ESERCIZIO 1

1)

$m = 150\ g$
$M = 50\ g$ $\quad R = 10\ cm$
$m' = 20\ g$ $\quad r' = 5\ cm$

(1)

$$\begin{cases} ma = mg - T_1 \\ Ma = T_2 - Mg\sin\alpha - A \\ AR = I\alpha/R = \alpha \\ (T_1 - T_2)r' = \frac{1}{2}m'r'^2\alpha' \end{cases}$$

$I_{sfera} = \frac{2}{5}MR^2$ $\quad I_{cilindro} = \frac{1}{2}mr^2$

$A = \frac{2}{5}M\cancel{R}\frac{a}{\cancel{R}} = \frac{2}{5}Ma$

$r'\alpha' = a$

$$\begin{cases} ma = mg - T_1 \\ \frac{2}{5}Ma = T_2 - Mg\sin\alpha \end{cases}$$

$$ma + \frac{2}{5}Ma = g(m - M\sin\alpha) - \frac{1}{2}m'r'\alpha'$$

(2)

$$\Rightarrow \quad a = g\,\frac{m - M\sin\alpha}{m + 2/5\,M + m'/2} = g\,\frac{m - M/2}{m + 2/5\,M + m'/2} = 5.33\ \frac{m}{s^2}$$

$$\Rightarrow \quad T_1 = m(g - a) = mg\,\frac{2/5M + m'/2 + M\sin\alpha}{m + 2/5M + m'/2} = mg\,\frac{19/40\,M + m'/2}{m + 2/5M + m'/2}$$

$= 0.67\,M$

$$\Rightarrow \quad T_2 = M(2/5\,a + g\sin\alpha) = Mg\,\frac{2/5\,m + m\sin\alpha + m'/2\sin\alpha}{m + 2/5\,M + m'/2} = \frac{19/40\,m + m'/2}{m + 2/5M + \dots}$$

$= 0.62\,M$

$a = \frac{125}{250}g = 5.33\ m/s^2$

$T_1 = 0.67\,M$

$T_2 = 0.62\,M$

$\alpha' = a/_{R'} = 10.66\ rad/s^2$

$$-mgh + \frac{1}{2}mv^2 + \frac{1}{2}I'\omega'^2 + \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh\sin\alpha = 0$$

$$\Rightarrow \frac{1}{2}(m+M)v^2 + \frac{1}{2}\frac{2}{5}MR^2\frac{v^2}{R^2} + \frac{1}{2}\frac{m'R^2}{2}\frac{v^2}{R^2} = gh(m - M\sin\alpha)$$

$$\omega = \frac{v}{R}, \quad \omega' = \frac{v}{R/2}$$

$$\frac{1}{2}(m + 2/5M + m'/2)v^2 = gh(m - M/2)$$

$$h = \frac{(m + 2/5M + m'/2)v^2}{2g(m - M/2)} = 0.376\ m$$

$$h' = h\sin\alpha = h/2 = \boxed{0.188\ m} \quad \text{(quota)}$$

(c)

$$A = \frac{2}{5}Ma = \frac{2}{5}Mg\frac{m - M/2}{m + 2/5M + m'/2}$$

$A \le A_{max}$ $\quad A_{max} = \mu Mg\cos\alpha = \mu Mg\frac{\sqrt{3}}{2}$

$$\frac{2}{5}Mg\frac{m - M/2}{m + 2/5M + m'/2} \le \mu Mg\frac{\sqrt{3}}{2}$$

$$\frac{2}{5}m - M/5 \le \mu\frac{\sqrt{3}}{2}m + \mu\frac{\sqrt{3}}{2}(2/5M + m'/2)$$

$$\Rightarrow m(2/5 - \mu\sqrt{3}/2) \le M/5 + \mu\sqrt{3}/2(2/5M + m'/2)$$

$$\boxed{m \le 220\ g}$$

a) $E_1 = \frac{1}{2} k \Delta x^2$

ESERCIZIO 2

b) $E_2 = mgh = mgd \sin\alpha = mgd/2$

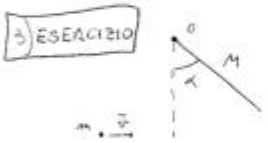
$d = \frac{k}{mg} \frac{1}{2} k \Delta x^2 = \frac{k}{mg} \Delta x^2 = \boxed{48\ cm}$

c) $E_3 = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v^2 + \frac{1}{2} \frac{2}{5} m v^2 = \frac{1}{2} \frac{7}{5} m v^2$

$\frac{1}{2} k \Delta x^2 = \frac{1}{2} \frac{7}{5} m v^2$

$v = \sqrt{\frac{5k}{7m}} \Delta x = \boxed{1.8\ m/s}$

3) ESERCIZIO



$m \cdot \frac{v}{2}$

$\omega = \sqrt{\frac{3g}{L}(\cos\vartheta - \cos\vartheta_0)}$

$\vartheta = \vartheta_0 \Rightarrow \vartheta = 0°$

$\omega_0 = \sqrt{\frac{3}{2}\frac{g}{L}}$

$\frac{1}{2} I \omega^2 = Mg \frac{L}{2}(1 - \cos\alpha) = Mg \frac{L}{4}$

$\omega = \sqrt{\frac{MgL}{2I}} = \sqrt{\frac{MgL}{\frac{2}{3}ML^2}} = \sqrt{\frac{3}{2}\frac{g}{L}}$ $\quad I_0 = \frac{1}{3}ML^2$

$L_1 = mvL = \underbrace{\frac{1}{3}ML^2\sqrt{\frac{3g}{2L}}}_{I_0\omega_0}$ Momento angolare prima urto

a) $L_2 = 0$ (dopo urto)

$\Rightarrow \frac{m}{M} = \frac{ML^2}{3} \cdot \sqrt{\frac{3g}{2L}} \cdot \frac{1}{mL^2} = \boxed{\frac{M}{m}\sqrt{\frac{3L}{2}}}$

(5) $\frac{1}{2} I' \omega'^2 = mg2L + MgL = MgL(1+2m/M)$ (4)

$I' = \frac{1}{3}ML^2 + mL^2 = \frac{1}{3}ML^2(1+3m/M)$

$\omega' = \sqrt{\frac{2MgL(1+2m/M)}{\frac{1}{3}ML^2(1+3m/M)}} = \sqrt{\frac{6g}{L}\frac{1+2m/M}{1+3m/M}}$

$L_f = I'\omega' = \frac{ML^2}{3}(1+3m/M)\sqrt{\frac{6g}{L}\frac{1+2m/M}{1+3m/M}}$

$mvL = \frac{ML^2}{3}\left(\sqrt{\frac{3g}{2L}} + \sqrt{\frac{6g}{L}(1+2m/M)(1+3m/M)}\right)$

$v = \frac{1}{3}\frac{M}{m}\sqrt{\frac{3gL}{2}}\left(1 + \sqrt{\frac{6g}{g}\frac{2L}{3L}(1+2m/M)(1+3m/M)}\right)$

$= \boxed{\frac{M}{m}\sqrt{\frac{gL}{6}}\left(1 + 2\sqrt{(1+2m/M)(1+3m/M)}\right)}$