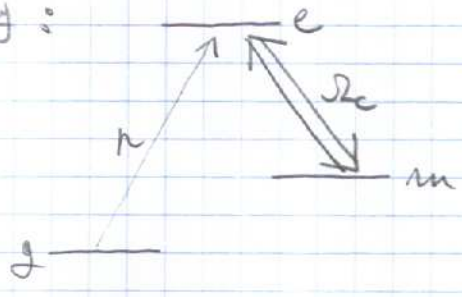
Slowing and trapping light via EIT

3-level atom with coherent dressing:



* strong beam on $m \rightarrow e$ transition, Rabi frequency Ω_c

* probe optical response of atoms around $g \rightarrow e$ transition, assume Rabi frequency of probe $\Omega_p \ll \Omega_c$

  ↳ linearised solution of O.B.E at lowest order in Ω_p.

* O.BE including $e \rightarrow g$ and $m \rightarrow g$ decays:
  - $\Gamma_e \longrightarrow$ spontaneous emission, generally ~ 10 MHz
  - $\Gamma_m \longrightarrow$ decoherence processes, much slower, almost zero in cold samples

$$\frac{d\rho}{dt} = -i\left[H,\rho\right] + \frac{\Gamma_e}{2}\left[2\sigma_e^-\rho\,\sigma_e^+ - \sigma_e^+\sigma_e^-\rho - \rho\,\sigma_e^+\sigma_e^-\right] +$$

$$+\frac{\Gamma_m}{2}\left[2\sigma_m^-\rho\,\sigma_m^+ - \sigma_m^+\sigma_m^-\rho - \rho\,\sigma_m^+\sigma_m^-\right]$$

with $\sigma^+_{e,m} = |e,m\rangle\langle g|$, $\sigma^-_{e,m} = |g\rangle\langle e,m|$.

Polarization on $g \rightarrow e$: $p_{eg} = \rho_{eg}\; d_{eg}$

Within RWA:

$$H = \hbar\omega_e |e\rangle\langle e| + \hbar\omega_m |m\rangle\langle m| + \hbar\omega_g |g\rangle\langle g| +$$
$$-\left(\hbar\Omega_p(t)|e\rangle\langle g| + \hbar\Omega_p^*(t)|g\rangle\langle e|\right) +$$
$$-\left(\hbar\Omega_c(t)|e\rangle\langle m| + \hbar\Omega_c^*(t)|m\rangle\langle e|\right)$$

$$\frac{d\rho_{eg}}{dt} = -i(\omega_e-\omega_g)\rho_{eg} + i\Omega_p(t)\left[\rho_{gg} - \rho_{ee}\right] + i\Omega_c(t)\rho_{mg} + -\frac{\Gamma_e}{2}\rho_{eg}$$

$$\frac{d\rho_{ee}}{dt} = i\left(\Omega_p(t)\rho_{ge} - \Omega_p^*(t)\rho_{eg}\right) + i\left(\Omega_c(t)\rho_{me} - \Omega_c^*(t)\rho_{em}\right) - \Gamma_e\rho_{ee}$$

$$\frac{d\rho_{mm}}{dt} = -i\left(\Omega_c(t)\rho_{me} - \Omega_c^*(t)\rho_{em}\right) - \Gamma_m\rho_{mm}$$

$$\frac{d\rho_{em}}{dt} = -i(\omega_e-\omega_m)\rho_{em} + i\Omega_c(t)(\rho_{mm}-\rho_{ee}) + i\Omega_p(t)\rho_{gm} + -\frac{\Gamma_m+\Gamma_e}{2}\rho_{em}$$

$$\frac{d\rho_{mg}}{dt} = -i(\omega_m-\omega_g)\rho_{mg} + i\left(\Omega_c^*(t)\rho_{eg} - \Omega_p(t)\rho_{me}\right) - \frac{\Gamma_m}{2}\rho_{mg}$$

for $\Omega_p = 0$: $\rho_{gg} = 1$, all other $= 0$

small Ω_p: $\rho_{eg} = O(\Omega_p)$, $\rho_{ee} = O(|\Omega_p|^2)$

$\rho_{em} = O(|\Omega_p|^2)$, $\rho_{mm} = O(|\Omega_p|^2)$

$\rho_{mg} = O(\Omega_p)$

$\longrightarrow$ keep only ρ_{eg}, ρ_{mg} (and $\rho_{gg} \approx 1$)!

$$\begin{cases} \dfrac{d\rho_{eg}}{dt} \simeq -i(\omega_e - \omega_g)\rho_{eg} + i\Omega_p(t)\cdot 1 + i\Omega_c(t)\rho_{mg} - \dfrac{\Gamma_e}{2}\rho_{eg} \\ \dfrac{d\rho_{mg}}{dt} = -i(\omega_m - \omega_g)\rho_{mg} + i\Omega_c^*(t)\rho_{eg} - \dfrac{\Gamma_m}{2}\rho_{mg} \end{cases}$$

Assume both $\Omega_p(t)$ and $\Omega_c(t)$ to be monochromatic

$$\Omega_p(t) = \Omega_p e^{-i\omega_p t} \quad , \quad \Omega_c = e^{-i\omega_c t}$$

Coupling beam Ω_c resonant with $m \rightarrow e$

$$\rho_{eg}(t) = \tilde{\rho}_{eg}(t)e^{-i\omega_p t} \quad , \quad \rho_{mg}(t) = \tilde{\rho}_{mg}(t)e^{-i(\omega_p - \omega_c)t}$$

$$\begin{cases} \dfrac{d}{dt}\tilde{\rho}_{eg} = -i(\omega_e - \omega_g - \omega_p)\tilde{\rho}_{eg} + i\Omega_p + i\Omega_c\tilde{\rho}_{mg} - \dfrac{\Gamma_e}{2}\tilde{\rho}_{eg} \\ \dfrac{d}{dt}\tilde{\rho}_{mg} = -i(\omega_m - \omega_g - \omega_p + \omega_c)\tilde{\rho}_{mg} + i\Omega_c^*\tilde{\rho}_{eg} - \dfrac{\Gamma_m}{2}\tilde{\rho}_{mg} \end{cases}$$

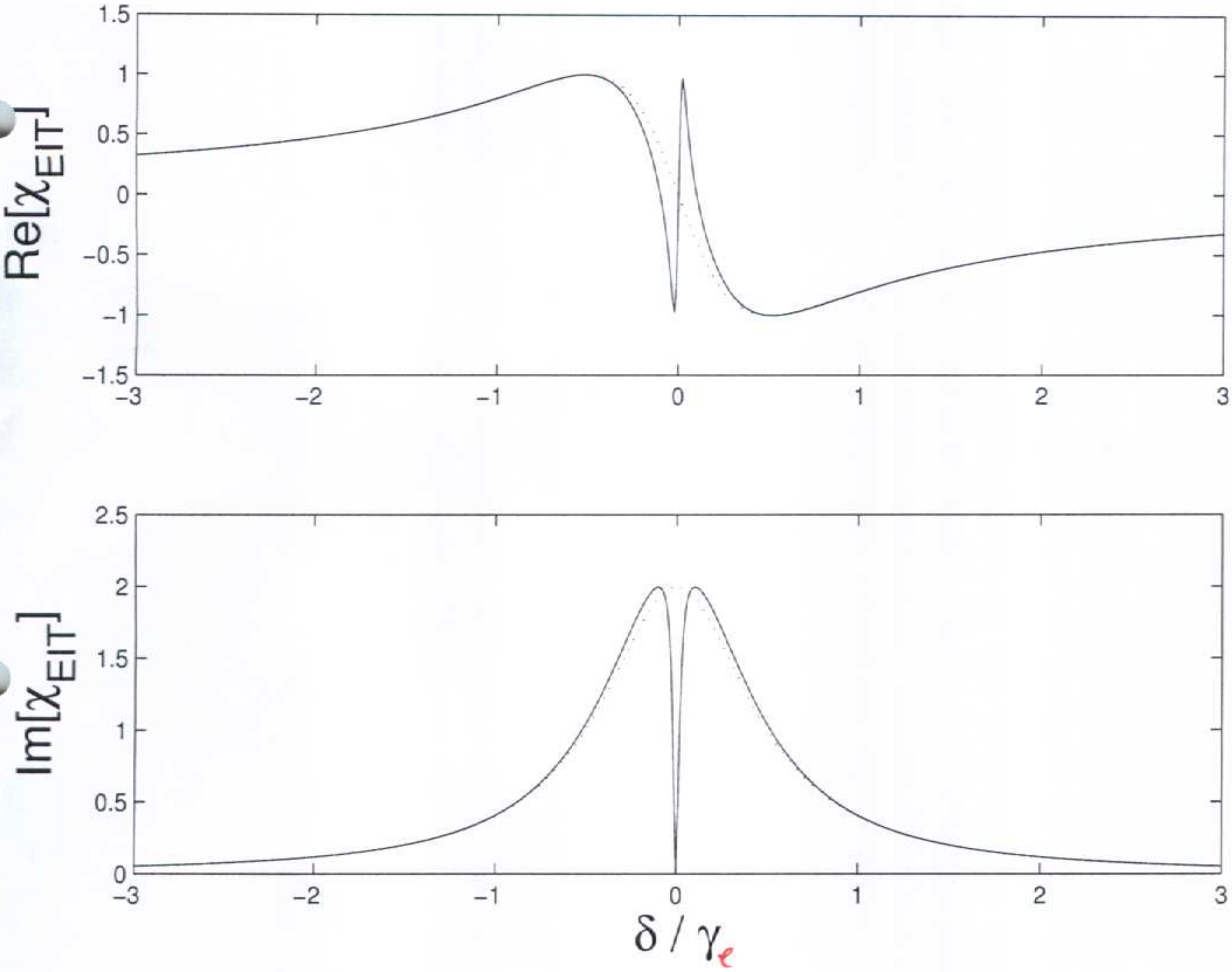
<u>steady state</u>

$$\tilde{\rho}_{mg} = \frac{\Omega_c^*\tilde{\rho}_{eg}}{\omega_m - \omega_g + \omega_c - \omega_p - i\Gamma_m/2}$$

$$\tilde{\rho}_{eg} = \frac{\Omega_p}{\omega_e - \omega_g - \omega_p - i\Gamma_e/2 - \dfrac{|\Omega_c|^2}{\omega_m + \omega_c - \omega_g - \omega_p - i\Gamma_m/2}}$$

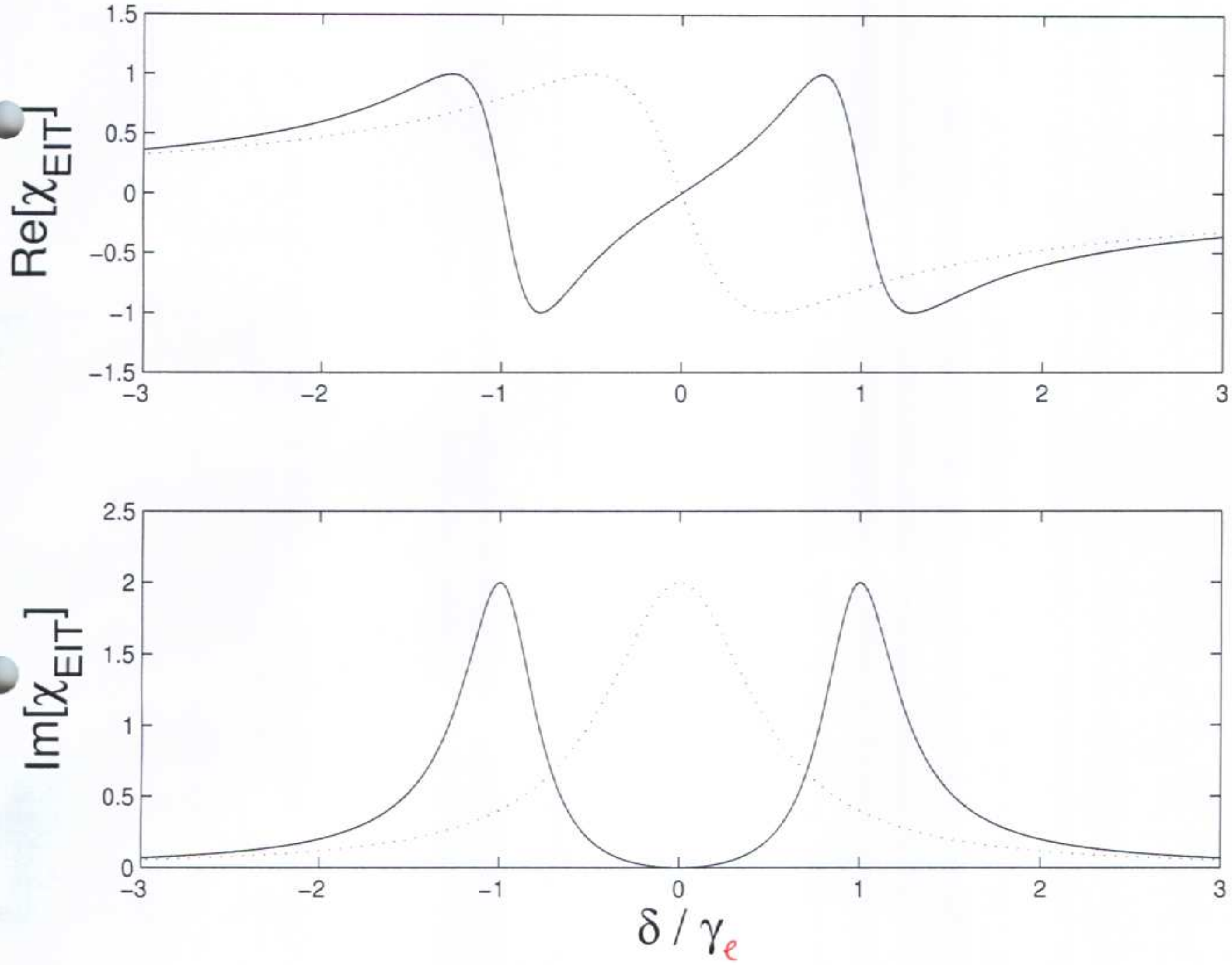
$\Omega_c = 0{,}1\ \gamma_e$

$\gamma_m = 10^{-3}\gamma_e$



$\Omega_c = 1 \cdot \gamma_e$

$\gamma_m = 10^{-3} \gamma_e$



gas of density n :

$$\chi(\omega_p) = \frac{n|d_{eg}|^2/\hbar}{\omega_e - \omega_g - \omega_p - i\Gamma_e/2 - \frac{|\Omega_c|^2}{\omega_m + \omega_c - \omega_g - \omega_p - i\Gamma_m/2}}$$

group velocity

$$v_g = \frac{d\omega}{dk} = \frac{c}{\frac{d}{d\omega}(\sqrt{\varepsilon}\,\omega)} \quad , \quad \varepsilon = 1 + 4\pi\chi \text{ assumed } \underline{\text{real}}$$

Assume :

* resonant dressing $\omega_m + \omega_c = \omega_e$
* work close to resonance $|\omega_p - (\omega_e - \omega_g)| << \Gamma_e$
* weak damping from m : $\Gamma_m << \Omega_c, \Gamma_e$
  but also : $\Gamma_m \cdot \Gamma_e << |\Omega_c|^2$

Under these assumptions :

$$\chi(\omega_p) \simeq \frac{n|d_{eg}|^2}{\hbar|\Omega_c|^2}\left(\omega_p - (\omega_e - \omega_g)\right) + i\, n|d_{eg}|^2 \frac{\Gamma_m}{\hbar|\Omega_c|^2}$$

to be compared with undressed atom, for which

$$\chi(\omega_p) \simeq 2i\,\frac{n|d_{eg}|^2}{\hbar\Gamma_e}$$

* suppressed absorption : $n|d_{eg}|^2 \frac{\Gamma_m}{|\Omega_c|^2}$ vs $\frac{2n|d_{eg}|^2}{\Gamma_e}$

$\longrightarrow$ Electromagnetically Induced Transparency effect

spontaneous emission rate $\Gamma_e = \frac{4}{3} \frac{(\omega_e-\omega_g)^3}{c^3} \frac{|d_{eg}|^2}{\hbar}$

Γ_m determined by extrinsic physics, spontaneous emission rate negligible.

* $\mathrm{Re}(\chi(\omega_p)) \simeq \frac{3}{32\pi^3} (n\lambda_{eg}^3) \frac{\Gamma_e}{|\Omega_c|^2} \cdot \delta_p$

Group velocity :

$$\frac{d}{d\omega_p}\sqrt{1+4\pi\chi} \simeq \frac{d}{d\omega_p}\left[2\pi \frac{3}{32\pi^3} (n\lambda_{eg}^3) \frac{\Gamma_e}{|\Omega_c|^2}(\omega_p-(\omega_e-\omega_g))\right]$$

$$= \frac{3}{16\pi^2}(n\lambda_{eg}^3)\frac{\Gamma_e}{|\Omega_c|^2}$$

$$\Big(\underbrace{\omega_p \frac{d}{d\omega_p}\sqrt{1+4\pi\chi}}_{\gg} + \underbrace{\sqrt{1+4\pi\chi}}\Big) \simeq \frac{3}{16\pi^2}\frac{\Gamma_e\omega_p}{|\Omega_c|^2} n\lambda_{eg}^3$$

as ω_p optical $\gg$ Γ_e, Ω_c by orders of magnitude

So :
$$\frac{v_g}{c} \simeq \frac{16\pi^2}{3}\frac{|\Omega_c|^2}{\Gamma_e\omega_p}\cdot\frac{1}{n\lambda_{eg}^3}$$

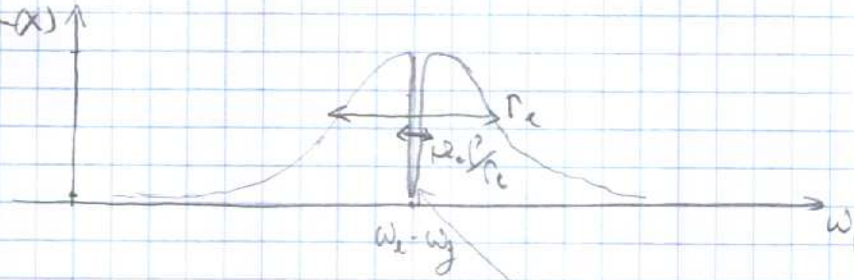
Inserting numerical parameters for ultracold atomic clouds:

$n \simeq 10^{14}\,\mathrm{cm}^{-3}$, $\lambda_{eg} \simeq 800\,\mathrm{nm}$, $\Omega_c \simeq \Gamma_e \simeq 10\,\mathrm{MHz}$, $\omega_p \simeq 2{,}5\ 10^{15}\,\mathrm{s}^{-1}$

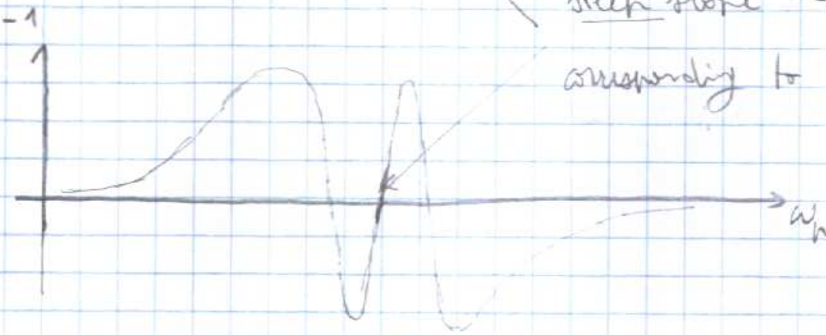
$\Longrightarrow \frac{v_g}{c} \simeq 1{,}24\ \mathrm{m s}^{-1}$!!!

Refractive index and absorption spectrum:

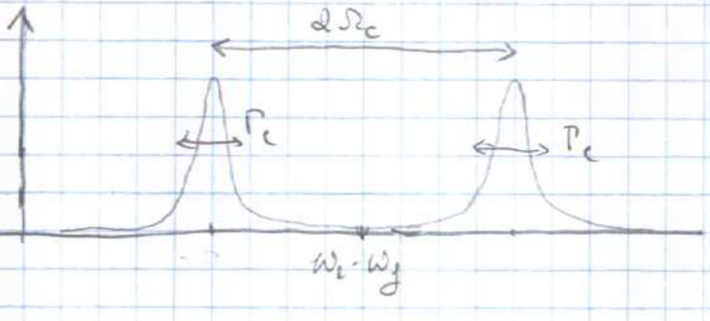
for $\Omega_c \ll \Gamma_e$



steep slope $\rightarrow v_g/c \ll 1$
corresponding to transparency dip



NOTE: for $\Omega_c \gg \Gamma_e$:



$\rightarrow$ Autler - Townes doublet: 2 independent lines $\rightarrow$ steepest slope corresponds to absorption peak

* EIT effect arises from more remarkable quantum interference effect than simple A.T. splitting

* can be seen as atoms being trapped in "dark state"

$$\frac{\Omega_p}{\sqrt{|\Omega_p|^2+|\Omega_c|^2}}\,|m\rangle - \frac{\Omega_c}{\sqrt{|\Omega_p|^2+|\Omega_c|^2}}\,|g\rangle = |\text{dark}\rangle$$

* dark state does not interact with light as:

$\Omega_p |m\rangle \xrightarrow{\Omega_c}$

$|e\rangle$ interfere destructively

$-\Omega_c |g\rangle \xrightarrow{\Omega_p}$

Much interesting physics connected to $|dark\rangle$:

* EIT, slow v_g
* effective magnetic fields for neutral atoms
* velocity selective coherent population trapping (one of first techniques for sub-recoil laser cooling → C. Cohen-Tannoudji's Nobel prize in 1997)

<u>history</u>
* originally discovered in Pisa by Alzetta, Gozzini.
* theoretically interpreted by Orriols and Arimondo
* independently rediscovered by Imamoglu and Harris from different point of view

Bibliography

* G Alzetta, A. Gozzini, L Moi, G. Orriols, Nuovo Cimento B 36, 5 (1976);
* E. Arimondo, G Orriols, Lett Nuovo Cimento 17, 333 (1976)

  ↳ first studies of coherent population trapping

* E. Arimondo in "Progress in Optics", ed. E. Wolf, vol. 35, pag 257 (1996)

  ↳ review from Pisa point of view

* S. Harris, Physics Today 50 (7), 36 (1997)

  ↳ the "rediscovery" of EIT : a review.

* A Aspect, E. Arimondo, R Kaiser, N Vansteenkiste, and C Cohen-Tannoudji, Phys. Rev. Lett 61, 826 (1988)

  ↳ the paper that assured Nobel prize to CCT.

* L.V Hau, S.E. Harris, Z Dutton, C.H. Behroozi, Nature 397, 594 (99)

  ↳ slow light in BEC at 17 m/s.

* Y.-J. Lin, R.L Compton, K. Jimenez-Garcia, J.V. Porto, I.B Spielman, Nature 462, 628 (2009)

  ↳ synthetic magnetic fields for ultracold neutral atoms.

Addenda on coherent driving

$$\begin{cases} \Omega_L(t) = \bar{\Omega}_L e^{-i\omega_L t} \\ \Omega_c(t) = \bar{\Omega}_c e^{-i\omega_c t} \end{cases}$$

$$H = \hbar\omega_e |e\rangle\langle e| + \hbar\omega_m |m\rangle\langle m| + \hbar\omega_g |g\rangle\langle g|$$
$$+ \hbar\Omega_c(t) |e\rangle\langle m| + \hbar\Omega_c^*(t) |m\rangle\langle e|$$
$$+ \hbar\Omega_L(t) |e\rangle\langle g| + \hbar\Omega_L^*(t) |g\rangle\langle e|$$

Schrödinger eq : $i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle$

Define $U(t) = \exp\left[+\frac{i}{\hbar}\hat{K}t\right]$ with $\hat{K} = \hbar\omega_L |e\rangle\langle e| + \hbar(\omega_L - \omega_c)|m\rangle\langle m|$

$$|\psi'(t)\rangle = U(t)|\psi(t)\rangle$$

$$\frac{d}{dt}|\psi'(t)\rangle = \frac{dU}{dt}|\psi(t)\rangle + U(t)\frac{d}{dt}|\psi(t)\rangle$$

$$i\hbar\frac{d}{dt}|\psi'\rangle = i\hbar\frac{dU}{dt}U^\dagger U|\psi\rangle + UH|\psi\rangle$$
$$= \underbrace{\left(i\hbar\frac{dU}{dt}U^\dagger + UHU^\dagger\right)}_{H'}|\psi'\rangle$$

$$H' = -\hat{K} + e^{\frac{i}{\hbar}\hat{K}t} H e^{-\frac{i}{\hbar}\hat{K}t} = -\hbar\omega_L|e\rangle\langle e| + \hbar(\omega_c - \omega_L)|m\rangle\langle m| +$$
$$+ \hbar\Omega_c(t)\, e^{i\omega_L t}|e\rangle\langle m| e^{-i(\omega_L-\omega_c)t} + h.c. +$$
$$+ \hbar\Omega_L(t)\, e^{i\omega_L t}|e\rangle\langle g| + h.c. + \sum_{i=g,e,m}\hbar\omega_i |i\rangle\langle i| =$$

$$H' = \hbar\omega_g |g\rangle\langle g| + \hbar(\omega_e - \omega_L)|e\rangle\langle e| +$$
$$+ \hbar(\omega_m + \omega_c - \omega_L)|m\rangle\langle m| +$$
$$+ \hbar\bar{\Omega}_c |e\rangle\langle m| + \hbar\bar{\Omega}_c^+ |m\rangle\langle e| +$$
$$+ \hbar\bar{\Omega}_L |e\rangle\langle g| + \hbar\bar{\Omega}_L^+ |g\rangle\langle e|$$

→ eliminated the dependence of $\Omega_{c,L}(t) = \bar{\Omega}_{c,L} e^{-i\omega_{c,L}t}$

→ at the expense of redefining state vector
$|\psi'(t)\rangle = U(t)|\psi(t)\rangle$.

→ Physical observables:

* population $p_i = \langle\psi|i\rangle\langle i|\psi\rangle = \langle\psi'|i\rangle\langle i|\psi'\rangle$
unchanged.

* dipole moment $\langle\psi(t)|d_{eg}(|e\rangle\langle g| + |g\rangle\langle e|)|\psi(t)\rangle =$

$$= \langle\psi' U| d_{eg}(|e\rangle\langle g| + |g\rangle\langle e|)|U^+\psi'\rangle =$$

$$= \Big(e^{+i\omega_L t}\langle\psi'|e\rangle\langle g|\psi'\rangle +$$
$$+ e^{-i\omega_L t}\langle\psi'|g\rangle\langle e|\psi'\rangle\Big) d_{eg}$$

$$= \left(e^{i\omega_L t}\rho'_{ge} + e^{-i\omega_L t}\rho'_{eg}\right) d_{eg}$$