# Abstracts

## Algebraic Geometry in Higher Dimensions

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**V. Alexeev**
*TBA*

**L. Bădescu**
*A Barth-Lefschetz theorem for submanifolds of a product of projective spaces*

This is a joint work with Flavia Repetto.

Let $X$ be a complex submanifold of dimension $d$ of $\mathbb{P}^m \times \mathbb{P}^n$ ($m \geq n \geq 2$) and denote by $\alpha\colon \operatorname{Pic}(\mathbb{P}^m \times \mathbb{P}^n) \to \operatorname{Pic}(X)$ the restriction map of Picard groups, by $\mathcal{N}_{X|\mathbb{P}^m\times\mathbb{P}^n}$ the normal bundle of $X$ in $\mathbb{P}^m \times \mathbb{P}^n$. Set $t := \max\{\dim \pi_1(X), \dim \pi_2(X)\}$, where $\pi_1$ and $\pi_2$ are the two projections of $\mathbb{P}^m \times \mathbb{P}^n$. We prove a Barth-Lefschetz type result as follows:

*Theorem. If $d \geq \frac{m+n+t+1}{2}$ then $X$ is algebraically simply connected, the map $\alpha$ is injective and* $\operatorname{Coker}(\alpha)$ *is torsion-free. Moreover $\alpha$ is an isomorphism if $d \geq \frac{m+n+t+2}{2}$, or if $d = \frac{m+n+t+1}{2}$ and $\mathcal{N}_{X|\mathbb{P}^m\times\mathbb{P}^n}$ is decomposable.*

Explicit examples show that these bounds are optimal. The main technical ingredients in the proof are: the Kodaira-Le Potier vanishing theorem in the generalized form of Sommese, the join construction and an algebraisation result of Faltings concerning small codimensional subvarieties in $\mathbb{P}^N$.

## I. Bauer

*Action of the absolute Galois group on moduli spaces of surfaces*

This is a joint work with F. Catanese, F. Grunewald.

Hilbert schemes are defined over the integers, hence it follows that the moduli spaces of surfaces of general type are also defined over the integers, and there is an action of the absolute Galois group Gal := $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ on the sets of its irreducible components, respectively of its connected components.

Considering varieties isogenous to a product (quotients of a product of curves by the free action of a finite group $G$) we show that the absolute Galois group acts faithfully on the set of connected components. In fact, given an element $\sigma \in \mathrm{Gal}$ different from the identity and from complex conjugation, there is a surface of general type $X$ such that $X$ and the conjugate variety $X^\sigma$ have non isomorphic fundamental groups. These results vastly generalize a phenomenon discovered by J.P. Serre in the 60's, that there exist Galois conjugate varieties which have non isomorphic fundamental groups, in particular are not homeomorphic.

## F. Campana

*Geometry of orbifolds and classification theory*

Geometric orbifolds are pairs $X/\Delta$ with $X$ complex projective (say) and $\Delta$ a ramification divisor. They interpolate between projective manifolds ($\Delta$ empty) and logarithmic manifolds ($\Delta$ reduced integral). They possess the usual geometric invariants of manifolds; in particular morphisms and birational maps can be naturally defined for them. Fibrations in this birational category enjoy additivity properties *not satisfied* by the birational category of manifolds without orbifold structure, which permit to express some invariants of the total space in term of those of the generic fibre and of the base, and so to inductively lift properties in towers of fibrations. For this reason in particular, this category appears to be the natural frame of classification theory of complex projective manifolds (without orbifold structure). We shall extend some propertier of manifolds to this larger class, and raise some questions about rational orbifold curves and orbifold universal covers.

**L. Caporaso**
*Degeneration of Brill-Noether varieties*

Brill-Noether varieties of smooth curves are well known subvarieties of the Picard scheme parametrizing line bundles with certain properties. When smooth curves degenerate to singular ones Brill-Noether varieties should degenerate to geometrically meaningful objects.

An approach to this problem, using Néron models, will be described together with some recent results.

**F. Catanese**
*TBA*

**L. Chiantini**
*On the interpolation problem on varieties*

We discuss the linear interpolation problem for multiple points and linear systems, on general projective varieties. Mainly we focus on the case of multiplicity 2 and 3.

**B. Fantechi**
*Moduli spaces with symmetric obstruction theories*

An obstruction theory is called symmetric if it admits a symmetric isomorphism with its dual. Moduli of sheaves on a Calabi Yau threefold and the intersection of two Lagrangians have a symmetric obstruction theory: as a result, they have expected dimension zero, and their virtual degree can be calculated by integrating a constructible function - in particular, it is defined even in the nonproper case. We discuss some properties of these spaces and their dg structures.

**S. Kebekus**

*On the structure of surfaces mapping to the moduli stack of canonically polarized varieties*

This is a joint work with Sándor Kovács.

Shafarevich's well-known hyperbolicity conjecture asserts that a family of curves over a quasi-projective 1-dimensional base is isotrivial unless the logarithmic Kodaira dimension of the base is positive. More generally, it has been conjectured by Viehweg that the base of a smooth family of canonically polarized varieties is of log general type if the family is of maximal variation. Using extension properties of logarithmic pluri-forms, we relate the minimal model program of the base to the moduli map, and give bounds for the variation of a family in terms of the logarithmic Kodaira dimension of the base. This gives an affirmative answer to Viehweg's conjecture for families parametrized by surfaces.

**J. Kollár**

*Moduli spaces for varieties of general type: What remains to be done?*

**S. Kovács**

*Characterizations of projective spaces and hyperquadrics*

This is a joint work with Carolina Araujo and Stéphane Druel.

We confirm the following conjecture of Beauville: Let $L$ be an ample line bundle on the smooth projective variety $X$ and suppose that the $p-$th exterior power of the vector bundle $T_X$ contains the $p-$th power of $L$. Then either $X$ is a projective space and $L = \mathcal{O}(1)$ or $p = \dim X$ and $X$ is a quadric hypersurface and (again) $L = \mathcal{O}(1)$.

**A. Lopez**

*A (new) Enriques-Fano threefold*
*with non $\mathbb{Q}-$smoothable canonical singularities*

This is a joint work with Roberto Muñoz and Andreas Knutsen.

An Enriques-Fano threefold $X$ is a projective threefold having as hyperplane section a smooth Enriques surface. If we assume that $X$ has cyclic quotient terminal singularities, there is a classification of such threefolds due to Bayle and Sano.

We proved that any Enriques-Fano threefold has genus at most 17 (proved also indipendently by Prokhorov). In the talk we will construct a new Enriques-Fano threefold $X$ in $\mathbb{P}^9$ with two properties:

1) it does not belong to Bayle-Sano's list nor is limit of them;

2) its normalization is a $\mathbb{Q}-$Fano threefold with canonical singularities that does not admit a $\mathbb{Q}-$smoothing (that is a flat limit of threefolds with cyclic quotient terminal singularities). By some results in deformation theory it is known that (Namikawa) Fano threefolds with Gorenstein terminal singularities are smoothable, while (Minagawa-Sano) $\mathbb{Q}-$Fano threefolds of index at least 1 with terminal singularities are $\mathbb{Q}-$smoothable. Therefore $X$ shows that the latter cannot be extended to the case of canonical singularities.

**J. McKernan**

*The Sarkisov Program*

Conjectural the output of the minimal model program is either a minimal model or a Mori fibre space. However in neither case is the output unique. Recently Kawamata has show that any two minimal models are connected by a sequence of flops.

The Sarkisov program aims to factorise any birational map between two Mori fibre spaces as a sequence of elementary links. In the case of surfaces these links are elementary transformations and the Sarkisov program provides a nice framework to prove that the birational automorphism of $\mathbb{P}^2$ is generated by a Cremona transformation and PGL(3).

In this talk I will describe recent work with Christopher Hacon where we extend Sarkisov's program to all dimensions.

## M. Mella

*Base loci of linear system and the Waring problem*

The Waring problem for forms is the quest for an additive decomposition of homogeneus polynomials into powers of linear ones. The subject has been widely considered in old times, (Sylvester, Hilbert, Richmond and Palatini) with special regards to the existence of a unique decomposition of this type. This would give a "canonical" decomposition. I do expect that the one described at the beginning of the $XX^{th}$ century are the only possible cases in which the decomposition is unique. The aim of this talk is to give evidence to this expectation.

## Y. Miyaoka

*Explicit bound of the canonical degree of a curve on a surface of general type with $K^2 > c_2$*

Let $X$ be a minimal projective surface of general type defined over the complex numbers and let $C \subset X$ be an irreducible curve of geometric genus $g$. Assume that $K_X^2$ is greater than the topological Euler number $c_2(X)$. Then we prove that the "canonical degree" $CK_X$ of $C$ is uniformly bounded in terms of the given invariants $g$, $K_X^2$ and $c_2(X)$, thus giving an effective version of a theorem of Bogomolov on the boundedness of the curves of fixed genus in $X$.

## S. Mukai

*Hilbert's $14^{th}$ problem for the Kronecker quiver*

I was led to the following question by a systematic study of Nagata's counterexamples to Hilbert's $14^{\text{th}}$ problem:

Is the ring of invariant polynomials finitely generated for a linear action of the 2-dimensional additive group?

I will explain the background and discuss the case where the action comes from the quiver $\bullet \rightrightarrows \bullet$ consisting of two vertices and two arrows of the same direction.

**G. Ottaviani**

*Higher secant varieties to Segre and Grassmann varieties*

This is a joint work with C. Peterson and H. Abo.

The dimension of the higher secant varieties to the Veronese varieties are the expected ones with a short list of exceptions, thanks to the Alexander-Hirschowitz theorem. Their equations are still unknown. We study the same kind of problems for Segre and Grassmann varieties. It corresponds to find the decomposition of a general (resp. skew-symmetric) tensor.

**T. Peternell**

*Coverings of projective and Kaehler manifolds: self-maps,deformations and classification*

In my talk I want to discuss the following three topics:

1. A self-map of a projective manifold $X$ of a surjective covering $f\colon X \to X$. Assume that $f$ has degree at least 2. What can be said on the structure of $X$ and $f$? Particularly interesting is the case when $X$ is a Fano manifold with Picard number 1.
2. The deformation theory of coverings of compact Kaehler manifolds in connection with "generic semi-positivity".
3. Given a special variety X and a covering $f\colon X \to Y$. What can be said on $Y$? Here I want in particular discuss the case when $X$ is a torus.

**P. Pirola**

*On the Iitaka Severi set of a projective surface*

We study dominant rational maps from a general surface of degree d of the projective space to surfaces of general type. We prove special properties of these rational maps. We show that for small degree the general surface has only the identity map.

**M. Reid**

*TBA*

## F. Russo

*Conic-connected manifolds and rationality via special families of rational curves*

This is a joint work with Paltin Ionescu.
During the talk I would like to discuss the following topics:

1) Classification of *conic-connected manifolds*, i.e. projective manifolds $X$ such that two general points may be joined by an irreducible conic contained in $X$;
2) A criterion of rationality asserting that the local ring of a smooth point $x$ in $X$ is isomorphic to the local ring of a point in $\mathbb{P}^n$ if and only if $X$ admits a covering family of rational 1-cycles, all passing through $x$, all smooth at $x$ and such that the general cycle of the family is uniquely determined by its tangent line at $x$. The last hypothesis can be omitted when the locus of reducible 1-cycles has codimension at least 2 and "more global" versions hold in the smooth case.
3) Rationality, strong constraints for the existence and finer classification of conic-connected projective manifolds with $b_2 = 1$ and for which the locus of conics through two general points has maximal dimension equal to the secant defect of $X$.

## E. Sernesi

*Moduli of rational fibrations*

We obtain some relations between the numerical invariants of a surface fibered over the projective line with general curves as fibres. We apply such relations to give upper bounds on the genus of a general curve varying in a non-trivial linear system on a non-ruled surface.

## V. Shokurov

*Complements on surfaces*

For algebraic surfaces, bounded complements exist for any boundary multiplicities, that is, in the whole segment $\{0, 1]$. Moreover, we can find complimentary indecies rather divisible. It is expected a higher dimensional generalization of this result and methods in future.

**A. Sommese**

*Recent Progress in Numerical Algebraic Geometry*

Following a short overview of Numerical Algebraic Geometry, some recent ongoing work will be presented:

a) Some of the numerical issues dealt in the design of software for Numerical Algebraic Geometry and some details about Bertini, the software package recently released by D. Bates, J. Hauenstein, C. Wampler, and myself.
b) work with C. Wampler on numerical computation of exceptional sets of algebraic maps (with regards to fiber dimension) by means of iterated fiber products.
c) work with D. Bates, C. Peterson, and C. Wampler on numerical computation of the geometric genus of curve components of algebraic sets.

**J. Starr**

*Rational simple connectedness*

This is a joint work with A.J. de Jong.

Rational simple connectedness is to simple connectedness as rational connectedness is to path connectedness. As time permits, I hope to discuss two results involving this notion.

(1) Hassett's result that every rationally simply connected fibration over a curve satisfies "weak approximation", i.e., power series sections are approximated to arbitrary order by polynomial sections.
(2) A theorem that a rationally simply connected fibration over a surface has a rational section, assuming the vanishing of a Brauer obstruction and some other hypotheses (which I will state precisely). In particular, joint work with A. J. de Jong and Xuhua He proves that if a general fiber is a minimal, projective homogeneous variety G/P, the Brauer obstruction is the only obstruction. As pointed out by Philippe Gille, this implies the last unknown case (i.e., the $E_8$ case) of Serre's Conjecture II in Galois cohomology for the function field of a surface.

**B. van Geemen**

*Real multiplication on K3 surfaces and Kuga Satake varieties*

The endomorphism algebra of a K3 type Hodge structure is a totally real field or a CM field. We give a low brow introduction to the case of a totally real field and show existence results for the Hodge structures, for their polarizations and for certain K3 surfaces. We discuss some examples of Kuga Satake varieties of these Hodge structures. Finally we indicate various open problems related to the Hodge conjecture.

**A. Verra**

*On the Prym moduli spaces in low genus*

A survey on the Kodaira dimension of Prym moduli spaces is given and new results of unirationality are proved for Prym moduli spaces of curves of low genus and for some related moduli spaces.

**G. Vezzosi**

*Derived algebraic geometry*

I will describe briefly how to do some geometry on commutative differential graded algebras, on simplicial algebras and on commutative ring spectra. Here the geometry has to take the intrinsic homotopical flavor of such objects into account. Then I will describe some applications of this 'homotopical geometry' to problems raised outside its framework such as the problem of constructing a universal elliptic cohomology theory and the so-called geometric Langlands correspondence.

At the end I will sketchily discuss some open problems and current directions in this field.

**J. Wiśniewski**

*TBA*