Gravity

An exercise in quantization

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Quantum Gravity: problem statement

- QG is a theory that satisfies these requirements:
    - Is a quantum theory (non-commutative algebra, representations).
    - Has a classical limit (Poisson algebra, symplectic manifold). The limit is General Relativity (GR).
    - Is a field theory (continuum, locality, causality).
    - Is minimal (non spurious degrees of freedom).
- The definition is non-vacuous. It is essentially the deformation quantization of the classical field theory of GR.
- It is known to exist formally (perturbatively) and it is plausible that it exists non-perturbatively.

QG construction sketch

Start with the Einstein-Hilbert action on a 4d manifold.

- Construct classical field theory (FT).
  - Phase space (with gauge).
  - (Pre-)symplectic structure.
  - Gauge reduction: physical phase space and Poisson structure.
  - Identify algebra of observables; associate phenomenological interpretations.
- Construct quantum field theory (QFT).
  - Apply deformation quantization to FT of GR.
  - Make sure locality, causality, gauge reduction all commute with quantization.
- This construction is just Field Quantization.
  In no way is it unique to GR.

Elephant in the room: Some steps are perturbative (formal power series in $\hbar$ and G) and no non-perturbative analog is know. Absolutely the same statement is true for the Standard Model.

Why is GR hard?

QG is still hard for many reasons. But much is known about each obstacle in isolation.

1. Non-linearity
   - $\lambda\phi^4$, QED, YM, fluids
2. Dynamical Causality
   - gas dynamics, fluids, quasilinear hyperbolic PDE
3. Singularities
   - fluid shocks, breaking waves, wave focusing
4. Gauge Redundancy
   - Maxwell, YM, TFT, string
5. Non-local Observables
   - Aharonov-Bohm, TFT, Cartan method
6. UV Renormalization
   - all interacting QFTs
7. IR Renormalization
   - all QFTs with massless fields
8. Non-perturbative Definition
   - all physical QFTs

Conservative philosophical position

- What is a physical theory?

  A highly efficient quantitative, method of summarizing past empirical data and predicting future empirical data, with quantifiable uncertainty.

  - ‘empirical data’: outcomes of controlled experiments
  - ‘summary’: Lagrangian
  - ‘controlled experiment’: state + observable
  - ‘prediction’: state + observable $\mapsto$ numbers
- No a priori reason for world to be ‘simple’ or ‘beautiful’.
- What is a problem for a theory?
  - Internal: consistency, failure of unambiguous prediction.
  - External: unambiguous predictions do not match empirical data.
- What is not a problem for a theory?
  - Fails to account for all coincidences.
  - Intractable by given approximation or formalism.
  - Theoretical prejudice.
  - Aesthetics.

Perceived problems of QG

- Timelessness
    - Failure of 3+1 canonical formalism. Covariant formalism has no such problem. Subjective passage of time modeled adquately by clock observables.
- Non-renormalizability
    - Power-counting renormalizability is a relic of outdated methods.
    - Modern renormalization: Lagrangian + renormalization scheme $\mapsto$ n-point functions. Change in renormalization scheme absorbed by $O(\hbar)$ change in Lagrangian (theorem).
    - All statements are perturbative. Non-perturbative statements await non-perturbative formulation.
- Black hole evaporation and unitarity
    - Intermediate times: analogous to room with open window.
    - Long times: simply unknown.
- Naturalness (cosmological constant)
    - The cosmological constant is a free parameter.
- Unification
    - Aesthetic or theoretical prejudice.

Covariant symplectic structure [✓]

- Obviates the need for 3+1 formalism.
- Lagrangian $\mapsto$ (pre-)symplectic form on solution space.
- Lagrangian $\mapsto$ Poisson bracket on observables (Peierls bracket).
- The two are compatible (up to partial gauge fixing or gauge reduction).
- 3+1 formalism completely recovered from the special Lagrangian $\mathcal{L}(\phi,\pi) = \pi\dot{\phi} - \mathcal{H}(\phi,\pi)$.

[K2] I.K. *Characteristics, Conal Geometry and Causality in Locally Covariant Field Theory* [arXiv:1211.1914]

BV-BRST and gauge reduction [✓]

- Convenient algebraic formalism for the construction of the Poisson algebra of gauge invariant observables.
- Algebraic analog the (geometric) gauge reduction of the solution space to the physical phase space.
- Main advantages:
  - Quantization is algebraic. BV-BRST survives quantization.
  - Is behind the ‘quantization commutes with gauge reduction’ (up to obstructions) theorem.
  - Obstructions are known as ‘gauge anomalies’.
  - Gauge anomalies are absent in 4d GR.

[H T] M.Henneaux, C.Teitelboim, *Quantization of Gauge Systems* (1994)

[R] K.Rejzner, PhD Thesis (Hamburg) [arXiv:1111.5130]

Observables [✓] and [✗]

- Gauge invariant observables in GR are non-local.
- Hard (but not impossible) to find examples that are both
  - mathematically tractable
  - phenomenologically meaningful
- Untapped connection to Cartan's method for solving geometric equivalence problems.
- Still of work to be done!

[W] R.P.Woodard, PhD Thesis (Harvard, 1984)

[P W] D.N.Page, W.K.Wootters PhysRev D**27** 2885 (1983)

[G P P T] R.Gambini, R.A.Porto, J.Pullin, S.Torterolo PhysRev D**79** 041501 (2009)

[K1] I.K. *Quantum astrometric observables I* [arXiv:1111.7127]

Deformation quantization [✓]

- Formalization of the correspondence principle and what quantization has meant in practice.
- Relies only on symplectic or Poisson geometry.
- Always possible, at least perturbatively. Non-perturbatively: active research program.
- Sheds new light on operator ordering ambiguities.
- Is behind the ‘quantization commutes with gauge reduction’ theorem.

[D F] M.Dütsch, K.Fredenhagen, *Perturbative Algebraic Field Theory, and Deformation Quantization* [arXiv:hep-th/0101079]

[H] S.Hollands (work in progress)

[B F R] R.Brunetti, K.Fredenhagen, P.L.Ribeiro, (work in progress)

UV renormalization [✓]

- Epstein-Glaser: local and regularization independent formulation of renormalization.
- Essentially always possible (there always exists a 'renormalization scheme').
  - Changes in renormalization scheme absorbed at order $O(\hbar)$ in local Lagrangian.
  - No 'power counting' restrictions.
- Works in curved backgrounds and applies to GR.

[B D F] R.Brunetti, M.Dütsch, K.Fredenhagen, *Perturbative algebraic quantum field theory and the renormalization groups*, [arXiv:0901.2038]

[R] K.Rejzner, PhD Thesis (Hamburg) [arXiv:1111.5130]

IR renormalization [✓] and [✗]

- IR divergences in perturbative scattering with massless fields. Must use inclusive cross-sections.
- External sources, thermodynamic limit, Haag's theorem. Must use non-Fock representation of algebra of observables.
- Essentially, it is known what to do, but not at the same level of polish as for UV renormalization.
- Still work to be done!

[E] G.G.Emch *Algebraic Methods in Statistical Mechanics and Quantum Field Theory* (Wiley, 1972)

Nonperturbative definition [✗]

- The real elephant in the room.
- QG exists as much as the Standard Model (perturbatively in $\hbar$ and λ).
- The Standard Model does not exist as much as QG (both lack a non-perturbative definition, existence proof).
- Without non-perturbative existence, no unambiguous, quantitative predictions can be made (no rigorous error bars).
- \$1 M Clay Institute prize for solving this problem.

Discussion

- The problem of Quantum Gravity can be precisely stated.
- There exists a perturbative solution: (modulo observables) QG exists as much as the Standard Model does.
- Solution is conservative: no new fields, no new dimensions, no radically different dynamics, no unneeded discretization.
- Advances in several active areas of mathematics automatically improve the construction.
- Many of the physical implications left to be explored.

Thank you for your attention!