Strong CP-Violation in External Magnetic Fields

Vacuum Polarization and Vacuum Birefringence

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Introduction

Quantum ChromoDynamics allows for CP-symmetry to be violated

$$\mathcal{L}_{QCD} = \sum_{f=1}^{N_f} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{\theta\, g^2}{64\pi^2} \tilde{G}^{\mu\nu} G_{\mu\nu}$$

Strong CP-violating processes have never been observed.

From neutron electric dipole moment measurements

$$d_n < 0.29 \times 10^{-25} \qquad \text{Baker et al.(2006)}$$

$$d_n \simeq 5 \times 10^{-16}\, \theta \; e\, cm \quad \Longrightarrow \quad \theta < 10^{-10}$$

"Strong CP problem": *it is not clear why strong interactions should conserve CP-symmetry to such a high precision.*

Vacuum polarization induced by an external Electromagnetic Field

$$\bar{\nabla} \cdot \bar{P} = \langle \Omega | J^0_{e.m.}(x) | \Omega \rangle_{A_\nu}$$

In QCD we have two qualitatively different contributions ($\bar{E} \cdot \bar{B} = 0$)

$$\langle \theta | J^0_{e.m.}(x) | \theta \rangle_{A_\mu, QCD} \quad = \langle \theta | J^0_{e.m.}(x) | \theta \rangle^{CPeven}_{A_\mu} + \langle \theta | J^0_{e.m.}(x) | \theta \rangle^{CPodd}_{A_\mu}$$

An asymmetry in quarks' density distribution ρ^u e ρ^d generates a "CP-odd" vacuum polarization, aligned along the magnetic field axis.

$$\bar{D}(t, \theta) = \int_V \mathrm{d}^3 x \, \bar{x} \, \langle \theta | J^0_{e.m.}(t, \bar{x}) | \theta \rangle^{CP-odd}_{A^\mu}$$

If the magnetic field $\bar{B}$ is very intense we expect to observe macroscopic effects of Strong CP-violation.

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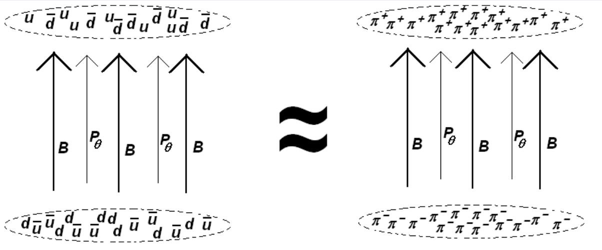
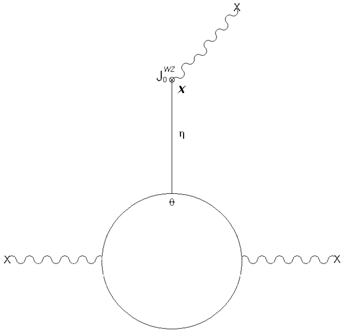
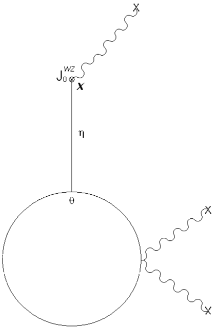


Diagrams contributing to the electromagnetic charge density to leading order in p and α

$$\langle\theta|J^0_{e.m.}(x)|\theta\rangle^{CP-odd}_{A_\mu} \text{ at } O(p^6) \text{ and } O(\alpha^2)$$





Uniform magnetic field, oscillating with frequency ω

External magnetic field: $\bar{B}(x) = B\cos(\omega t)\hat{z}$

$$\bar{P}(t) = \frac{5}{6}\frac{\alpha^2\theta B^3}{m_\eta^2(4\pi F)^2}\left\{\left[\frac{1}{2} - \frac{1}{3}\frac{\omega^2}{m_\eta^2} + \frac{11}{45}\frac{\omega^2}{m_\pi^2}\right]\cos(\omega t) + \right.$$
$$\left. + \left[\frac{1}{6} + \frac{\omega^2}{m_\eta^2} + \frac{1}{10}\frac{\omega^2}{m_\pi^2}\right]\cos(3\omega t)\right\}\hat{z}$$

R. Millo , P. Faccioli , Phys.Rev.**D77**(2008) 065013.

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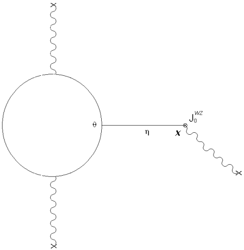
The QCD vacuum has a non-linear response:

- exchanges a single photon with the external field;
- exchanges three photons with the external field.



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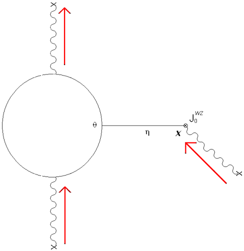

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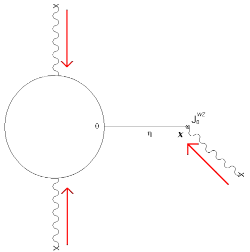

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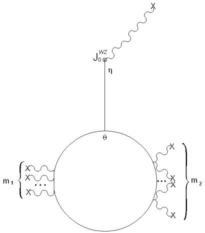
If we had considered diagrams contributing at order $O(\alpha^{n>2})$, we would have had terms with higher frequency.

$$(2n+1)\omega \iff \alpha^{n+1}$$



Uniform and constant magnetic field $\bar{B} = B\hat{z}$

In the static field limit $\omega = 0$ the polarization is

$$\bar{P}_\theta = \frac{5}{9} \frac{\alpha^2 \; \theta \; B^3}{m_\eta^2 \, (4\pi F)^2} \, \hat{z} \;\; \propto \;\; B^3 \, \theta$$

We can estimate the strength of this effect

Electric dipole moment induced by $1 Tesla$ in $1 m^3$

$$D_\theta = \int_V \mathrm{d}^3 x \bar{P}(x) = 1.52 \times 10^{-18} \, \theta \, \mathrm{e\,cm}$$

The effect is extremely weak.

$$d_n \simeq 5 \times 10^{-16} \theta \, \mathrm{e\,cm} \simeq 3 \times 10^2 D_\theta$$

Very Intense Field

Magnetars

Neutron stars whose magnetic field $\geq 10^{15}G$ (up to $10^{17}G$).

The typical diameter of a neutron star is $\sim 10^4$;

Difference of potential close to a magnetar

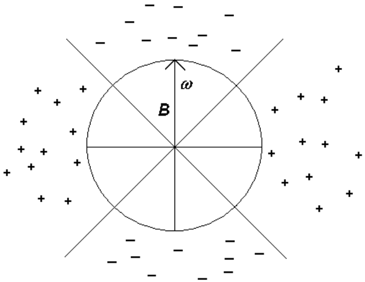
$$V_\theta = 3 \times 10^6 \left(\frac{B}{10^{15}G}\right)^3 \left(\frac{L}{1m}\right) \theta \; V$$

An electron at rest may be accelerated at energies of the order of 3eV (3MeV), assuming $B = 10^{15}G$ ($B = 10^{17}G$) and $\theta \sim 10^{-10}$.

The typical energies of electrons on a neutron star surface is 1keV.

Magnetar

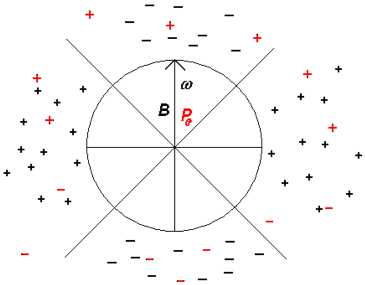
Due to magnetar rotation we have an induced quadrupolar electric field.



$$\rho = -\frac{1}{2\pi}\bar{\omega} \cdot \bar{B}$$

Magnetar

Due to CP-violating interactions we have an induced dipolar field



$$M = 10^{27} G\ m^3 \Longrightarrow \boxed{D_\theta = 3 \times 10^{26} \theta\, \mathrm{e\,cm}}$$

Low Energy ElectroMagnetic dynamics

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a}{2}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{b}{2}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2 + O\left(\frac{p^8}{m_e^8}\right)$$

Natural energy threshold: $\Lambda_\gamma = m_e$.

(30's, Heisenberg-Euler)

$$a = \frac{2\alpha^2}{45m_e^4}; \quad b = 7a$$

Low Energy ElectroMagnetic dynamics

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We have an analogy with material mediums

$$\mathcal{L} = \frac{1}{2}\left(\bar{E}\cdot\bar{D} - \bar{B}\cdot\bar{H}\right)$$

we can easily calculate vacuum polarization and magnetization in the presence of an external e.m. field

$$\bar{P}^{even} = \bar{D} - \bar{E} = \frac{\partial\mathcal{L}_{eff}}{\partial\bar{E}} - \bar{E}$$

$$\bar{M}^{even} = \bar{B} - \bar{H} = \bar{B} + \frac{\partial\mathcal{L}_{eff}}{\partial\bar{B}}$$

'CP-odd" Lagrangian

$$\mathcal{L}_{CP-odd} = \mathcal{L}_{eff} + \frac{d}{4}\left(F_{\mu\nu}F^{\mu\nu}\right)\left(F_{\alpha\beta}\tilde{F}^{\alpha\beta}\right)$$

We have a CP-violating correction

$$\bar{P}^{odd} = dB^2\bar{B} \; ; \qquad \bar{E} = 0$$

$$\bar{M}_\theta = -dE^2\bar{E} \; ; \qquad \bar{B} = 0$$

Now we can deternine the QCD contribution to the coefficient d

$$\bar{P}_\theta = \frac{5}{9}\frac{\alpha^2\,\theta}{m_\eta^2\,(4\pi F)^2}B^2\bar{B} \quad \Longrightarrow \quad \boxed{d_\theta = \frac{5}{9}\frac{\alpha^2\,\theta}{m_\eta^2\,(4\pi F)^2}}$$

The vacuum is also magnetized by an external electric field.

Vacuum Birefringence

When the vacuum is permeated by an external magnetic field, it is no longer isotropic and two different refraction indices n_1 and n_2 can be identified.

$$\begin{aligned} \psi^{even} &= \pi(n_1 - n_2)\frac{L}{\lambda}\sin(2\beta); \\ n_1 - n_2 &= b - 4a \end{aligned}$$

Polarized light shined through such a medium may acquire ellipticity.

CP-odd vacuum birefringence:QCD contribution

$$\begin{aligned} \psi^{odd} &= \psi^{even} + 2\pi d\frac{L}{\lambda}B^2\cos(2\beta) \\ &= \psi^{even}\left(1 + 10^{-10}\theta\,\mathrm{coth}(2\beta)\right) \end{aligned}$$

R.Millo , P.Faccioli, hep-ph/arXiv:0811.4689v2

Conclusions

- The vacuum is polarized (magnetized) by an external magnetic (electric) field and behaves like a medium with a non-linear response;
- the polarization vector is aligned along the axis of the external field;
- Magnetic field close to a Magnetar may give birth to potentially observable effects: "Soft Gamma Repeaters"; and "hot spots";
- This result enables us to determine the QCD contribution to the leading CP-odd term in the effective QED lagrangian;
- "CP-odd" interactions modify the polarization of photons in external magnetic fields, nonetheless the effect is too small to be detected. Deviations from the expacted value may be connected to physics beyond the Standard Model.

THANK YOU!