Addendum (11/12) : more on the RWA

σ^+ field : $\vec{B}(t) = B_0 \hat{\sigma}_+ e^{-i\omega_L t} + c.c$

$\left[\hat{\sigma}_\pm = \frac{1}{\sqrt{2}}\left(\hat{x} \pm i\hat{y}\right)\right]$

$$= \sqrt{2}\, B_0 \left[\hat{x} \cos\omega_L t + \hat{y} \sin\omega_L t\right]$$

$\vec{B}$ field rotates at ω_L

linear polarization

$$\vec{B}(t) = B_0 \hat{x} \left[e^{-i\omega_L t} + e^{i\omega_L t}\right]$$

$$= \frac{1}{\sqrt{2}} \left[B_0 \hat{\sigma}_+ e^{-i\omega_L t} + B_0 \hat{\sigma}_- e^{-i\omega_L t} + c.c.\right]$$

$$= B_0 \left[R_z(\omega_L t)\hat{x} + R_z(-\omega_L t)\hat{x}\right]$$

Unitary Transform to rotating frame :

$$H_{eff} = (\mu_B B_{ext} - \omega_L) S_z - \mu_B B_0 \hat{S}_x +$$

$$- \mu_B B_0 \underbrace{\left\{R_z(-2\omega_L t)\, \hat{x}\right\}}_{\text{rotates fast at } 2\omega_L \text{ angular speed}} \cdot \vec{S}$$

RWA means that in rotating frame you only keep wave rotating at zero angular speed ω_L. It is exact for a circularly polarized σ_+ field

anti-RWA terms rotate very fast and average to 0 over period
↳ relevant only if $\omega_L \simeq \Omega, \Delta\omega, \gamma$