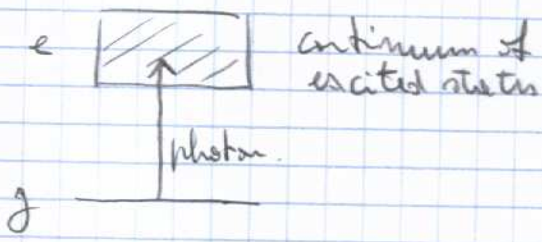
Lecture 9 : Statistical properties of radiation. Photodetection signals and coherence.

Model of photo-detector



$$H = \int \frac{d^3k}{(2\pi)^3} \sum_{\varepsilon} \hbar c k \, a^{\dagger}_{k\varepsilon} a_{k\varepsilon} + V(R) \left(+ \frac{1}{2m}\left(P - \frac{q}{c} A(R)\right)^2\right.$$

$H_{det} = \frac{P^2}{2m} + V(R)$ has ground state + continuum of excited states

At lowest order in q: $H_{int} = -\frac{q}{2mc}\left(P \cdot A(R) + A(R) \cdot P\right) \simeq$

$\simeq -\frac{q}{mc} P \cdot A(0)$ in the dipole approxim

Transition rate to manifold of e states:

Fermi golden rule $\Gamma = \frac{2\pi}{\hbar} \int df \, |M_{fi}|^2 \, \delta(E_f - E_i)$

final states: $\int df = \int df_{field} \cdot \int de_{atom}$

$$\Gamma = \frac{2\pi}{\hbar} \int df_{field} \int de_{atom} \left| \langle f_{field}, e_{atom} | P \cdot A(0) | i_{field}, g \rangle \right|^2 \cdot \left(\frac{q}{mc}\right)^2 \cdot$$

$$\cdot \, \delta\left(E_{f_{field}} + E_{e_{atom}} - E_{i_{field}} - E_{g_{at}}\right) =$$

$$= \frac{2\pi}{\hbar} \int df_{field} \cdot \int de_{atom} \; |\langle f_{field} | A(0) | i_{field} \rangle|^2$$

$$\cdot \, |\langle e_{atom} | p | g \rangle|^2 \cdot \left(\frac{q}{mc}\right)^2 \delta\left(E_{f_{field}} + E_{e_{atom}} - E_{i_{field}} - E_g\right)$$

$$\underline{A}(R) = \int \frac{d^3k}{(2\pi)^3} \cdot \sum_{\varepsilon} \sqrt{\frac{2\pi\hbar c}{k}} \left[\hat{\varepsilon}\, e^{ikR} a(k,\varepsilon) + \hat{\varepsilon}\, e^{-ikR} a^{+}(k,\varepsilon) \right]$$

$$= A^{(+)}(R) + A^{(-)}(R)$$

where : $A^{(+)}(R)$ involves $a(k, \varepsilon)$

evolves in time as e^{-ickt}

$\Rightarrow$ "positive frequency" component.

$A^{(-)}(R) \longrightarrow$ "negative frequency" component.

$\langle f | A^{(+)} | i \rangle \neq 0$ only if $E_f < E_i \rightarrow$ destroys photon

$\langle f | A^{(-)} | i \rangle \neq 0$ only if $E_f > E_i \rightarrow$ creates photon.

Here, $E_{e_{atom}} > E_g$ imposes $E_{f_{field}} < E_{i_{field}}$

$\Rightarrow$ only $A^{(+)}$ part gives contribution ~~to~~

$\langle f_{field} | A(0) | i_{field} \rangle$.

Assume that detector bandwidth >> bandwidth of photon states

$$\Gamma = \frac{2\pi}{\hbar}\left(\frac{q}{mc}\right)^2 \int df_{field} \, |\langle f_{field} | A^{(+)}(0) | i_{field} \rangle|^2 \cdot$$

$$\cdot \int de_{atom} \, |\langle e_{atom} | P | g \rangle|^2 \, \delta(E_{f_{field}} + E_{e_{atom}} - E_{i_{field}} - E_g)$$

$$\simeq \frac{2\pi}{\hbar}\left(\frac{q}{mc}\right)^2 \int df_{field} \, |\langle f_{field} | A^{(+)}(0) | i_{field} \rangle|^2 \cdot |P_{eg}|^2 \, \rho_{at}$$

where $P_{eg} = \langle e_{atom} | P | g \rangle$ for the specific region of final states that is excited, i.e.

$E_{e_{atom}} = E_g + E_{field}$

$\hookrightarrow$ energy of photons in beam.

($A^{(+)}(0)$ destroys a photon)

ρ_{at} = density of atomic states at $E_{e_{atom}}$.

$$= \frac{2\pi}{\hbar}\left(\frac{q}{mc}\right)^2 |P_{eg}|^2 \rho_{at} \cdot \langle i_{field} | A^{(-)}(0) \, A^{(+)}(0) | i_{field} \rangle = \Gamma$$

Photodetector signal is proportional to <u>positive energy part</u> of the <u>field intensity</u>

Several photo-detectors : coincidence counting

Γ_2 for detectors at r_1, r_2 simultaneously clicking :

$$\Gamma_2 = \frac{2\pi}{\hbar}\left(\left(\frac{q}{mc}\right)^2 |P_{vg}|^2 \rho_{at}\right)_1 \left(\left(\frac{q}{mc}\right)^2 |P_{vg}|^2 \rho_{at}\right)_2 \cdot$$

$$\cdot \langle i_{field} | A^{(-)}(r_1)\, A^{(-)}(r_2)\, A^{(+)}(r_2)\, A^{(+)}(r_1) | i_{field} \rangle$$

$\rightarrow$ same time clicks.

[More details in 9.4 bis]

More generally : joint detection rates at (r_1, t_1), (r_2, t_2)

$$\Gamma_2 = c^{te} \times \langle i_{field} | A^{(-)}(r_1, t_1)\, A^{(-)}(r_2, t_2)\, A^{(+)}(r_2, t_2)\, A^{(+)}(r_1, t_1) | i_{field} \rangle$$

where $t_2 > t_1$: earlier times have to sit in the outer positions of the $\langle \ldots \rangle$

for same time $t_1 = t_2$: order does not matter.

In summary : photodetection signal is proportional to expectation value of field correlation function

* normally ordered
* earlier times $\rightarrow$ inner positions

Normalized quantities allow to get rid of model-dependent factors:

$$g^{(2)}(r_1,t_1;r_2,t_2) = \frac{P_2(r_1,t_1;r_2,t_2)}{P(r_1,t_1)\,P(r_2,t_2)} =$$

$$= \frac{\langle A^{(-)}(r_1,t_1)\,A^{(-)}(r_2,t_2)\,A^{(+)}(r_2,t_2)\,A^{(+)}(r_1,t_1)\rangle}{\langle A^{(-)}(r_1,t_1)\,A^{(+)}(r_1,t_1)\rangle\,\langle A^{(-)}(r_2,t_2)\,A^{(+)}(r_2,t_2)\rangle}$$

$\longrightarrow$ "second-order" coherence function.

on coherent state: $\hat{A}^{(+)}(r_1,t_1)\,|i_{field}\rangle = a(r_1,t_1)\,|i_{field}\rangle$

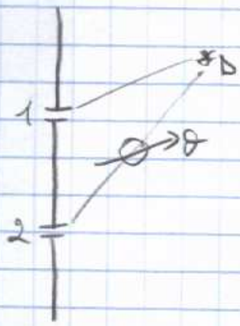
$$\Longrightarrow \quad g^{(2)}(r_1,t_1;r_2,t_2) = 1.$$

* photo-detection signals are statistically independent

Another useful coherence function

$$g^{(1)}(r_1,t_1;r_2,t_2) = \frac{\langle A^{(-)}(r_1,t_1)\,A^{(+)}(r_2,t_2)\rangle}{\left[\langle A^{(-)}(r_1,t_1)\,A^{(+)}(r_1,t_1)\rangle\,\langle A^{(-)}(r_2,t_2)\,A^{(+)}(r_2,t_2)\rangle\right]^{1/2}}$$

* appears in interference pattern:



$$A_D = A_1 + e^{i\vartheta} A_2$$

$$\langle A_D^{(-)} A_D^{(+)}\rangle = \langle (A_1^{(-)} + e^{-i\vartheta} A_2^{(-)})\,(A_1^{(+)} + e^{i\vartheta} A_2^{(+)})\rangle$$

$$= \langle A_1^{(-)} A_1^{(+)}\rangle + \langle A_2^{(-)} A_2^{(+)}\rangle +$$

$$+ 2\,\mathrm{Re}\left[e^{i\vartheta}\langle A_1^{(-)} A_2^{(+)}\rangle\right]$$

* for balanced fields $\langle A_1^{(-)} A_1^{(+)} \rangle = \langle A_2^{(-)} A_2^{(+)} \rangle$,

<u>visibility of fringes</u> as a function of θ is determined by $g^{(1)}(r_1,t_1; r_2,t_2)$

* for coherent states $g^{(1)}(r_1,t_1; r_2,t_2) = 1$.

Coherent states are indeed <u>FULLY COHERENT</u>,
i.e. $g^{(n)}(1, 2 \ldots n) = 1$

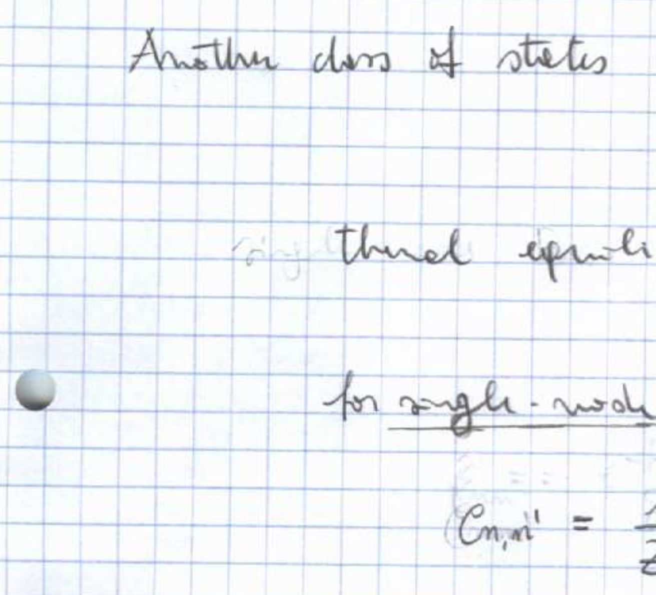
Another class of states : <u>thermal (or chaotic) light</u>

thermal equilibrium state $\hat{\rho} = \frac{1}{Z} \exp(-\beta \hat{H})$
with $\beta = 1/k_B T$

for <u>single-mode field</u> at ω_0 :

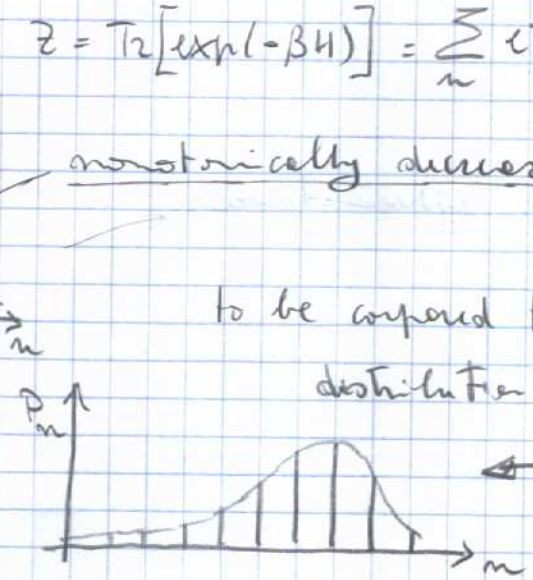
$$\rho_{n,n'} = \frac{1}{Z} \delta_{n,n'} e^{-\beta \hbar \omega_0 n}$$

$$\text{with } Z = \mathrm{Tr}\left[\exp(-\beta H)\right] = \sum_n e^{-\beta \hbar \omega_0 n} = \frac{1}{1 - e^{-\beta \hbar \omega_0}}$$



<u>monotonically decreasing</u> with n.

to be compared to <u>Poissonian</u> distribution of coherent state



Average occupation $\langle n \rangle = \frac{1}{e^{\beta\hbar\omega} - 1}$

with variance $\langle \Delta n^2 \rangle = \frac{1}{e^{\beta\hbar\omega} - 1} + \left(\frac{1}{e^{\beta\hbar\omega} - 1} \right)^2$

$$g^{(2)} = \frac{\langle a^\dagger a^\dagger a a \rangle}{\langle a^\dagger a \rangle^2} = 2$$

$\longrightarrow$ suggests correlations in photo-detection signal

Many independent modes, thermally occupied:

$$\hat{\rho} = \frac{1}{Z} \exp\left(-\sum_n \beta_i \hbar\omega_n a_i^\dagger a_i\right)$$

$\hookrightarrow$ different modes may have different "temperature"

* models "grey-body" radiation, e.g. the emission from a lamp

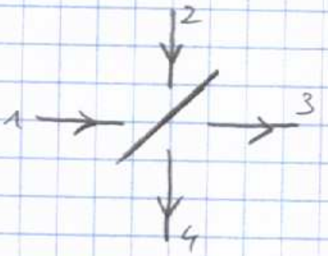
Wick theorem   proof in Y. Castin, lecture notes available at cond-mat/0407118.

single the average of field operators factors into sum of "contractions"

$$\langle A_1 A_2 A_3 A_4 \rangle = \langle A_1 A_2 \rangle \langle A_3 A_4 \rangle + \langle A_1 A_3 \rangle \langle A_2 A_4 \rangle + \langle A_1 A_4 \rangle \langle A_2 A_3 \rangle$$

where A's can be creation or destruction operators

Remarks on beam-splitter :



* coherent states input $\alpha_1, \alpha_2 \longrightarrow$ coherent states output

$$\begin{pmatrix}\alpha_3\\ \alpha_4\end{pmatrix} = S \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}$$

* thermal states input $\bar{n}_1, \bar{n}_2 \longrightarrow$?

$$\hat{a}_3 = t\hat{a}_1 + ir\hat{a}_2$$

$$\hat{a}_4 = t\hat{a}_2 + ir\hat{a}_1$$

$$\langle \hat{a}_3^\dagger a_3\rangle = |t|^2\langle a_1^\dagger a_1\rangle + \left(irt \cancelto{0}{\langle a_1^\dagger a_2\rangle} + c.c.\right) + |r|^2\langle a_2^\dagger a_2\rangle$$

$$= |t|^2 \bar{n}_1 + |r|^2 \bar{n}_2$$

$$\langle a_4^\dagger a_4\rangle = |t|^2\bar{n}_2 + |r|^2 \bar{n}_1$$

$$\langle a_3^\dagger a_3^\dagger a_3 a_3\rangle = 2\langle a_3^\dagger a_3\rangle^2 + \cancelto{0}{\langle a_3^\dagger a_3^\dagger\rangle\langle a_3 a_3\rangle} = 2\left[|t|^2\bar{n}_1 + |r|^2 n_2\right]$$

$$\Longrightarrow g^{(2)}(3,3) = 2$$

idem for $\langle a_4^\dagger a_4^\dagger a_4 a_4\rangle$

$$\langle a_3^\dagger a_4\rangle = (t\, a_1^\dagger - ir\, a_2^\dagger)(t a_2 + ir\, a_1) = irt\,\bar{n}_1 - irt\,\bar{n}_2 =$$

$$= irt(\bar{n}_1 - \bar{n}_2)$$

$\longrightarrow$ some correlations appear in $g^{(2)}(3,4)$ if inputs are not symetric

$$\langle a_3^\dagger a_4^\dagger a_4 a_3 \rangle = \langle a_3^\dagger a_3 \rangle \langle a_4^\dagger a_4 \rangle + \langle a_3^\dagger a_4 \rangle \langle a_4^\dagger a_3 \rangle +$$

$$+ \cancel{\langle a_3^\dagger a_4^\dagger \rangle \langle a_4 a_3 \rangle} \to 0 =$$

$$= \left(|t|^2 \bar{n}_1 + |r|^2 \bar{n}_2\right)\left(|t|^2 \bar{n}_2 + |r|^2 \bar{n}_1\right) +$$

$$+ irt\,(\bar{n}_1 - n_2)\,(-irt)\,(\bar{n}_1 - \bar{n}_2) =$$

$$= |t|^2|r|^2\,\bar{n}_1^2 + |t|^2|r|^2\,\bar{n}_2^2 + \left(|t|^4 + |r|^4\right)\bar{n}_1\bar{n}_2 +$$

$$+ r^2t^2\left(\bar{n}_1^2 + \bar{n}_2^2 - 2\bar{n}_1\bar{n}_2\right)$$

$$= \left(t^4 + r^4 - 2r^2t^2\right)\bar{n}_1\bar{n}_2 + 2r^2t^2\left(\bar{n}_1^2 + \bar{n}_2^2\right)$$

$$g^{(2)}(3,4) = 1 + \frac{r^2t^2(\bar{n}_1 - \bar{n}_2)^2}{(t^2\bar{n}_1 + r^2\bar{n}_2)(t^2\bar{n}_2 + r^2\bar{n}_1)}$$

$\hookrightarrow$ some partial correlation exist

From general result $g^{(2)}(3,3) = 2$, it follows that attenuated thermal light remains thermal, yet with a lower intensity:

$$\bar{n}_2 = 0 \implies \bar{n}_3 = t^2 n_1 \leq n_1$$

This result can be used to support "grey-body" light as a thermodynamical black body filtered with material-dependent "greyness factor".

example:

* single mode $\langle a^+ a^+ a\, a \rangle = 2 \langle a^+ a \rangle^2 + \cancel{\langle a^+ a^+ \rangle \langle a\, a \rangle}$

$$= 2 \langle a^+ a \rangle^2$$

$$\Rightarrow g^{(2)} = 2$$

* $\langle A^{(-)}(r_1)\, A^{(-)}(r_2)\, A^{(+)}(r_2)\, A^{(+)}(r_1) \rangle =$

$$= \langle A^{(-)}(r_1)\, A^{(+)}(r_1) \rangle \langle A^{(-)}(r_2)\, A^{(+)}(r_2) \rangle +$$

$$+ \langle A^{(-)}(r_1)\, A^{(+)}(r_2) \rangle \langle A^{(-)}(r_2)\, A^{(+)}(r_1) \rangle +$$

$$+ \underbrace{\cancel{\langle A^{(-)}(r_1)\, A^{(-)}(r_2) \rangle}}_{0} \underbrace{\cancel{\langle A^{(+)}(r_2)\, A^{(+)}(r_1) \rangle}}_{0}$$

* $\langle A^{(-)}(r_2)\, A^{(+)}(r_1) \rangle = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \frac{2\pi \hbar c}{\sqrt{k k'}} \langle a_k^+ e^{-ik r_2}\, a_{k'} e^{ik' r_1} \rangle$

for thermal state $\langle a_k^+ a_{k'} \rangle = \bar{n}_k (2\pi)^3 \delta^{(3)}(k-k')$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{2\pi \hbar c}{k} \bar{n}_k\, e^{ik(r_1 - r_2)} = g^{(1)}(r_2, r_1)$$

g localized around $r_1 \simeq r_2$ with
spatial extension $\simeq 1/\Delta k = l_c$, with
$\Delta k \sim \Delta\omega / c$ linewidth of light

$$g^{(2)}(r_1, r_2) = 1 + \frac{|\langle A^{(-)}(r_2)\, A^{(+)}(r_1)\rangle|^2}{\langle A^{(-)}(r_1) A^{(+)}(r_1)\rangle \langle A^{(-)}(r_2) A^{(+)}(r_2)\rangle}$$

$$= \begin{cases} 2 & \text{for } r_1 = r_2 \\ \text{tends to } 1 & \text{for } |r_1 - r_2| \gg 1/\Delta k = \ell_c \end{cases}$$

ℓ_c = "coherence length"

NOTE : same behaviour is obtained if the A's are considered to be classical, independent random variables with gaussian distribution

Similar behaviour in time on a time scale $\tau_c \simeq \ell_c/c$.

→ very short for experiments

→ pseudo-chaotic light obtained by diffusing coherent light on suspension of polystyrene balls in water. Here $\tau_c \simeq 0{,}1$ sec.

↳ time-varying speckle patterns

→ classical light has $g^{(2)}(r_1 = r_2) \geq 1$

quantum mechanics allows for "antibunching" effects $g^{(2)}(r_1 = r_2) < 1$.

Ex: single photon wavepacket has $g^{(2)}(r_1, r_2) = 0$ as

$\langle A^{(-)}(r_1) A^{(-)}(r_2) A^{(+)}(r_2) A^{(+)}(r_1)\rangle = 0$.

Hanbury-Brown and Twiss experiment

How to measure the angular radius of a star?

$\theta = 2R/d$

Syrius (canis major):

$R = 1{,}68\ R_\odot$ [solar radius $R_\odot = 6{,}95\ 10^8\,m$]

$d = 5{,}24$ ly [1 light year ly $= 9{,}46\ 10^{15}\,m$]

$\theta \simeq 2R/d = 4{,}7 \cdot 10^{-8}$ rad $\simeq 2{,}7 \cdot 10^{-6}$ degrees

i.e. angular size of a person (2m) at $4 \cdot 10^7\,m = 40.000$ km

i.e the earth circumference !!!

A principal difficulty: diffraction broadening:

optical collection device of radius R_D, e.g lens or mirror.

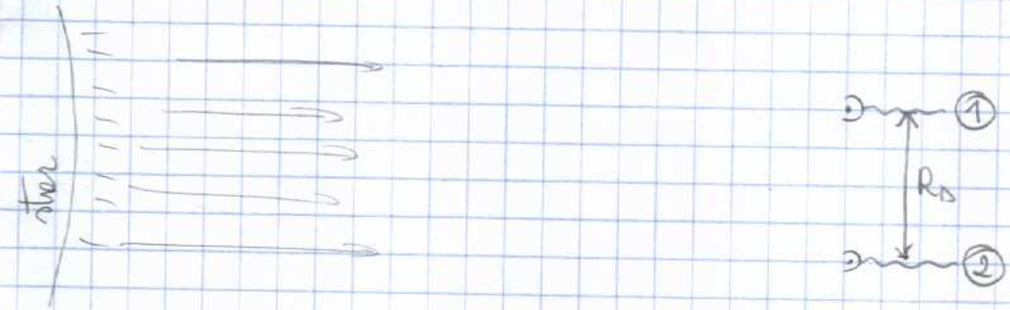
incident plane wave is diffracted into a cone of angular aperture $\theta_D = \lambda / R_D$.

in order for $\theta_D \lesssim \theta$, one needs $\frac{\lambda}{R_D} \lesssim \theta$,

i.e. $R_D \gtrsim \lambda/\theta$ For visible light $\lambda \simeq 600$ nm

$\Longrightarrow \underline{R_D \gtrsim 13\,m}$ very large!!

Consider two photodetectors spread by R_D :



* each of them observes some average photocurrent $\bar{J}_{1,2}$

* what about noise ? can one extract useful information from correlations of noise

$$C = \frac{\langle J_1 J_2 \rangle}{\bar{J}_1 \bar{J}_2} \quad ?$$

Simple model of photodetector :

* photocurrent $J(t)$ proportional to instantaneous field intensity $\langle A^\dagger(r_1,t) A(r_1,t) \rangle$

* neglect for simplicity shot noise in photocurrent (uncorrelated for two detectors)

$$C = \frac{\langle A^\dagger(r_1,t) A^\dagger(r_2,t) A(r_1,t) A(r_2,t) \rangle}{\langle A^\dagger(r_1,t) A(r_1,t) \rangle \langle A^\dagger(r_2,t) A(r_2,t) \rangle} = g^{(2)}(r_1,t;r_2,t)$$

Light emission from star can be modelled as :

* thermal light

* narrow emission line (exercise : relax this hypothesis !)

9.12

$$g^{(2)}(r_1,t;r_2,t) = 1 + \frac{|\langle A^{\dagger}(r_1,t)\,A(r_2,t)\rangle|^2}{\langle A^{\dagger}(r_1,t)\,A(r_1,t)\rangle\langle A^{\dagger}(r_2,t)\,A(r_2,t)\rangle}$$

↳ expect $|r_1-r_2|$-dependent bunching

Electric field at r = sum of field emitted by all points of star.

Emission from different points uncorrelated

$$A(r) = \int d^2x\; A_s(x)\; e^{i\frac{\omega}{c}|x-r|}$$

↳ surface of star

$$\langle A^{\dagger}(r_1,t)\,A(r_2,t)\rangle = \int d^2x \int d^2x' \,\langle A_s^{\dagger}(x)\,A_s(x')\rangle\, e^{-i\frac{\omega}{c}|x-r_1|}\, e^{i\frac{\omega}{c}|x'-r_2|}$$

$$= \int d^2x\; \langle A_s^{\dagger}(x)\,A_s(x)\rangle \cdot e^{i\frac{\omega}{c}|r_1-r_2|\phi(x)}$$

* if $\frac{\omega}{c}|r_1-r_2|\phi(x) \ll$ $\longrightarrow$ $\langle A^{\dagger}(r_1,t)\,A(r_2,t)\rangle \simeq$

$$\simeq \langle A^{\dagger}(r_1,t)\,A(r_1,t)\rangle = \langle A^{\dagger}(r_2,t)\,A(r_2,t)\rangle$$

* if $\frac{\omega}{c}|r_1-r_2|\phi(x)$ can be $\approx 2\pi$ $\longrightarrow$ $\langle A^{\dagger}(r_1,t)\,A(r_2,t)\rangle$ quickly drops.

Maximum value of $\phi(x) \simeq \Theta = 2R/d$

$\Rightarrow$ critical $|r_1-r_2| \simeq \frac{2\pi c}{\omega\Theta} = \lambda/\Theta \simeq 13\,\text{m}$ for Syrius.

Experiment :

* measure $C(|z_1 - z_2|)$ and looks for characteristic distance ρ at which it drops

↳ angular radius of star can be extracted as $\theta \simeq \lambda / \rho$.

→ Similar experiment can be performed to obtain angular radius of μ-wave astrophysical sources

→ Related expts using :

- matter waves from ultracold atomic clouds of bosons (Aspect, 2005)

-- inverted "antibunching" effect for Fermi fields, e.g. electrons in mesoscopic conductors or ultra-cold fermionic atoms (Aspect, 2006)

- many other expts (and proposals) exploit the idea of extracting physical information from noise.

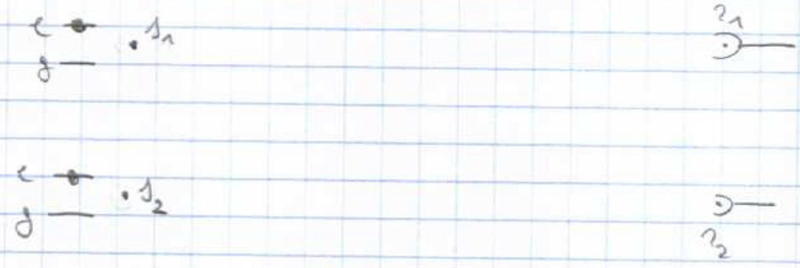
All present calculations are based on classical theory :

thermal radiation $\Longleftrightarrow$ stochastic Gaussian variables.

↳ clearly OK for μ-waves

what about optical fields where emission process is quantized ?

Feynman interpretation of HB-T correlation in purely quantum terms:



two atoms in excited state located at s_1, s_2

$\downarrow$

emission of single photons at ω_0

two photodetectors at r_1, r_2

What is probability of observing both photons on detector 1 (or 2)?

$$A_1 = A(r_1) = \eta \cdot \left[e^{i\frac{\omega}{c}|r_1 - s_1|} A_{em}(s_1) + e^{i\frac{\omega}{c}|r_1 - s_2|} A_{em}(s_2) \right]$$

$$A_2 = A(r_2) = \eta \left[e^{i\frac{\omega}{c}|r_2 - s_1|} A_{em}(s_1) + e^{i\frac{\omega}{c}|r_2 - s_2|} A_{em}(s_2) \right]$$

$$\langle A_1^\dagger A_1 \rangle = |\eta|^2 \Big[\underbrace{\langle A_{em}^\dagger(s_1) A_{em}(s_1) \rangle}_{1} + \langle A_{em}^\dagger(s_2) A_{em}(s_2) \rangle + \ldots \underbrace{\langle A_{em}^\dagger(s_1) A_{em}(s_2) \rangle}_{0} + h.c. \Big] = 2|\eta|^2$$

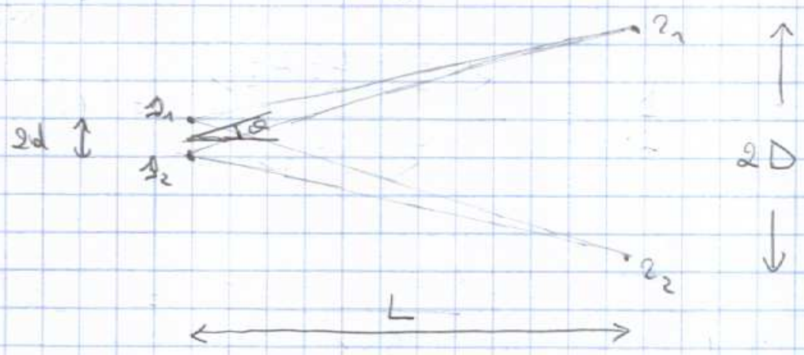
$$\langle A_1^\dagger A_1^\dagger A_1 A_1 \rangle = 4|\eta|^4$$

$$\langle A_1^\dagger A_2^\dagger A_2 A_1 \rangle = 2|\eta|^4 + 2|\eta|^4 \cos\phi = 2|\eta|^4 \left[1 + \cos\phi\right]$$

$$\text{with } \phi = \frac{\omega}{c}\left[|r_1 - s_1| + |r_2 - s_2| - |r_1 - s_2| - |r_2 - s_1| \right]$$

$$g^{(2)}(1,1) = 1, \quad \text{but } g^{(2)}(1,2) = \frac{1}{2}(1 + \cos\phi)$$

For detectors far away from sources:



$$|r_1 - s_1| - |r_1 - s_2| = 2d \, D/L \qquad \left[\theta \simeq D/L\right]$$

$$|r_2 - s_1| - |r_2 - s_2| = -2d \, D/L$$

$$\Rightarrow \phi \simeq \frac{\omega}{c}\frac{4dD}{L} \longrightarrow \text{fringes as funct. } D \text{ with period } \frac{\lambda L}{4d}$$

even if two sources are incoherent at 1-photon level
[$\langle A_1^\dagger A_1 \rangle$ do not show any fringe]

$\longrightarrow$ interference fringes are visible in 2-photon correlation signal

ex: Ou-Mandel interferometer PRL 62, 2941 ('89)

Interferometric detection of BCS pairing in Fermi gases.
PRL 94, 223202 ('05)

Classical model:

$A_{in}(s_1)$ and $A_{in}(s_2)$ are classical variables

i) $A_{in}(s_1)$ and $A_{in}(s_2)$ deterministic (coherent fields):

$$|A_1|^2 = |\eta|^2 \Big[|A_{in}(s_1)|^2 + |A_{in}(s_2)|^2 + 2\,\mathrm{Re}\Big[A^*_{in}(s_1)\, A_{in}(s_2) \cdot$$
$$\cdot \exp\left(\frac{i\omega}{c}\big[|r_1 - s_2| - |r_1 - s_1|\big]\right)\Big]\Big] \simeq$$

$$\simeq |\eta|^2 \Big\{ |A_{in}(s_1)|^2 + |A_{in}(s_2)|^2 + 2\,\mathrm{Re}\Big[A^*_{in}(s_1)\, A_{in}(s_2) \cdot$$
$$\cdot \exp\left(-\frac{i\omega}{c} 2d \frac{y}{L}\right)\Big]\Big\}$$

$\longrightarrow$ usual Young double-slit fringes.

$$g^{(2)}(1,1) = \frac{|A_1|^4}{(|A_1|^2)^2} = 1 = g^{(2)}(1,2).$$

$\longrightarrow$ standard full coherence.

ii) $A_{in}(s_1, s_2)$ = independent thermal fields

$$\langle A_1^+ A_1 \rangle = |\eta|^2 \Big[\langle A^+_{in}(s_1) A_{in}(s_1) \rangle + \langle A^+_{in}(s_2) A_{in}(s_2) \rangle \Big] = 2|\eta|^2$$

$$\langle A_1^+ A_2 \rangle = |\eta|^2 \Big\{ \langle A^+_{in}(s_1) A_{in}(s_1) \rangle \cdot \exp\left[i\frac{\omega}{c}\big(|r_2 - s_1| - |r_1 - s_1|\big)\right] +$$
$$+ \langle A^+_{in}(s_2) A_{in}(s_2) \rangle \cdot \exp\left[i\frac{\omega}{c}\big(|r_2 - s_2| - |r_1 - s_2|\big)\right]\Big\}$$

$$\langle A_1^+ A_1^+ A_1 A_1 \rangle = 2 \langle A_1^+ A_1 \rangle^2 = 8 |\eta|^4$$

$$\langle A_1^+ A_2^+ A_2 A_1 \rangle = \langle A_1^+ A_1 \rangle \langle A_2^+ A_2 \rangle + \langle A_1^+ A_2 \rangle \langle A_2^+ A_1 \rangle$$

$$= 4|\eta|^4 + |\eta|^4 \Big[1 + 1 + 2 \operatorname{Re} \Big\{ \exp \Big[i \frac{\omega}{c} \times$$

$$\times \left(|r_2 - s_1| - |r_1 - s_1| - |r_2 - s_2| + |r_1 - s_2| \right) \Big] \Big\} \Big] =$$

$$= 4|\eta|^4 + |\eta|^4 \left[2 + 2 \cos \frac{\omega}{c} 4\, dD/_L \right] =$$

$$= |\eta|^4 \left[6 + 2 \cos \frac{4\omega}{c} dD/_L \right]$$

→ fringe visibility is no longer complete

$$v = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{4}{12} = \frac{1}{3} < 1$$

→ general bound for classical states with random phases is $v \leq 1/2$.

see e.g. Scully - Zubairy, "Quantum Optics"

<u>Experiments</u> have observed $v > 3/4$ ruling out classical theories.

see e.g. Ou - Mandel PRL 62, 2941 ('89)