Trento, 5th December 2008
Physics PhD Workshop

Francesco Bariani

DYNAMIC PHOTONIC STRUCTURES USING ELECTROMAGNETICALLY INDUCED TRANSPARENCY

Supervisor: Dr. I. Carusotto

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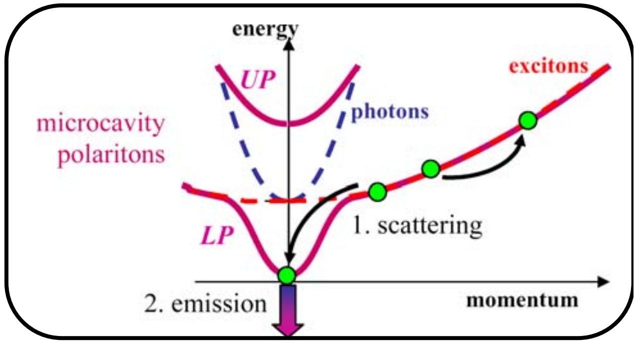
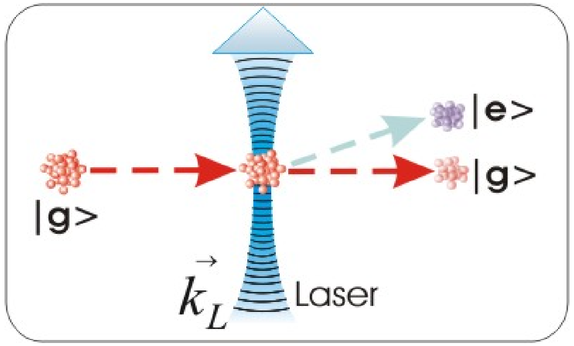
CARUSOTTO @ BEC

1) Exciton-polaritons in semiconductors: Bose-Einstein Condensation (BEC) and SuperFluidity (SF) in collaboration with Wouters, Sarchi (now coming in Trento) (EPFL - Losanna), Ciuti (Denis Diderot - Paris), lattices of quantum dots with Gerace (Pavia) and Imamoglu's group (ETH - Zurich).

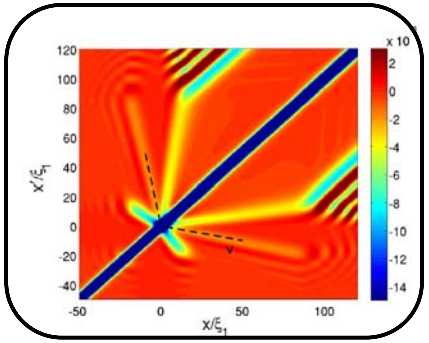
Collaboration with experimental groups: Lagoudakis (EPFL - Losanna), Luis Vina (Universidad Autonoma - Madrid), Alberto Bramati (LKB - Paris).





2) Atom optics, spectroscopy and detection of quantum phases of cold gases with BEC group (Trento), Castin (ENS - Paris), Kollath, Dao, Georges (Polytechnique - Paris).

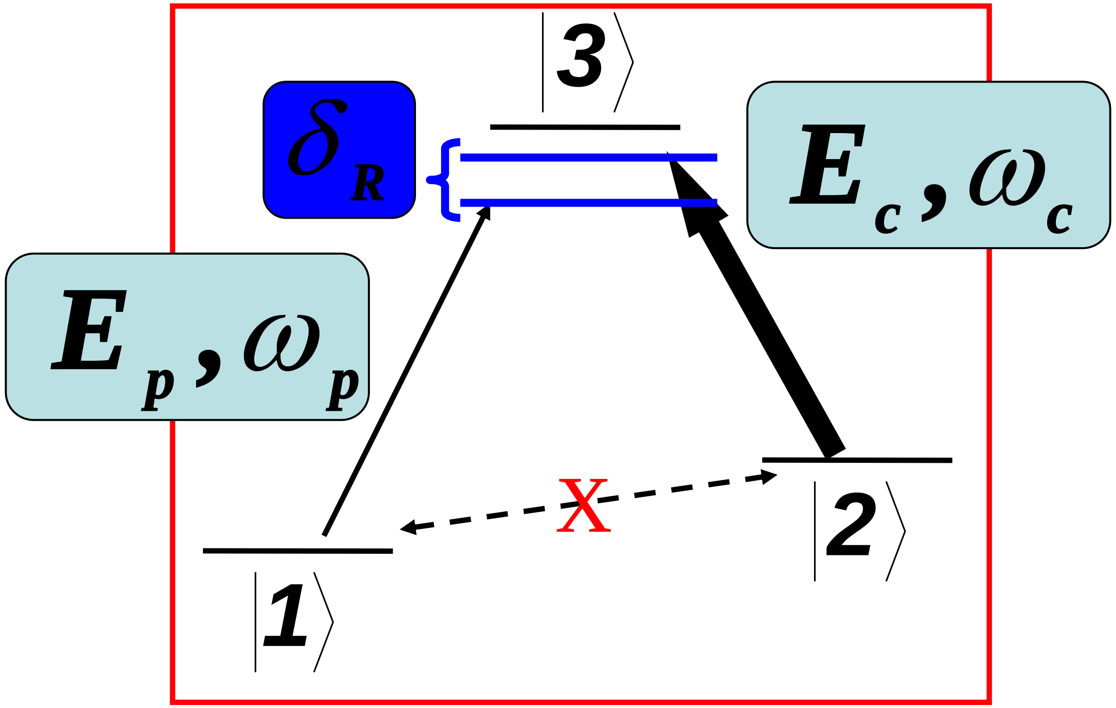
3) Zero-point fluctuations: Ultrastrong coupling regime in radiation-matter interaction and Dynamical Casimir effect with De Liberato, Ciuti, (Denis Diderot - Paris), Hawking radiation in moving BEC with Recati (BEC - Trento) and cosmology group in Bologna.



OUTLINE

- Introduction to EIT
- Polaritons in cold atoms: Static properties
- Polariton in cold atoms: Time-dependent picture

THREE-LEVEL DRESSED ATOM



Dressing field

$$\Omega_c = -\frac{\mu_{23} E_c}{\hbar}$$

Probe field

$$\Omega_p = -\frac{\mu_{13} E_p}{\hbar}$$

Atomic Hamiltonian

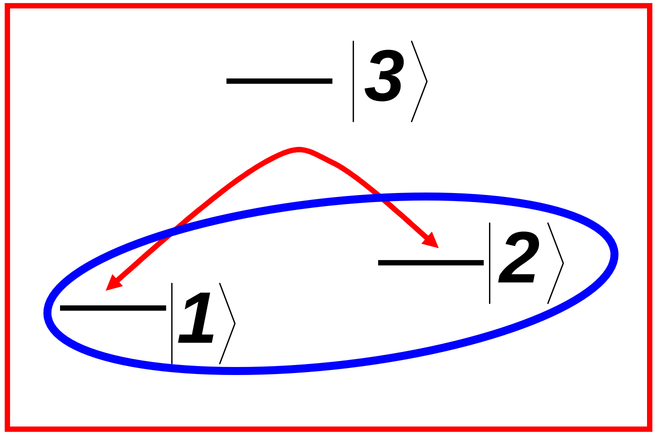
$$\hat{H}_0 = \sum_i \hbar \omega_i |i\rangle\langle i|$$

Atom – Laser interaction Hamiltonian

$$\hat{V}_{AL} = \hbar \frac{\Omega_p}{2} e^{-i\omega_p t} |3\rangle\langle 1| + \hbar \frac{\Omega_c}{2} e^{-i\omega_c t} |3\rangle\langle 2| + h.c.$$

ELECTROMAGNETICALLY INDUCED TRANSPARENCY

RAMAN RESONANCE $\delta_R = 0$



1) No Absorption

$$|NC\rangle \propto \Omega_p |2\rangle + \Omega_c |1\rangle$$

Coherent superposition of ground and metastable state, decoupled from the excited.

2) Tunability of group velocity of light

$$v_{gr} = \frac{c}{1 + \frac{\omega_p}{2} \frac{d\chi}{d\omega_p}} \propto c\Omega_c^2$$

3) Reflectivity dip

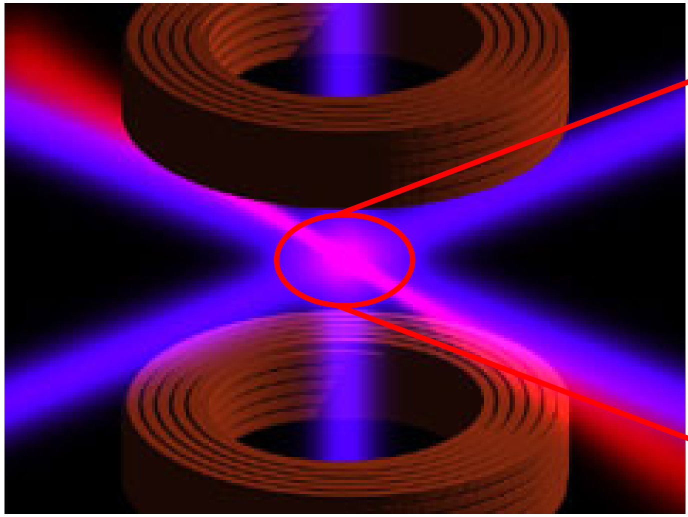
$$v_{ph} = \frac{c}{n} = \frac{c}{\sqrt{1 + \Re[\chi]}} = c$$

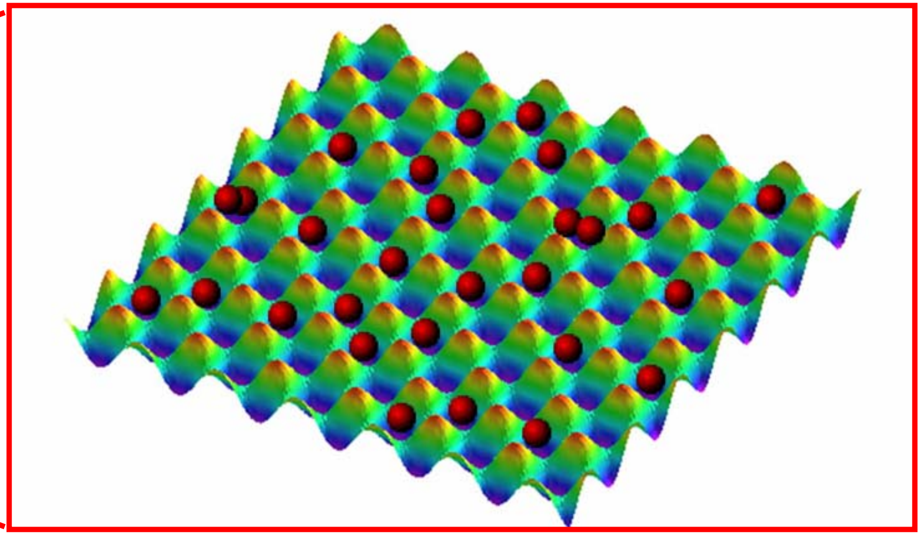
Spectral window $\Delta\omega \propto \Omega_c$

OPTICAL LATTICES & ULTRACOLD ATOMS

Countepropagating laser beams
generate a stationary wave:
• Induced atomic dipole moment
• AC Stark shift

Atomic lattices realized
in periodic optical potential:
• High regularities
• No impurities

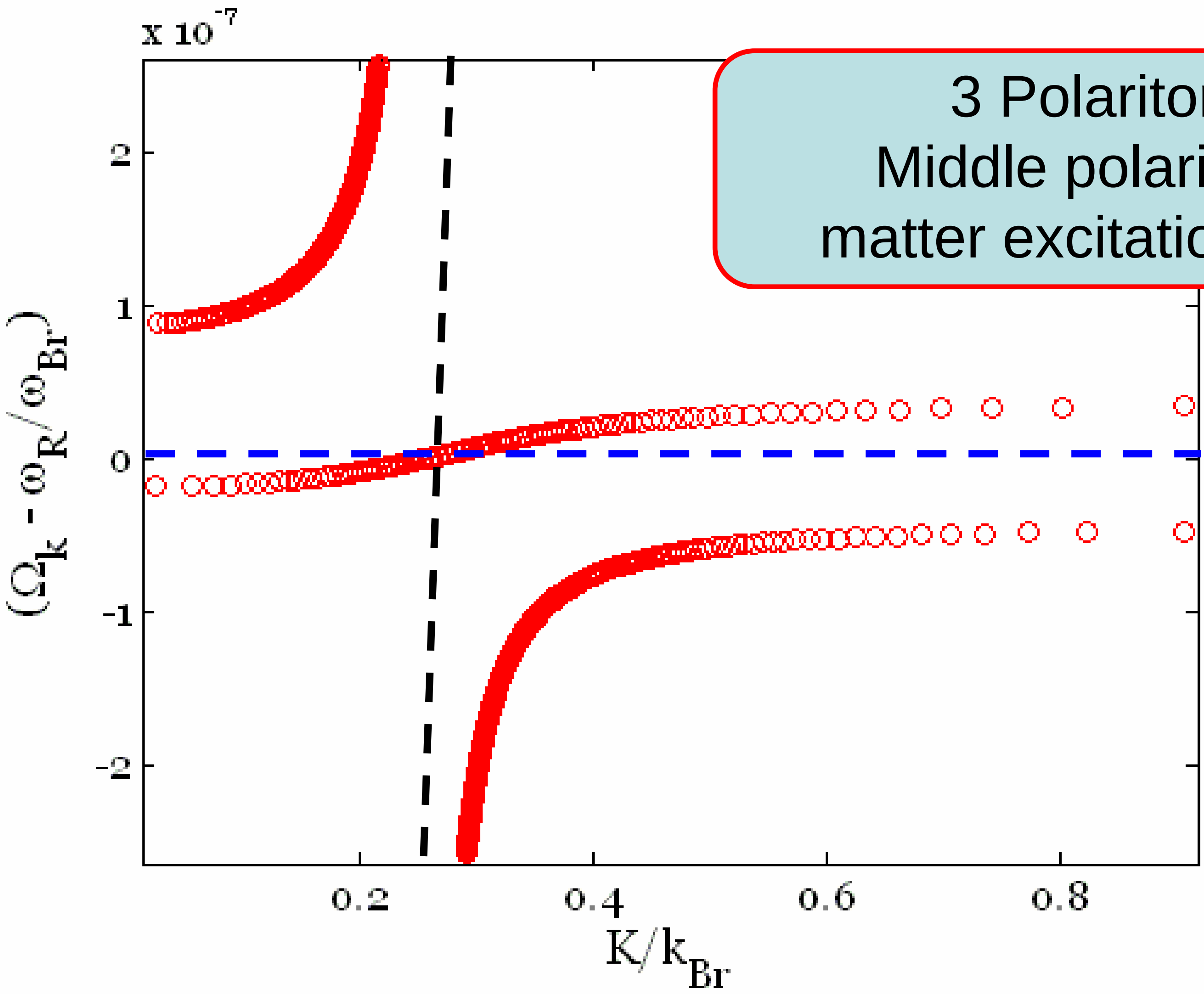




BEC into an optical lattice, raising
adiabatically the lattice intensity:
QUANTUM PHASE TRANSITION

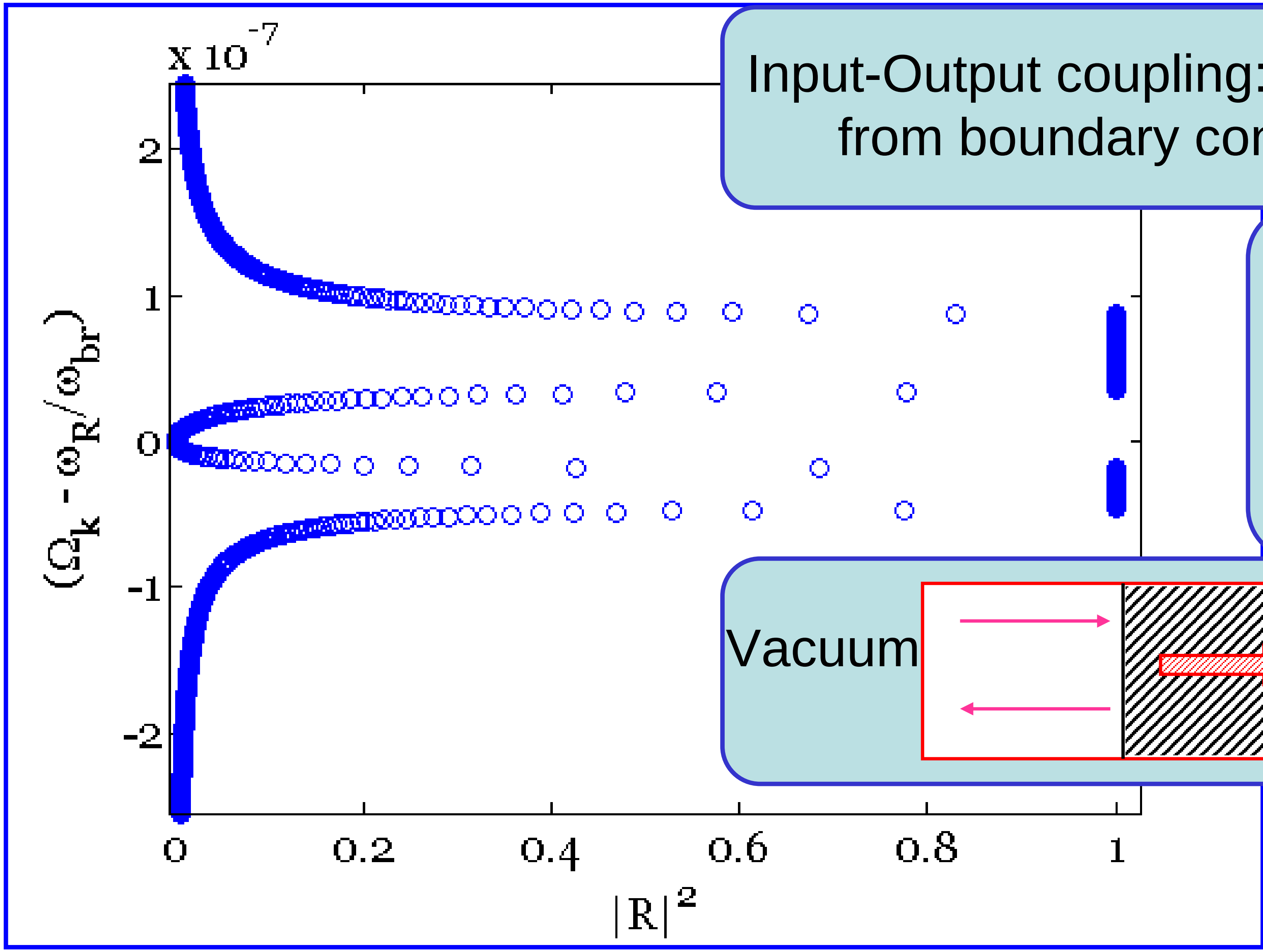
MOTT INSULATOR PHASE:
• Fixed number of atoms per site
• Gap in the excitation spectrum:
No phonons

1D DISPERSION NEAR RESONANCE



3 Polariton bands.
Middle polariton is slow:
matter excitation dominates.

1D REFLECTIVITY NEAR RESONANCE



Input-Output coupling:Reflectivity from boundary conditions

Polariton wave

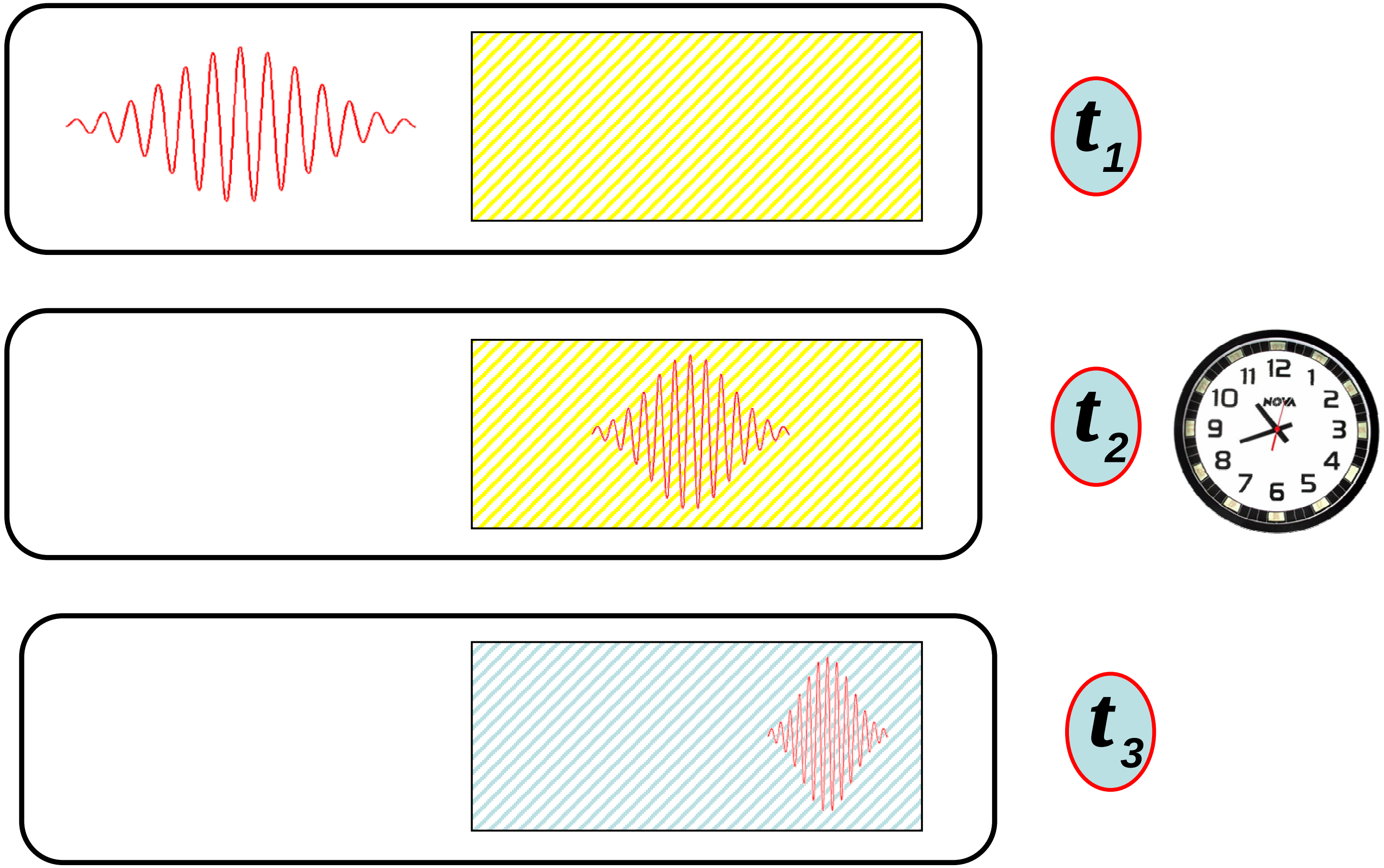
Plane wave

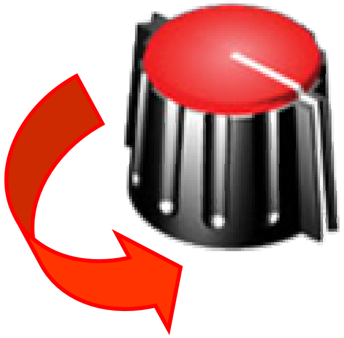
Vacuum

Semi-infinite medium

DYNAMIC PHOTONIC STRUCTURES

Couple light inside the medium

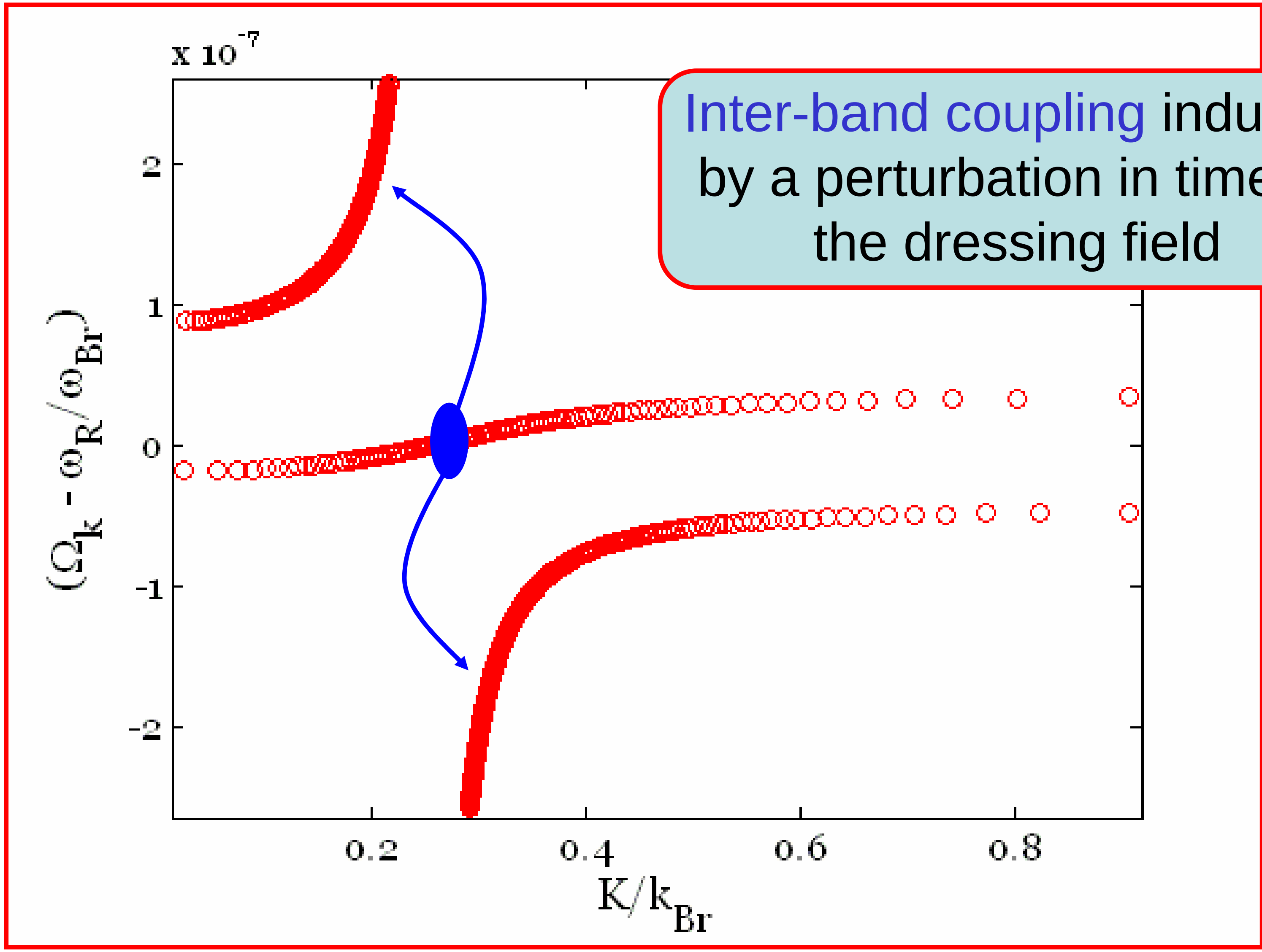




Change in time of the dielectric properties.

Modify the spectrum
Preserve Quantum coherence

BEYOND STEADY-STATE



Inter-band coupling induced by a perturbation in time of the dressing field

ADIABATIC TRANSITION THEORY

$$\begin{pmatrix} |1\rangle & 0 & \sqrt{D} \\ 0 & |2\rangle & \Omega_c(t) \\ \sqrt{D} & \Omega_c(t) & |3\rangle \end{pmatrix}$$

Coupling from state i to j for an adiabatic transition (Messiah)

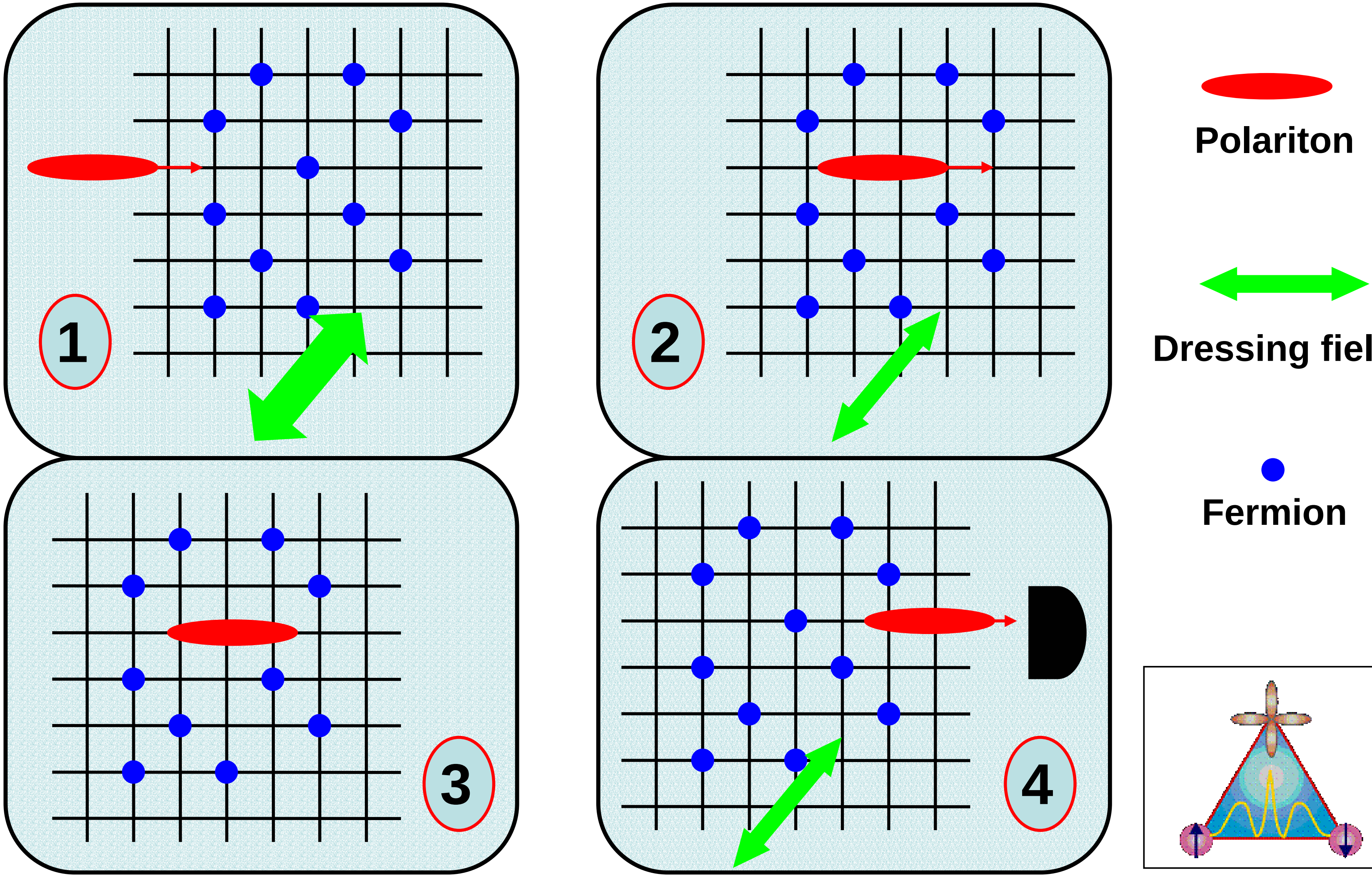
$$p_{i\to j} \cong \left| \int_{t_0}^{t_1} \alpha_{ij}(t) \exp\left(\int_{t_0}^{t} i\omega_{ij}(t')\right) \right|^2$$

$$\alpha_{ij}(t) = \left\langle \frac{d}{dt} i(t) \middle| j(t) \right\rangle$$

Adiabatic condition

$$\frac{\delta\Omega_c}{\sqrt{D}} \frac{1}{\tau} << \sqrt{D}$$

PROBING ULTRACOLD FERMIONS



Condensed Matter Group
CPHT Ecole Polytechnique

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OPTICAL BLOCH EQUATIONS

Optical Bloch equations for the atomic density matrix

$$i\hbar\partial_t\hat{\sigma}(t) = [\hat{H}_0 + \hat{V}_{AL}, \hat{\sigma}(t)] + R\hat{\sigma}(t)$$

Dissipative term

Linear Polarization of an atomic ensemble

$$P(t) = \frac{N_{atom}}{V} 2\mu_{13}\sigma_{31}(t) = \varepsilon_0\chi E_p(t)$$

Steady-state, linear solution for the Susceptibility

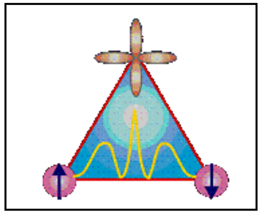
$$\chi(\omega_p) = -\frac{f}{\left(\omega_{31} - \omega_p - i(\gamma_{31}/2) + \dfrac{(\Omega_c/2)^2}{(\omega_{21} + \omega_c) - \omega_p - i(\gamma_{21}/2)}\right)}$$

δ_R Raman 2-photon detuning

PERSPECTIVES

- Effect of Dynamical Photonic Structures on the Scattering
- EIT as probe of quantum state of ultracold gases *

  * Collaboration with C. Kollath, T.L. Dao (Paris)



- Extend the description in 2D and 3D: scattering, polarization effects *

  * Collaboration with D. Gerace (Pavia)

- Polariton-polariton scattering and non-linear effects (full quantum model)

TRANSFER MATRIX
FOR A CHAIN OF ATOMIC SHEETS

Matrix algorithm: useful to describe the propagation
of electric field in 1D arbitrarily complex structure.

$$E(z) = E_1 \exp\left(in\frac{\omega}{c}z\right) + E_2 \exp\left(-in\frac{\omega}{c}z\right) \quad \Rightarrow \quad v = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

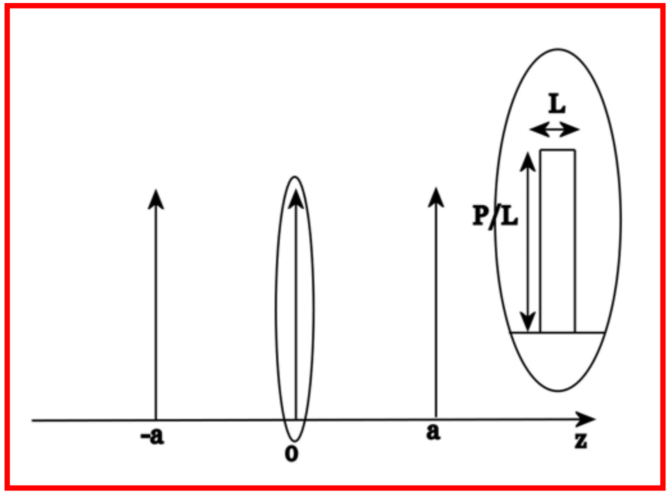
Homogenous layer with index n1

Interface between media with indices n1 and n2

v

$$M_{n_1,d} = \begin{pmatrix} \exp\left(in_1\frac{\omega}{c}d\right) & 0 \\ 0 & \exp\left(-in_1\frac{\omega}{c}d\right) \end{pmatrix}$$

$$M_{n_1 \to n_2} = \frac{1}{2n_2}\begin{pmatrix} (n_1+n_2) & (n_2-n_1) \\ (n_2-n_1) & (n_1+n_2) \end{pmatrix}$$



Elementary cell Transfer Matrix

$$P(\omega) = \chi(\omega)\, a$$

$$M(\omega) = \begin{pmatrix} e^{ika}\left(1+\frac{iP(\omega)}{2}\frac{\omega}{c}\right) & e^{-ika}\left(\frac{iP(\omega)}{2}\frac{\omega}{c}\right) \\ e^{ika}\left(-\frac{iP(\omega)}{2}\frac{\omega}{c}\right) & e^{-ika}\left(1-\frac{iP(\omega)}{2}\frac{\omega}{c}\right) \end{pmatrix}$$

FANO-HOPFIELD QUANTUM MODEL

J.J. Hopfield, *Phys. Rev.* *112*, 5 (1958)

"Cubic array of identical atoms separated by distances large enough that the overlap of wave functions of nearest atoms can be neglected."

$$H_M = \sum_{j,\vec{L}} \hbar\omega_j b^+_{j,\vec{L}} b_{j,\vec{L}} = \sum_{j,\vec{k}\in fBz} \hbar\omega_j b^+_{j,\vec{k}} b_{j,\vec{k}}$$

$$H_R = \sum_{\vec{k}\in fBz} \sum_{\vec{G}} \hbar\omega_{\vec{k}+\vec{G}} a^+_{\vec{k}+\vec{G}} a_{\vec{k}+\vec{G}}$$

Minimal coupling replacement

$$H = H_M + H_R + H_{INT} \longrightarrow b_{j,\vec{L}'} \to b_{j,\vec{L}'} + i\frac{\mu}{\hbar}\vec{A}(\vec{L}')$$

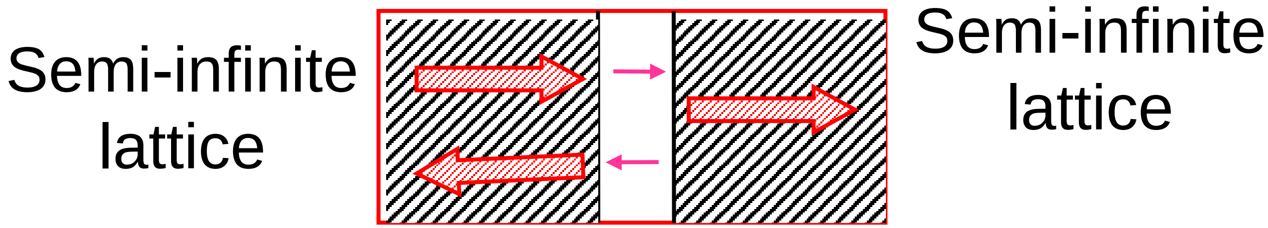
Quadratic Hamiltonian: EXCITON – PHOTON mixing ⟶ POLARITON

$$\alpha_{\vec{k},n} = \sum_j \left(u^{\vec{k}}_{j,n} b_{j,\vec{k}} + v^{\vec{k}}_{j,n} b^+_{j,-\vec{k}} \right) + \sum_{\vec{G}} \left(u^{\vec{k}}_{\vec{G},n} a_{\vec{k}+\vec{G}} + y^{\vec{k}}_{\vec{G},n} a^+_{-\vec{k}+\vec{G}} \right)$$

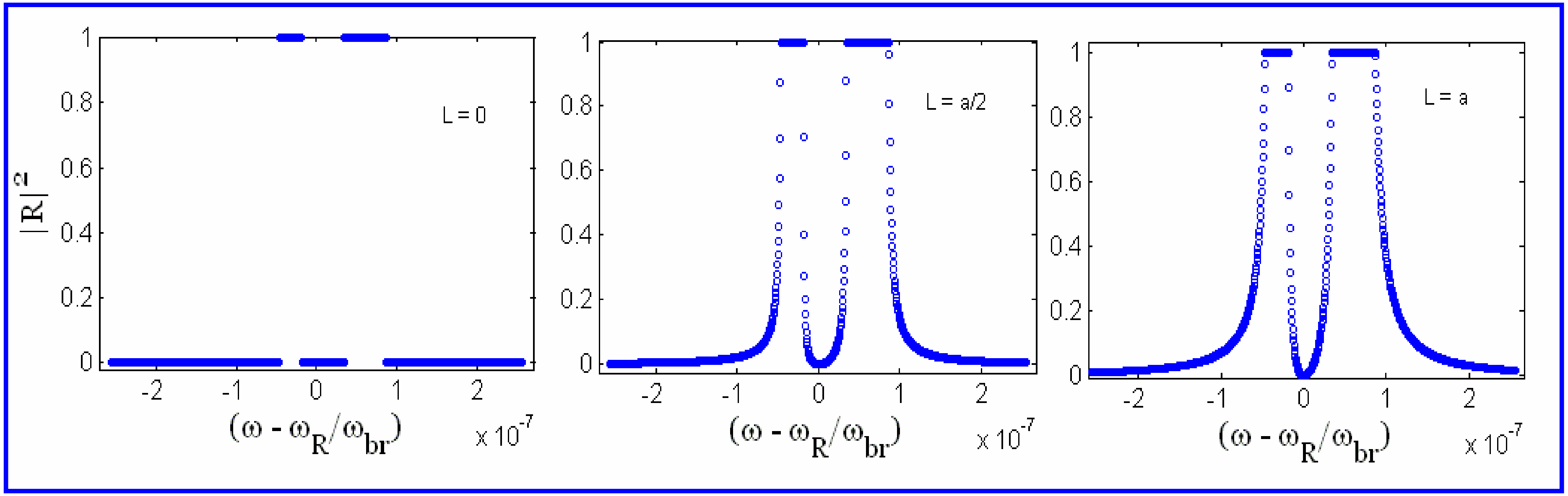
$$H = \sum_{\vec{k},n} \hbar\Omega_{\vec{k},n} \alpha^+_{\vec{k},n} \alpha_{\vec{k},n}$$

For the complete formalism see
I. Carusotto, M. Antezza, FB, S. De Liberato e C. Ciuti, Physical Review A, **77** 063621 (2008)

SCATTERING ON DEFECTS



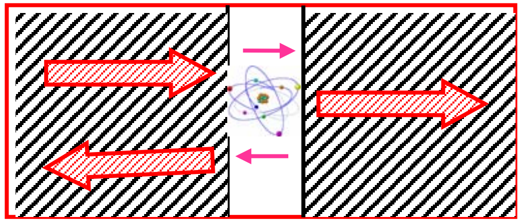
Defect: vacuum



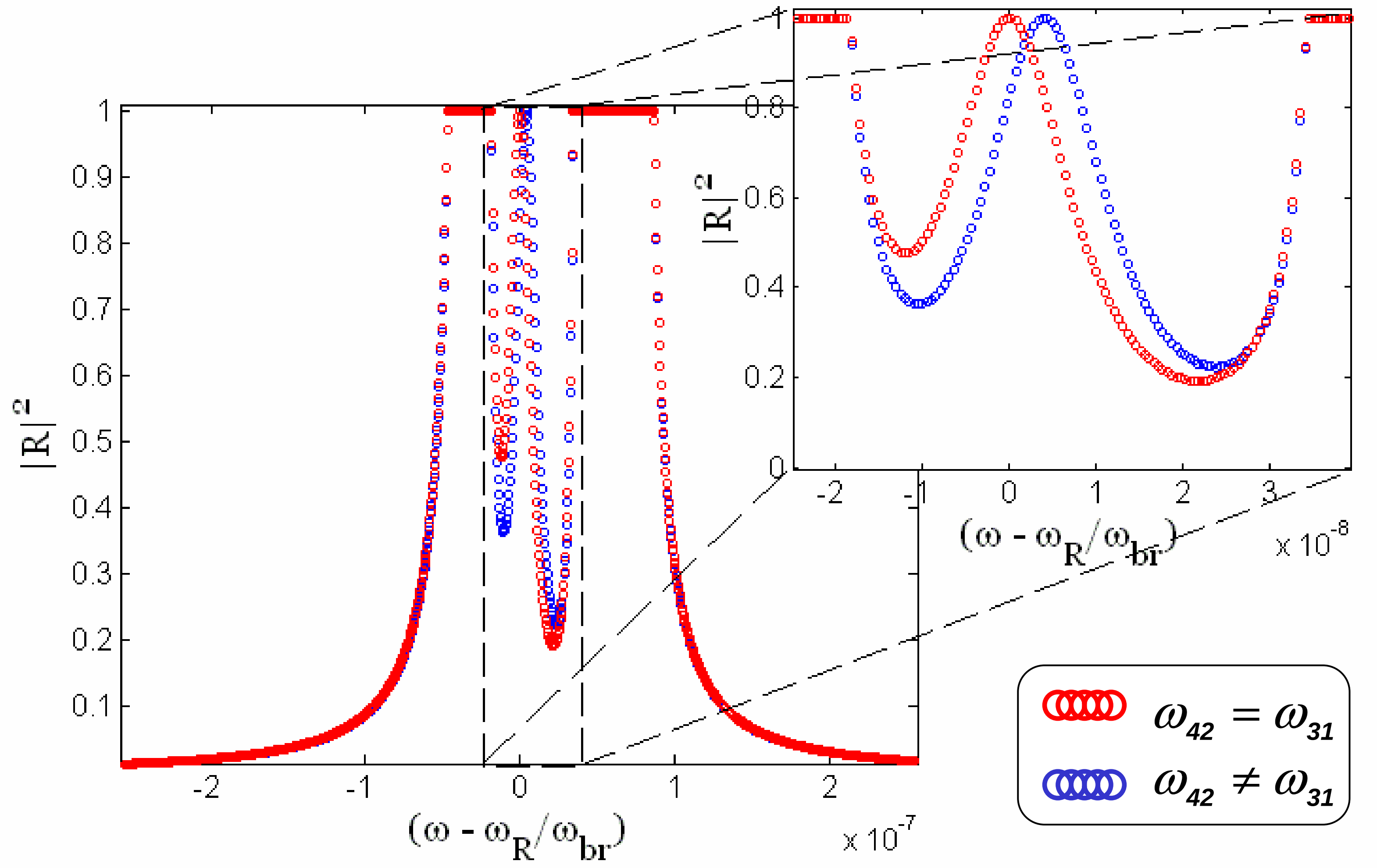
In 1D Reflection is the equivalent of Scattering

SCATTERING ON DEFECTS

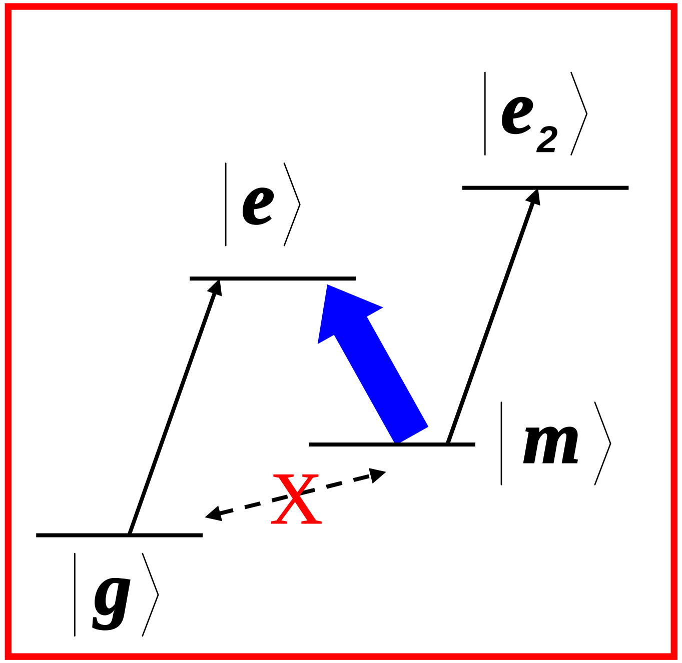
Defect:
2-level atom



In 1D Reflection is the equivalent of Scattering



PHOTON-PHOTON RESONANCE ?



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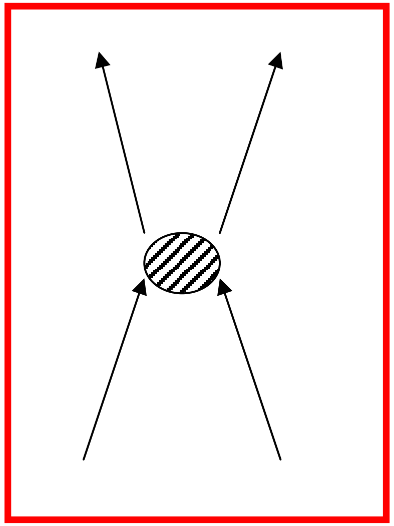
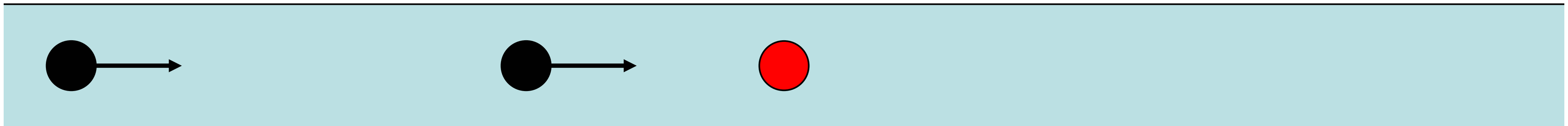
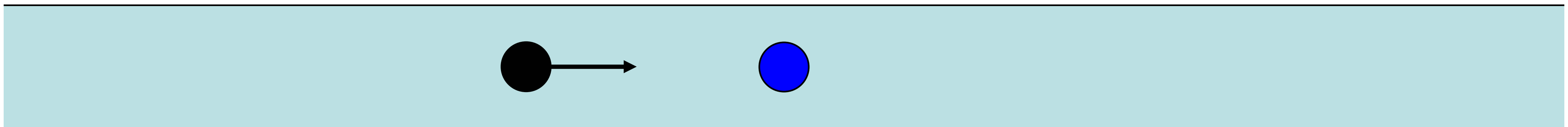


Similar to
Imamoglu et al.,
PRL **79** 1467 (1997)

The first photon transfer the atom in the metastable state



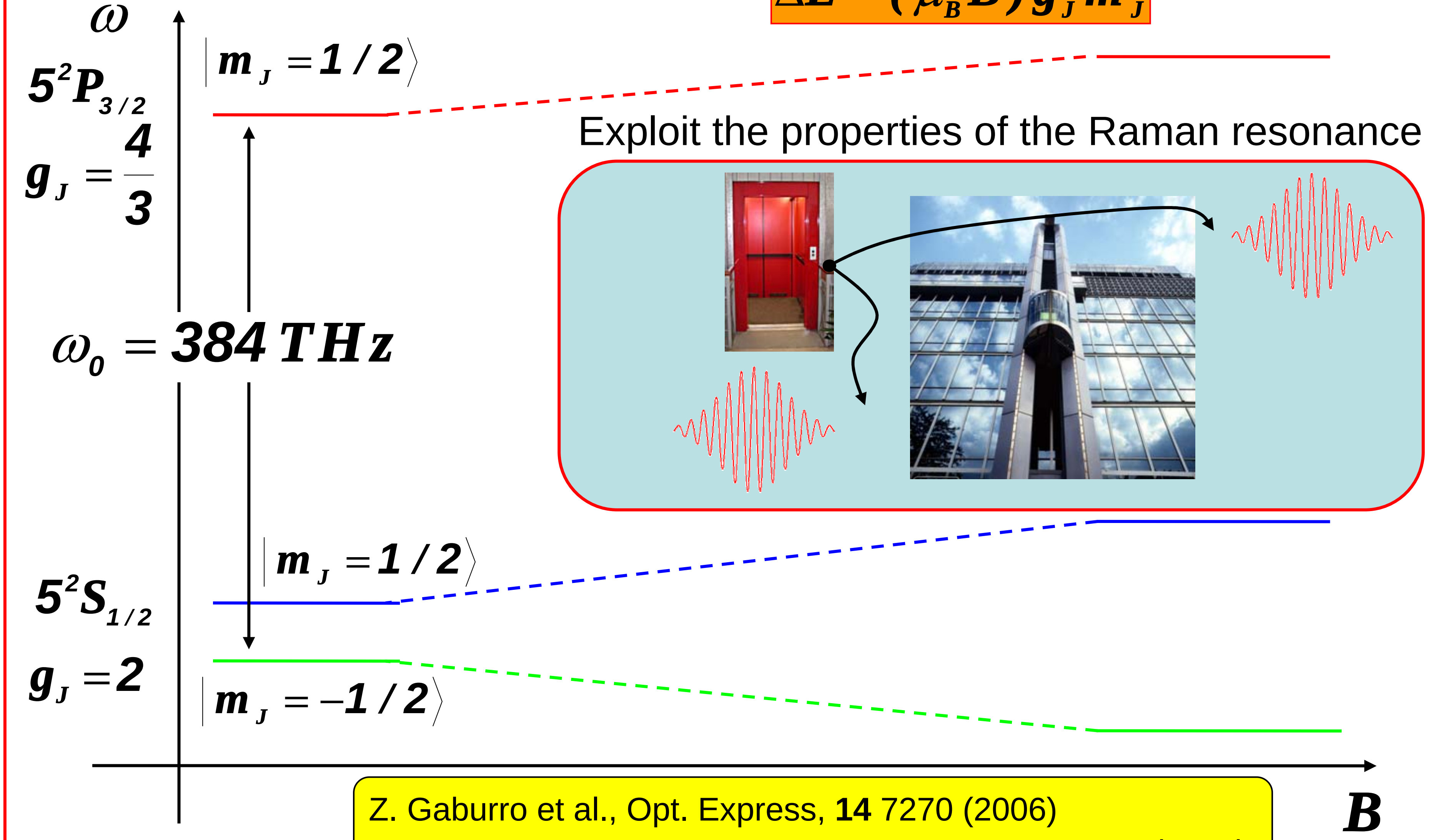
The second photon “sees” a different atomic response



PHOTON ENERGY LIFTER

D2 line of the Rubidium 87 + **ZEEMAN EFFECT** with strong fields

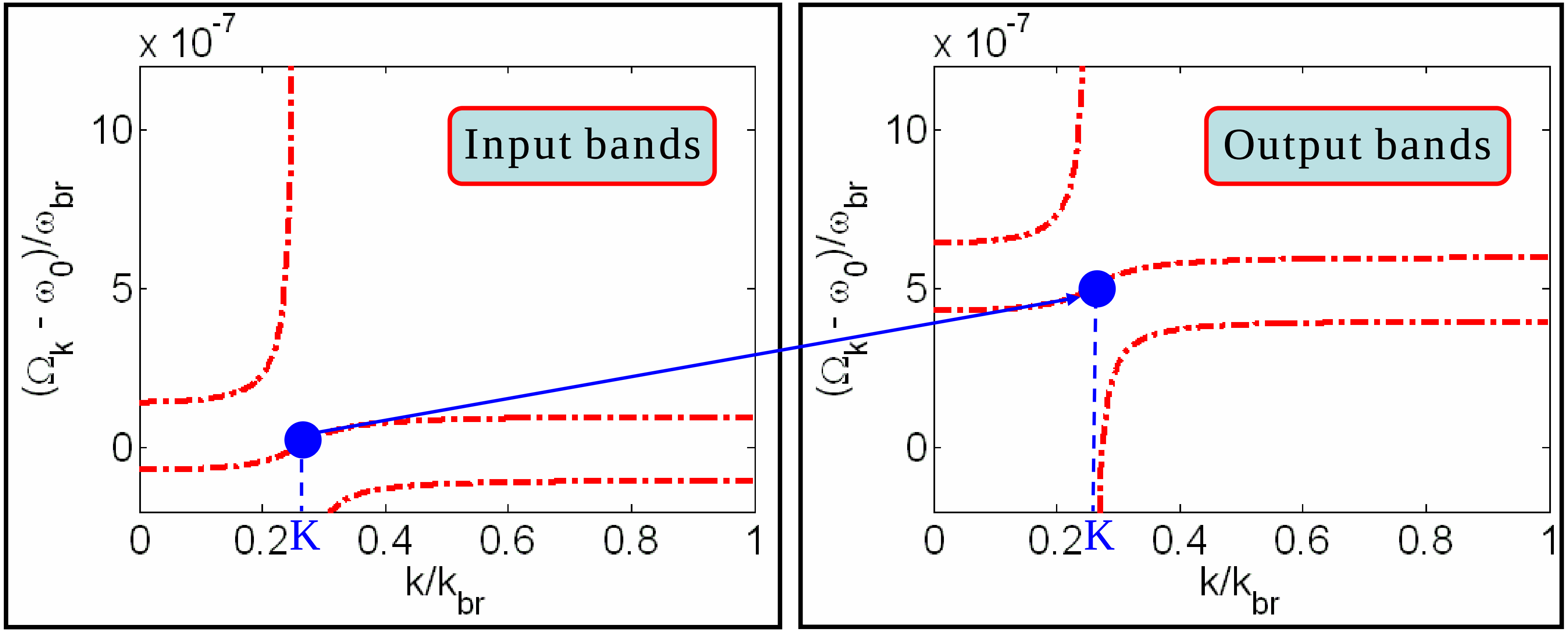
$$\Delta E = (\mu_B B) g_J m_J$$



Z. Gaburro et al., Opt. Express, **14** 7270 (2006)
FB e I. Carusotto, Jour. Eur. Opt. Soc. – RP, **3** 08005 (2008)

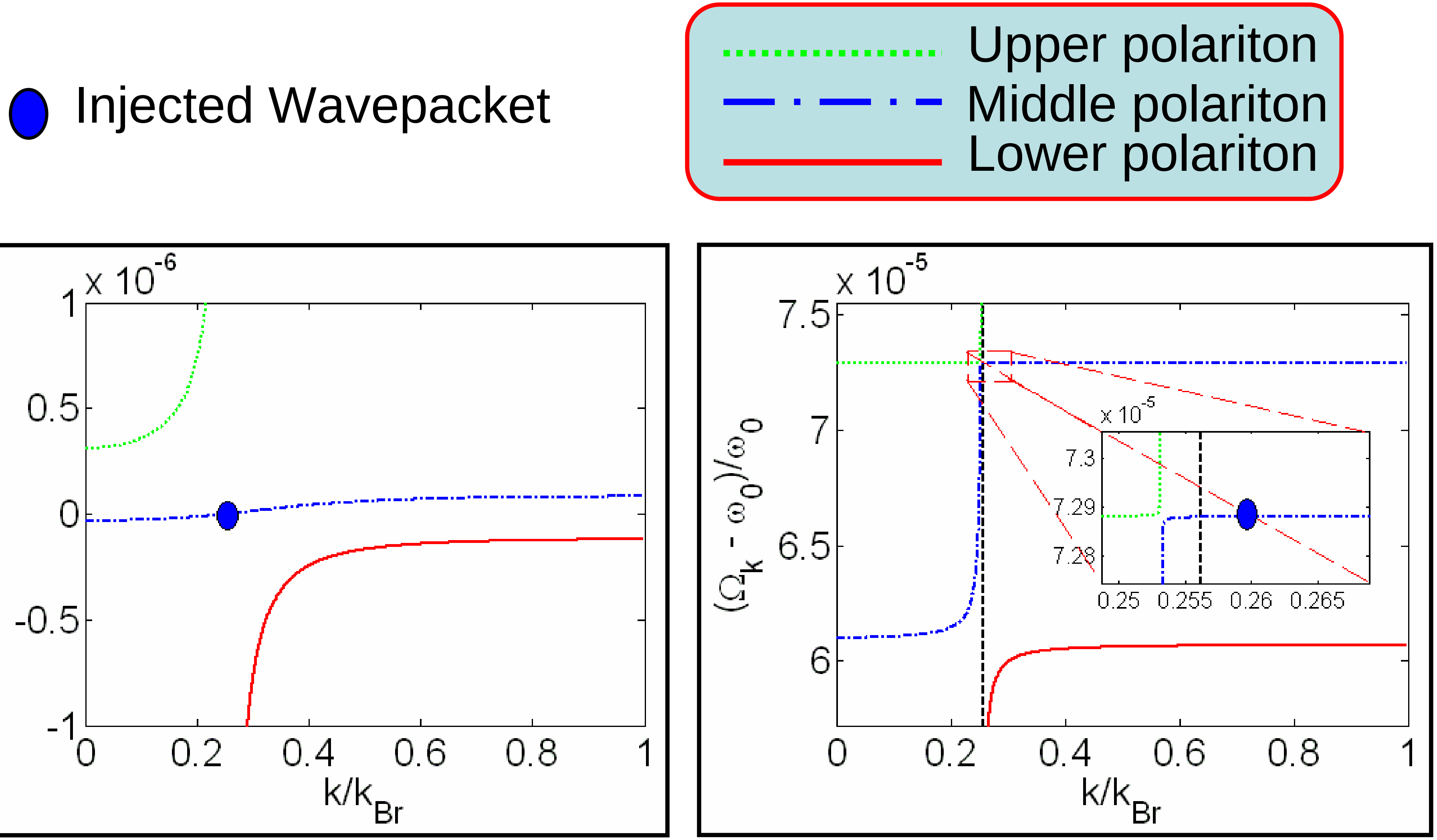
PHOTON LIFTER EFFECT

BLOCH WAVEVECTOR CONSERVATION



Because of the exciton-photon mixing
It's possible to change the radiation frequency
by acting on the matter degrees of freedom

PHOTON LIFTER EFFECT



MAXWELL-BLOCH EQUATIONS

To study the pulse propagation
for a time-dependent dressing field:
Maxwell-Bloch Equations
in Slowly Varying Envelope approximation

$$\begin{cases} \partial_t E = \dfrac{i}{2}(\partial_x^2 + 1)E + i\sqrt{D}\sigma_{31} \\ \partial_t \sigma_{31} = -\dfrac{\gamma_{31}}{2}\sigma_{31} + i\sqrt{D}E - i\dfrac{\Omega_c(t)}{2}\sigma_{21} \\ \partial_t \sigma_{21} = -\dfrac{\gamma_{21}}{2}\sigma_{21} - i\dfrac{\Omega_c(t)}{2}\sigma_{31} \end{cases}$$

Maxwell equation coupling electric field and atomic polarization

Optical Bloch Equations

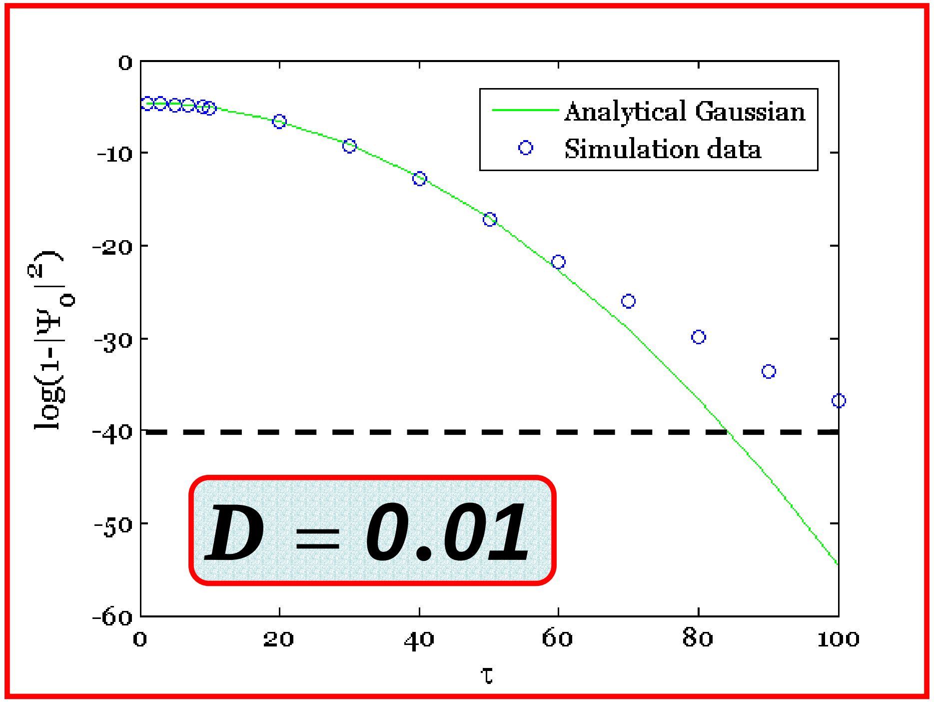
INFINITE PERTURBATION

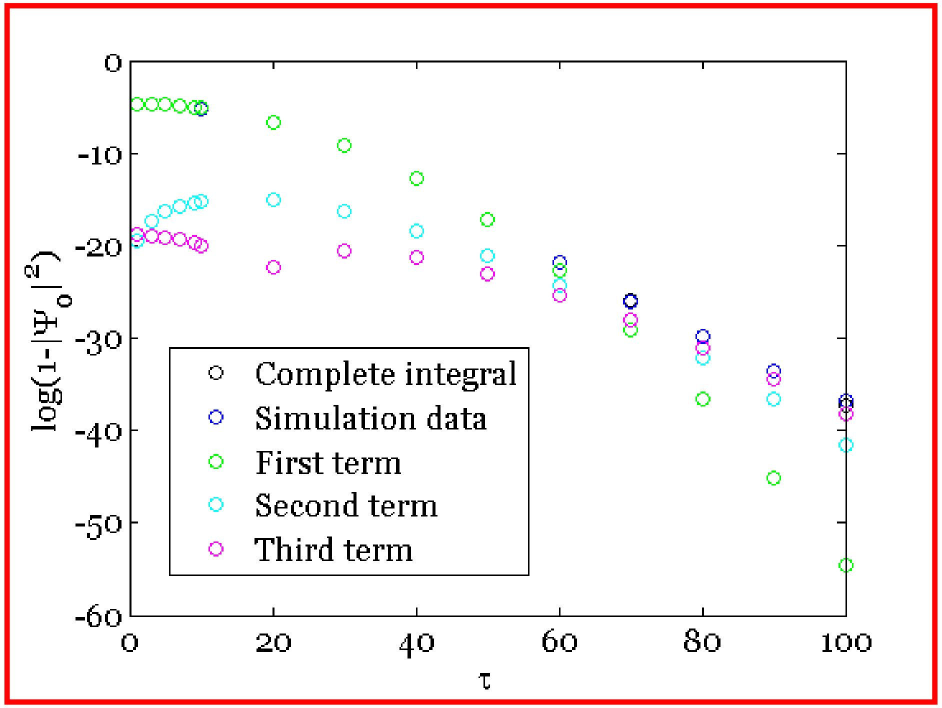
$$\Omega_c(t) = \Omega_0 + \frac{\delta\Omega}{2} Erf\left(\frac{t}{\tau}\right)$$

Long time behavior:
Gaussian – exponential?

$$\Delta\Omega^2(t) \to D \quad \Rightarrow \quad p_\infty(\tau) \propto exp\left(-\frac{D\tau^2}{4}\right)$$

We need to use the whole integral to map the coupling





ADIABATIC TRANSITION THEORY

The Messiah's integral in our case

$$p_{0\to\pm} \cong \left| \int_{t_0}^{t_1} \frac{\sqrt{D}}{\Delta\Omega^2(t)} \frac{d}{dt}\left(\frac{\Omega_c(t)}{2}\right) \exp\left(\pm i\int_{t_0}^{t} \Delta\Omega(t')\right) \right|^2$$

$$\Delta\Omega = \sqrt{D + (\Omega_c(t)/2)^2}$$

Adiabatic condition

$$\frac{\delta\Omega_c}{\sqrt{D}} \frac{1}{\tau} << \sqrt{D}$$