^2 \simeq \frac{\omega^2}{c^2}\varepsilon_0(\omega) \quad ; \quad 2i\, k_i k_r \simeq \frac{i\omega^2\gamma}{2c^2}\frac{d\varepsilon_0}{d\omega}$$

$$\Rightarrow k_i = \frac{\omega\gamma}{4c\sqrt{\varepsilon_0}}\frac{d\varepsilon_0}{d\omega}$$

* If $\omega\varepsilon_0' \gg \varepsilon_0$ (strong dispersion) $\Longrightarrow v_g = \dfrac{c\sqrt{\varepsilon_0}}{\varepsilon_0 + \omega\varepsilon_0'/2} \simeq \dfrac{2c\sqrt{\varepsilon_0}}{\omega\varepsilon_0'} \ll c$

then $2k_i = \dfrac{\gamma}{v_g}$.

factor 2 in decay of intensity vs amplitude

* What about general case ?

Polariton lifetime $\rightarrow \Gamma$ of complex solution $\hat{\omega} = \omega - i\Gamma/2$

of $\dfrac{\tilde{\omega}^2}{c^2}\varepsilon(\tilde{\omega}) = k^2$ with k real.

$$\frac{(\omega - i\Gamma/2)^2}{c^2}\,\varepsilon(\omega - i\Gamma/2) = k^2$$

$$\frac{\omega^2}{c^2}\left(\varepsilon_0(\omega) + \frac{i}{2}(\gamma - \Gamma)\,\varepsilon_0'\right) - i\,\frac{\Gamma\omega}{c^2}\,\varepsilon_0 = k^2$$

$$\frac{\cancel{i}}{2}\,\frac{\omega^{\cancel{2}}(\gamma - \Gamma)}{\cancel{c^2}}\,\varepsilon_0' = \cancel{i}\,\frac{\Gamma\cancel{\omega}}{\cancel{c^2}}\,\varepsilon_0$$

$$\frac{\omega\varepsilon_0'}{2}\,\gamma = \Gamma\,\varepsilon_0 + \frac{\omega\Gamma\varepsilon_0'}{2}$$

$$\Gamma = \underbrace{\frac{\omega\varepsilon_0'/2}{\varepsilon_0 + \omega\varepsilon_0'/2}}_{\text{Hopfield weight of matter excitation}} \cdot \gamma$$

non-radiative decay of matter excitation.

$$\frac{\Gamma}{v_g} = \frac{\omega\varepsilon_0'/2}{\varepsilon_0 + \omega\varepsilon_0'/2}\,\gamma\,\frac{1}{c\sqrt{\varepsilon_0}}\left(\varepsilon_0 + \omega\varepsilon_0'/2\right) = \gamma\,\frac{\omega\varepsilon_0'}{2c\sqrt{\varepsilon_0}} = 2k_i$$

factor 2 in decay of intensity vs amplitude.

<u>Dilute medium</u> with $\varepsilon_0 \simeq 1$:

$$v_g \simeq \frac{c}{1 + \omega\varepsilon_0'/2} \quad ; \quad k_i = \frac{\omega\varepsilon_0'}{4c}$$

$$\Gamma = \frac{\omega\varepsilon_0'/2}{1 + \omega\varepsilon_0'/2}\cdot\gamma \simeq \left(1 - \frac{v_g}{c}\right)\gamma .$$