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A characterization of complete flag manifolds

Gianluca Occhetta

with R. Muñoz, L.E. Solá Conde, K. Watanabe and J. Wiśniewski

Busan, January 2016

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 \boldsymbol{X} smooth complex projective variety.

Theorem [Mori (1979)]

 $T_X \text{ ample } \Leftrightarrow X = \mathbb{P}^{\mathfrak{m}}.$



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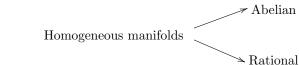
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X smooth complex projective variety.

Theorem [Mori (1979)]

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- Examples:



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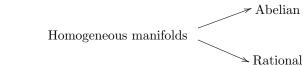
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- $T_X \text{ nef} \Rightarrow ??$
- Examples:



Theorem [Demailly, Peternell and Schneider (1994)] $T_X \text{ nef} \Rightarrow \begin{cases} X \overset{\text{`etale}}{\longleftarrow} X' \overset{F}{\longrightarrow} A \\ A \text{ Abelian, F Fano, } T_F \text{ nef} \end{cases}$

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Campana-Peternell Conjecture (1991)

Every Fano manifold with nef tangent bundle (CP manifold) is homogeneous.

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Campana-Peternell Conjecture (1991)

Every Fano manifold with nef tangent bundle (CP manifold) is homogeneous.

Results:

 \checkmark dim X = 3 [Campana & Peternell (1991)]

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- $\mathbf{V} \dim X = 4$ [CP (1993), Mok (2002), Hwang (2006)]
- $\checkmark \dim X = 5 \text{ and } \rho_X > 1 \text{ [Watanabe (2012)]}$

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- \checkmark dim X = 5 and $\rho_X > 1$ [Watanabe (2012)]
- $\mathbf{\nabla} \dim X = 5$ [Kanemitsu (2015)]

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- $\mathbf{Z} \dim \mathbf{X} = \mathbf{5} [\text{Kanemitsu} (2015)]$
- $\mathbf{V} \subset \mathbf{T}_{\mathbf{X}}$ big and 1-ample [Solá-Conde & Wiśniewski (2004)]

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- $\checkmark \dim X = 5$ [Kanemitsu (2015)]
- \checkmark T_X big and 1-ample [Solá-Conde & Wiśniewski (2004)] \checkmark X horospherical [Li (2015)]

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A vector bundle \mathcal{E} on a smooth complex projective variety X is a Fano bundle iff $\mathbb{P}_X(\mathcal{E})$ is a Fano manifold.

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Later the assumption on b_4 was removed by Watanabe (2013).

Finally the assumption " \mathbb{P}^1 -bundle" was replaced by "smooth \mathbb{P}^1 -fibration" (MOSWa 2013).

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Theorem

A Fano manifold with Picard number 2 whose elementary contractions are \mathbb{P}^1 -fibrations is isomorphic to one of the following • $\mathbb{P}_{\mathbb{P}^1}(\mathcal{O} \oplus \mathcal{O})$

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- $\mathbb{P}_{\mathbb{P}^1}(\mathcal{O}\oplus\mathcal{O})$
- $\mathbb{P}_{\mathbb{P}^2}(\mathsf{T}_{\mathbb{P}^2})$

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Theorem

- $\mathbb{P}_{\mathbb{P}^1}(\mathcal{O}\oplus\mathcal{O})$
- $\mathbb{P}_{\mathbb{P}^2}(\mathsf{T}_{\mathbb{P}^2})$
- $\mathbb{P}_{\mathbb{P}^3}(\mathcal{N}) = \mathbb{P}_{\mathbb{Q}^3}(\mathcal{S})$ \mathcal{N} Null-correlation , \mathcal{S} Spinor

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Theorem

- $\mathbb{P}_{\mathbb{P}^1}(\mathcal{O}\oplus\mathcal{O})$
- $\mathbb{P}_{\mathbb{P}^2}(\mathsf{T}_{\mathbb{P}^2})$
- $\mathbb{P}_{\mathbb{P}^3}(\mathcal{N}) = \mathbb{P}_{\mathbb{Q}^3}(\mathcal{S}) \mathcal{N}$ Null-correlation , \mathcal{S} Spinor
- $\mathbb{P}_{\mathbb{Q}^5}(\mathcal{C}) = \mathbb{P}_{K(G_2)}(\mathcal{Q}) \mathcal{C}$ Cayley, \mathcal{Q} universal quotient.

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- $\mathbb{P}_{\mathbb{P}^3}(\mathcal{N}) = \mathbb{P}_{\mathbb{Q}^3}(\mathcal{S})$ \mathcal{N} Null-correlation , \mathcal{S} Spinor
- $\mathbb{P}_{\mathbb{Q}^5}(\mathcal{C}) = \mathbb{P}_{K(G_2)}(\mathcal{Q}) \mathcal{C}$ Cayley, \mathcal{Q} universal quotient.

All the varieties appearing in the list are rational homogeneous!

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 Classify Fano manifolds whose elementary contractions are ¹-bundles - or just smooth P¹-fibrations.

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- Classify Fano manifolds whose elementary contractions are \mathbb{P}^1 -bundles or just smooth \mathbb{P}^1 -fibrations.
- The vector bundle approach seems difficult to apply to this more general situation.

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- Classify Fano manifolds whose elementary contractions are ¹-bundles - or just smooth P¹-fibrations.
- The vector bundle approach seems difficult to apply to this more general situation.
- Is it possible to prove directly that these varieties are rational homogeneous?

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Lie Algebras

• G semisimple Lie group,

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- $\bullet~G$ semisimple Lie group,
- \mathfrak{g} associated Lie algebra,
- n rank of g.

Lie Algebras

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- G semisimple Lie group,
- g associated Lie algebra,
- \mathfrak{n} rank of \mathfrak{g} .

With \mathfrak{g} is associated a square $n \times n$ matrix $A = [\mathfrak{a}_{ij}]$, called Cartan matrix, which encodes the structure of \mathfrak{g} ;

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With \mathfrak{g} is associated a square $n \times n$ matrix $A = [\mathfrak{a}_{ij}]$, called Cartan matrix, which encodes the structure of \mathfrak{g} ;

A and all its principal minors are positive definite and moreover

- $a_{ii} = 2$ for every i,
- $a_{ij} = 0$ iff $a_{ji} = 0$,
- if $a_{ij} \neq 0, i \neq j$, then $a_{ij}, a_{ji} \in \mathbb{Z}^-$ and $a_{ij}a_{ji} = 1, 2$ or 3.

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- if $a_{ij} \neq 0$, $i \neq j$, then a_{ij} , $a_{ji} \in \mathbb{Z}^-$ and $a_{ij}a_{ji} = 1, 2$ or 3.

Example (n=2)

The Cartan matrices of rank 2 semi simple Lie algebras are

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

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With the matrix A is associated a finite Dynkin diagram \mathcal{D} , in the following way

• \mathcal{D} is a graph with n nodes,

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- \mathcal{D} is a graph with \mathfrak{n} nodes,
- the nodes i and j are joined by $a_{ij}a_{ji}$ edges,

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Dynkin diagrams

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With the matrix A is associated a finite Dynkin diagram $\mathcal{D},$ in the following way

- \mathcal{D} is a graph with \mathfrak{n} nodes,
- the nodes i and j are joined by $a_{ij}a_{ji}$ edges,
- if $|a_{ij}| > |a_{ji}|$ the edges are directed towards the node i.

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Example (n=2)

The Dynkin diagrams of rank 2 Lie algebras are

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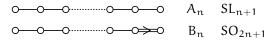
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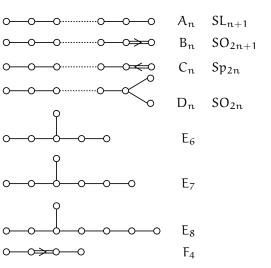
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Subgroups $P \subset G$ s.t. G/P is projective are called parabolic.

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Subgroups $P\subset G$ s.t. G/P is projective are called parabolic.

G = SL(4)			

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Subgroups $\mathsf{P}\subset\mathsf{G}$ s.t. G/P is projective are called parabolic.

G = SL(4)
$ \begin{array}{c} \bullet \\ \mathbb{P}^3 \end{array} \\ \hline \mathbb{G}(1,3) \end{array} \\ O \\ \mathbb{G}^3)^* \end{array} $

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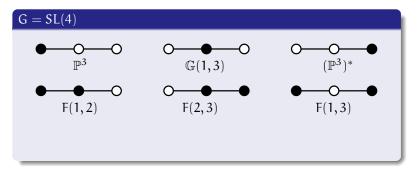
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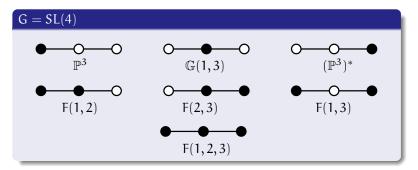
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Subgroups $\mathsf{P}\subset\mathsf{G}$ s.t. G/P is projective are called parabolic.



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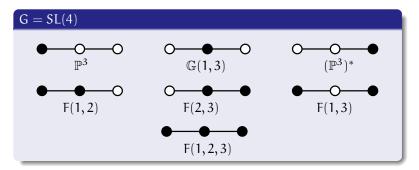
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Rational homogeneous manifolds

Subgroups $\mathsf{P}\subset\mathsf{G}$ s.t. G/P is projective are called parabolic.

A parabolic subgroup is given by the choice of a set of nodes, and the variety G/P is denoted by marking these nodes.



So a rational homogeneous (RH) manifold is given by a marked Dynkin diagram $(\mathcal{D}, \mathcal{I})$.

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X RH given by $(\mathcal{D}, \mathcal{I})$.

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X RH given by $(\mathcal{D}, \mathcal{I})$.

1 X is a Fano manifold;

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X RH given by $(\mathcal{D}, \mathcal{I})$.

1 X is a Fano manifold;

2 The Picard number ρ_X of X is #I;

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X RH given by $(\mathcal{D}, \mathcal{I})$.

- **1** X is a Fano manifold;
- $\ensuremath{ 2 } \ensuremath{ {\rm The Picard number } \rho_X {\rm ~of} ~X {\rm ~is} ~\# I;}$
- 3 The cone NE(X) is simplicial, and its faces correspond to proper subsets J ⊊ I;

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- X RH given by $(\mathcal{D}, \mathcal{I})$.
 - **1** X is a Fano manifold;
 - $\ensuremath{ 2 } \ensuremath{ {\rm The Picard number } \rho_X {\rm ~of} ~X {\rm ~is} ~\# I;}$
 - 3 The cone NE(X) is simplicial, and its faces correspond to proper subsets J ⊊ I;
 - (4) Every contraction $\pi: X \to Y$ is of fiber type and smooth.

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- X RH given by $(\mathcal{D}, \mathcal{I})$.
 - **1** X is a Fano manifold;
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 - 3 The cone NE(X) is simplicial, and its faces correspond to proper subsets J ⊊ I;
 - $\textbf{@ Every contraction $\pi: X \to Y$ is of fiber type and smooth. }$
 - **6** Y is RH with marked Dynkin diagram $(\mathcal{D}, \mathcal{J})$,

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- $X \; \mathrm{RH} \; \mathrm{given} \; \mathrm{by} \; (\mathcal{D}, \mathcal{I}).$
 - **1** X is a Fano manifold;
 - $\ensuremath{ 2 } \ensuremath{ {\rm The Picard number } \rho_X {\rm ~of} ~X {\rm ~is} ~\# I;}$
 - **③** The cone NE(X) is simplicial, and its faces correspond to proper subsets $J \subsetneq I$;
 - $\textbf{@ Every contraction $\pi: X \to Y$ is of fiber type and smooth. }$
 - **6** Y is RH with marked Dynkin diagram $(\mathcal{D}, \mathcal{J})$,
 - **6** Every fiber is RH with marked Dynkin diagram $(\mathcal{D} \setminus \mathcal{J}, \mathcal{I} \setminus \mathcal{J})$.

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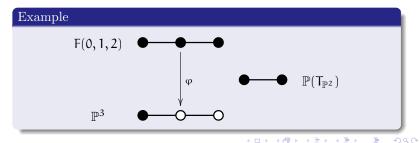
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- $X \; \mathrm{RH} \; \mathrm{given} \; \mathrm{by} \; (\mathcal{D}, \mathcal{I}).$
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 - **6** Y is RH with marked Dynkin diagram $(\mathcal{D}, \mathcal{J})$,
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Complete flag manifolds

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A complete flag manifold is a RH manifold with a diagram in which all the nodes are marked. The corresponding parabolic subgroup B is called a Borel subgroup.

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• Every RH manifold is dominated by a complete flag manifold.

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- Every RH manifold is dominated by a complete flag manifold.
- $p_i: G/B \to G/P^i$ contractions corresponding to the unmarking of one node are \mathbb{P}^1 -fibrations.

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- If Γ_i is a fiber of p_i , and K_i the relative canonical, the intersection matrix $[-K_i \cdot \Gamma_j]$ is the Cartan matrix.

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- If Γ_i is a fiber of p_i , and K_i the relative canonical, the intersection matrix $[-K_i \cdot \Gamma_j]$ is the Cartan matrix.

Example (A_n)

If $\mathcal{D} = A_n$, then G/B is the manifold parametrizing complete flags of linear subspaces in \mathbb{P}^n .

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Theorem

A Fano manifold with Picard number 2 whose elementary contractions are \mathbb{P}^1 -fibrations is isomorphic to one of the following

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Theorem

A Fano manifold with Picard number 2 whose elementary contractions are \mathbb{P}^1 -fibrations is isomorphic to one of the following

• •	• •	⊷	₩
$\mathbb{P}_{\mathbb{P}^1}(\mathcal{O}\oplus\mathcal{O})$	$\mathbb{P}_{\mathbb{P}^2}(T_{\mathbb{P}^2})$	$\mathbb{P}_{\mathbb{P}^3}\left(\mathcal{N} ight)$	$\mathbb{P}_{\mathbb{Q}^{5}}(\mathcal{C})$

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Theorem

A Fano manifold with Picard number 2 whose elementary contractions are \mathbb{P}^1 -fibrations is isomorphic to one of the following

$\mathbb{P}_{\mathbb{P}^1}(\mathcal{C})$	$\mathcal{O}\oplus\mathcal{O})$	$\mathbb{P}_{\mathbb{P}^2}(T_{\mathbb{P}^2})$	$\mathbb{P}_{\mathbb{P}^{3}}\left(\mathcal{N} ight)$	$\mathbb{P}_{\mathbb{Q}^5}(\mathcal{C})$
•	•	••	€€	€

Theorem

A Fano manifold with Picard number 2 whose elementary contractions are \mathbb{P}^1 -fibrations is a complete flag manifold.

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$\pi: M \to Y \mbox{ smooth } \mathbb{P}^1\mbox{-fibration}. \ \Gamma \mbox{ fiber}, \ K \ {\rm relative \ canonical}$

Lemma

Let D be a divisor on M and set $l:=D\cdot\Gamma+1.$ Then, $\forall i\in\mathbb{Z}$

$$\begin{split} & H^{i}(M,D) \cong \quad H^{i-1}(M,D+lK) \quad \textit{if } l < 0 \\ & H^{i}(M,D) \cong \quad \{0\} \qquad \textit{if } l = 0 \\ & H^{i}(M,D) \cong \quad H^{i+1}(M,D+lK) \quad \textit{if } l > 0 \end{split}$$

 $\label{eq:interm} \textit{In particular} \quad X(M,D) = -X(M,D+lK) \quad \textit{for any } D.$

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• X Fano manifold with Picard number 2.

Idea of Proof

I - Finding the intersection matrix

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- $\pi_i: X \to X_i$ elementary contration.

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- K_i relative canonical, Γ_i fiber of π_i .

Intersection matrix $[-K_i \cdot \Gamma_j]$:

$$\mathsf{A} := \left(egin{array}{cc} 2 & a \ b & 2 \end{array}
ight) \qquad a,b \leq 0$$

Idea of Proof

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$$\mathsf{A} := \left(egin{array}{cc} 2 & a \ b & 2 \end{array}
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 $\begin{array}{l} \text{Claim} \\ \det A > 0. \end{array}$

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ight) \qquad a,b \leq 0$$

Idea of Proof

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 $\begin{array}{l} \text{Claim} \\ \det A > 0. \end{array}$

 ${\rm Assume } \det A < 0.$

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• X Fano manifold with Picard number 2.

- $\pi_i: X \to X_i$ elementary contration.
- K_i relative canonical, Γ_i fiber of π_i .

Intersection matrix $[-K_i \cdot \Gamma_j]$:

$$\mathsf{A}:=\left(egin{array}{cc} 2 & a \ b & 2 \end{array}
ight) \qquad \mathfrak{a},\mathfrak{b}\leq \mathfrak{0}$$

 $\begin{array}{l} \text{Claim} \\ \det A > 0. \end{array}$

Assume det A < 0.

If H is an ample line bundle and we write

$$H = \alpha K_1 + \beta K_2.$$

from det A < 0 we get $\alpha, \beta > 0$.

Idea of Proof

I - Finding the intersection matrix

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Let D_0 be a very ample line bundle, set $d_0 = D_0 \cdot \Gamma_1 + 1$ and apply relative duality with respect to π_1

 $h^0(D_0) = h^1(D_0 + d_0K_1)$

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$$h^{0}(D_{0}) = h^{1}(D_{0} + d_{0}K_{1})$$

Set
$$E_1 := D_0 + d_0 K_1$$
.

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 $\mathrm{Set}\ E_1:=D_0+d_0K_1.$

By construction $E_1 \cdot \Gamma_1 < 0$, thus $e_1 := E_1 \cdot \Gamma_2 + 1 > 0$,

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$$h^{0}(D_{0}) = h^{1}(D_{0} + d_{0}K_{1})$$

Set $E_1 := D_0 + d_0 K_1$.

By construction $E_1 \cdot \Gamma_1 < 0$, thus $e_1 := E_1 \cdot \Gamma_2 + 1 > 0$,

otherwise $-\mathsf{E}_1$ would be ample, but its coefficients on K_1 and K_2 are negative.

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By construction $E_1 \cdot \Gamma_1 < 0$, thus $e_1 := E_1 \cdot \Gamma_2 + 1 > 0$,

otherwise $-\mathsf{E}_1$ would be ample, but its coefficients on K_1 and K_2 are negative.

Apply relative duality with respect to π_2

$$h^{1}(E_{1}) = h^{2}(E_{1} + e_{1}K_{2})$$

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otherwise $-\mathsf{E}_1$ would be ample, but its coefficients on K_1 and K_2 are negative.

Apply relative duality with respect to π_2

$$h^{1}(E_{1}) = h^{2}(E_{1} + e_{1}K_{2})$$

Set $D_1 := E_1 + e_1 K_2$ and apply relative duality with respect to π_1 .

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$$\label{eq:matrix} \begin{split} 0 \neq h^0(D_0) = h^1(E_1) = h^2(D_1) = \dots \\ \cdots = h^{2k-1}(E_k) = h^{2k}(D_k) = \dots \end{split}$$

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$0 \neq h^{0}(D_{0}) = h^{1}(E_{1}) = h^{2}(D_{1}) = \dots$ $\dots = h^{2k-1}(E_{k}) = h^{2k}(D_{k}) = \dots$

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getting a contradiction for $2k > \dim X.$

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So we have

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 $\dots = h^{2k-1}(E_{k}) = h^{2k}(D_{k}) = \dots$

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getting a contradiction for $2k > \dim X$.

The proof in the case det A = 0 uses the same idea.

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So we have

$$0 \neq h^{0}(D_{0}) = h^{1}(E_{1}) = h^{2}(D_{1}) = \dots$$

 $\dots = h^{2k-1}(E_{k}) = h^{2k}(D_{k}) = \dots$

getting a contradiction for $2k > \dim X$.

The proof in the case det A = 0 uses the same idea.

So the possible intersection matrices are (up to transposition):

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix} \quad \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

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Using the intersection matrices we write K_X (which has degree -2 on Γ_1 and Γ_2) as a combination of K_1 and K_2 :

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Using the intersection matrices we write K_X (which has degree -2 on Γ_1 and Γ_2) as a combination of K_1 and K_2 :

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$
$$K_1 + K_2 \qquad 2K_1 + 2K_2 \qquad 4K_1 + 3K_2 \qquad 10K_1 + 6K_2$$

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$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

$$K_1 + K_2 \qquad 2K_1 + 2K_2 \qquad 4K_1 + 3K_2 \qquad 10K_1 + 6K_2$$

The dimension of X is the only positive integer \mathfrak{m} such that

 $h^{\mathfrak{m}}(K_X)\neq 0.$

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Using the intersection matrices we write K_X (which has degree -2 on Γ_1 and Γ_2) as a combination of K_1 and K_2 :

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$
$$K_1 + K_2 \qquad 2K_1 + 2K_2 \qquad 4K_1 + 3K_2 \qquad 10K_1 + 6K_2$$

The dimension of X is the only positive integer \mathfrak{m} such that

 $h^{\mathfrak{m}}(K_X)\neq 0.$

The idea is to get to K_X starting from \mathcal{O}_X using relative duality.

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For example in the case

$$\left(\begin{array}{cc} 2 & -1 \\ -1 & 2 \end{array}\right)$$



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For example in the case

$$\left(\begin{array}{rrr}2 & -1\\-1 & 2\end{array}\right)$$

$$1 = h^0(\mathcal{O}_X) \stackrel{1}{=} \qquad h^1(K_1) \qquad \qquad \mathcal{O}_X \cdot \Gamma_1 + 1 = 1$$

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For example in the case

$$\left(\begin{array}{rrr}2 & -1\\ -1 & 2\end{array}\right)$$

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 $1 = h^{0}(\mathcal{O}_{X}) \stackrel{1}{=} h^{1}(K_{1}) \qquad \qquad \mathcal{O}_{X} \cdot \Gamma_{1} + 1 = 1$ $\stackrel{2}{=} h^{2}(K_{1} + 2K_{2}) \qquad \qquad K_{1} \cdot \Gamma_{2} + 1 = 2$

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For example in the case

$$\left(\begin{array}{cc} 2 & -1 \\ -1 & 2 \end{array}\right)$$

$$\begin{split} 1 &= h^0(\mathcal{O}_X) \quad \stackrel{1}{=} \quad h^1(K_1) & \mathcal{O}_X \cdot \Gamma_1 + 1 = 1 \\ &\stackrel{2}{=} \quad h^2(K_1 + 2K_2) & K_1 \cdot \Gamma_2 + 1 = 2 \\ &\stackrel{1}{=} \quad h^3(2K_1 + 2K_2) & (K_1 + 2K_2) \cdot \Gamma_1 + 1 = 1 \end{split}$$

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For example in the case

$$\left(\begin{array}{cc} 2 & -1 \\ -1 & 2 \end{array}\right)$$

$$\begin{array}{rcl} = h^0(\mathcal{O}_X) & \stackrel{1}{=} & h^1(K_1) & \mathcal{O}_X \cdot \Gamma_1 + 1 = 1 \\ & \stackrel{2}{=} & h^2(K_1 + 2K_2) & K_1 \cdot \Gamma_2 + 1 = 2 \\ & \stackrel{1}{=} & h^3(2K_1 + 2K_2) & (K_1 + 2K_2) \cdot \Gamma_1 + 1 = 1 \\ & = & h^3(K_X) \end{array}$$

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The dimension of X is three.

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For example in the case

1

$$\left(\begin{array}{rrr} 2 & -1 \\ -1 & 2 \end{array}\right)$$

$$\begin{array}{rcl} = h^0(\mathcal{O}_X) & \stackrel{1}{=} & h^1(K_1) & \mathcal{O}_X \cdot \Gamma_1 + 1 = 1 \\ & \stackrel{2}{=} & h^2(K_1 + 2K_2) & K_1 \cdot \Gamma_2 + 1 = 2 \\ & \stackrel{1}{=} & h^3(2K_1 + 2K_2) & (K_1 + 2K_2) \cdot \Gamma_1 + 1 = 1 \\ & = & h^3(K_X) \end{array}$$

The dimension of X is three.

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

$$2 \qquad 3 \qquad 4 \qquad 6$$

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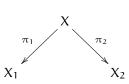
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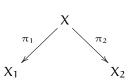
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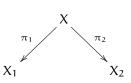
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Set
$$L_2 = -2K_1 - K_2$$
. Then $L_2 \cdot \Gamma_2 = 0$.

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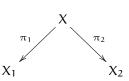
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Set $L_2 = -2K_1 - K_2$. Then $L_2 \cdot \Gamma_2 = 0$.

So there exists $H_2 \in Pic(X_2)$ such that $L_2 = \pi_i^* H_2$.

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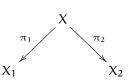
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Set $L_2 = -2K_1 - K_2$. Then $L_2 \cdot \Gamma_2 = 0$.

So there exists $H_2 \in Pic(X_2)$ such that $L_2 = \pi_i^* H_2$.

We can write $-\pi_1^* K_{X_2} = -K_X + K_2 = 5L_2 = 5\pi_1^* H_2$.

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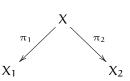
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Let's do the case

 $\left(\begin{array}{rrr}2 & -1\\ -3 & 2\end{array}\right)$

 $\mathrm{Set}\ L_2=-2K_1-K_2.\ \mathrm{Then}\ L_2\cdot\Gamma_2=0.$

So there exists $H_2 \in Pic(X_2)$ such that $L_2 = \pi_i^* H_2$.

We can write $-\pi_1^* K_{X_2} = -K_X + K_2 = 5L_2 = 5\pi_1^* H_2$.

So the index of X_2 , which has dimension 5, is a multiple of 5; therefore the index is 5 and $X_2 \simeq \mathbb{Q}^5$.

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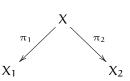
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So the index of X_2 , which has dimension 5, is a multiple of 5; therefore the index is 5 and $X_2 \simeq \mathbb{Q}^5$.

Set $L_1 = -3K_1 - 2K_2$. Then $L_1 \cdot \Gamma_2 = 1$, hence π_2 is a projective bundle.

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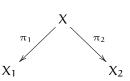
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Set $L_2 = -2K_1 - K_2$. Then $L_2 \cdot \Gamma_2 = 0$.

So there exists $H_2 \in Pic(X_2)$ such that $L_2 = \pi_i^* H_2$.

We can write $-\pi_1^* K_{X_2} = -K_X + K_2 = 5L_2 = 5\pi_1^* H_2$.

So the index of X_2 , which has dimension 5, is a multiple of 5; therefore the index is 5 and $X_2 \simeq \mathbb{Q}^5$.

Set $L_1=-3K_1-2K_2.$ Then $L_1\cdot\Gamma_2=1,$ hence π_2 is a projective bundle.

The proof is finished by computing the Chern classes of the bundle.

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The intersection matrix

$\begin{array}{l} X \ {\rm Fano} \ {\rm of} \ {\rm Picard} \ {\rm number} \ n \ {\rm with} \ {\rm nef} \ {\rm tangent} \ {\rm bundle} \ {\rm whose} \\ {\rm elementary} \ {\rm contractions} \ {\rm are} \ \mathbb{P}^1 \mbox{-fibrations} \ ({\rm FT}\mbox{-manifold} \ {\rm for} \ {\rm short}). \end{array}$

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$\begin{array}{l} X \ {\rm Fano} \ {\rm of} \ {\rm Picard} \ {\rm number} \ n \ {\rm with} \ {\rm nef} \ {\rm tangent} \ {\rm bundle} \ {\rm whose} \\ {\rm elementary} \ {\rm contractions} \ {\rm are} \ \mathbb{P}^1 \mbox{-fibrations} \ ({\rm FT}\mbox{-manifold} \ {\rm for} \ {\rm short}). \end{array}$

• Every contraction $\pi: X \to Y$ is of fiber type, i.e. $\dim(Y) < \dim(X)$;

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X Fano of Picard number n with nef tangent bundle whose elementary contractions are \mathbb{P}^1 -fibrations (FT-manifold for short).

• Every contraction $\pi: X \to Y$ is of fiber type, i.e. $\dim(Y) < \dim(X);$

• Fibers of every contraction $\pi: X \to Y$ are FT-manifolds;

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- Fibers of every contraction $\pi: X \to Y$ are FT-manifolds;
- The Picard number of $\pi^{-1}(y)$ is $\rho_X \rho_Y$;

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X Fano of Picard number n with nef tangent bundle whose elementary contractions are \mathbb{P}^1 -fibrations (FT-manifold for short).

- Every contraction $\pi: X \to Y$ is of fiber type, i.e. $\dim(Y) < \dim(X);$
- Fibers of every contraction $\pi: X \to Y$ are FT-manifolds;
- The Picard number of $\pi^{-1}(y)$ is $\rho_X \rho_Y$;
- The cone NE(X) is simplicial.

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The Cartan matrix of X is the $n \times n$ matrix M(X) defined by $M(X)_{ij} = -K_i \cdot \Gamma_j$.

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Theorem (MOSWa 2013)

The Cartan matrix of an FT-manifold with nef tangent bundle is the Cartan matrix of a semi simple Lie algebra.

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Let X be an FT-manifold, $I \subset \{1, ..., n\}$ any nonempty subset, and let $\pi_I : X \to X_I$ be the contraction of the corresponding face R_I .

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Speculations

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The Cartan matrix of an FT-manifold with nef tangent bundle is the Cartan matrix of a semi simple Lie algebra.

Let X be an FT-manifold, $I \subset \{1, ..., n\}$ any nonempty subset, and let $\pi_I : X \to X_I$ be the contraction of the corresponding face R_I .

Then every fiber of π_I is an FT-manifold whose Cartan matrix is the $|I| \times |I|$ principal submatrix of M(X) obtained from M(X) by subtracting rows and columns corresponding to indices which are not in I.

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Idea of Proof

In particular any 2×2 principal submatrix is the Cartan matrix of an FT-manifold of Picard number 2. So

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In particular any 2×2 principal submatrix is the Cartan matrix of an FT-manifold of Picard number 2. So

- $m_{ii} = 2$ for every i,
- $m_{ij} = 0$ iff $a_{ji} = 0$,
- if $\mathfrak{m}_{ij} \neq 0$, $i \neq j$, then $\mathfrak{m}_{ij}, \mathfrak{m}_{ji} \in \mathbb{Z}^-$ and $\mathfrak{m}_{ij}\mathfrak{m}_{ji} = 1, 2$ or 3.

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In particular any 2×2 principal submatrix is the Cartan matrix of an FT-manifold of Picard number 2. So

- $\mathfrak{m}_{\mathfrak{i}\mathfrak{i}}=2$ for every \mathfrak{i} ,
- $m_{ij} = 0$ iff $a_{ji} = 0$,
- if $\mathfrak{m}_{ij} \neq 0, i \neq j$, then $\mathfrak{m}_{ij}, \mathfrak{m}_{ji} \in \mathbb{Z}^-$ and $\mathfrak{m}_{ij}\mathfrak{m}_{ji} = 1, 2$ or 3.

This implies that M(X) is a generalized Cartan matrix (as in the theory of Kac-Moody algebras).

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This implies that M(X) is a generalized Cartan matrix (as in the theory of Kac-Moody algebras).

- By induction we may assume M(X) is
 - Of finite type (all the principal minors are positive definite) or
 - Of affine type (all the proper principal minors are positive definite, but det A = 0).

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$\mathsf{M}(X)$ affine implies that there exists a linear combination

$$\Gamma = \sum_{1}^{n} \mathfrak{m}_{i} \Gamma_{i}$$

with $\mathfrak{m}_i\in\mathbb{Z}_{>0},$ satisfying that $K_i\cdot\Gamma=0$ for all i.

Idea of Proof

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Idea of Proof

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$$\Gamma = \sum_{1}^{n} \mathfrak{m}_{i} \Gamma_{i}$$

with $\mathfrak{m}_i\in\mathbb{Z}_{>0},$ satisfying that $K_i\cdot\Gamma=0$ for all i.

Since the Γ_i 's are free curves the cycle $\sum_{1}^{n} \mathfrak{m}_i \Gamma_i$ is smoothable, and therefore numerically equivalent to an irreducible rational curve Γ .

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 $\mathsf{M}(X)$ affine implies that there exists a linear combination

$$\Gamma = \sum_{1}^{n} \mathfrak{m}_{i} \Gamma_{i}$$

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, satisfying that $K_i \cdot \Gamma = 0$ for all i.

Since the Γ_i 's are free curves the cycle $\sum_{i=1}^{n} m_i \Gamma_i$ is smoothable, and therefore numerically equivalent to an irreducible rational curve Γ .

Using $K_i \cdot \Gamma = 0$ one can prove that, for every $\ell = (l_1, \ldots, l_r)$, if $x \in \Gamma$, then $Z_{\ell}(\Gamma) = Z_{\ell} \times \mathbb{P}^1$.

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Take a reduced sequence $\ell = (l_1, \ldots, l_t)$ such that $\operatorname{Ch}(\ell) = X$

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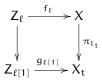
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Take a reduced sequence $\ell = (l_1, \dots, l_t)$ such that $Ch(\ell) = X$

Idea of Proof

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The map $g_{\ell[1]}$ is surjective, hence generically finite.

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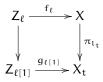
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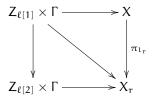
Take a reduced sequence $\ell = (l_1, \ldots, l_t)$ such that $Ch(\ell) = X$

Idea of Proof

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The map $g_{\ell[1]}$ is surjective, hence generically finite.



The diagonal map $g_{\ell[1]} \times \pi_{l_r}$ is of fiber type, and this is a contradiction.

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Theorem (OSWW 2014, OSWi 2015)

Let X be a smooth projective variety of Picard number n, such that there exist $\Gamma_i \in N_1(X)$, $i = 1, \ldots, n$, independent K_X -negative classes generating n extremal rays, whose associated elementary contractions $\pi_i : X \to X_i$ are smooth \mathbb{P}^1 -fibrations. Then X is isomorphic to a flag manifold G/B, for some semisimple group G.

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In the paper in which the conjecture is introduced, Campana and Peternell proposed the following approach:

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In the paper in which the conjecture is introduced, Campana and Peternell proposed the following approach:

1 Prove the conjecture for CP-manifolds of Picard number one.

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In the paper in which the conjecture is introduced, Campana and Peternell proposed the following approach:

1 Prove the conjecture for CP-manifolds of Picard number one.

② Show that, given a CP-manifold X and a contraction $f: X \to Y$, from the homogeneity of Y and of the fibers of f one can reconstruct the homogeneity of X.

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The Picard number one case turned out to be very hard.

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In the paper in which the conjecture is introduced, Campana and Peternell proposed the following approach:

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A possible different approach is the following:

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A possible different approach is the following:

• Prove the conjecture for a CP-manifold with "maximal" Picard number.

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In the paper in which the conjecture is introduced, Campana and Peternell proposed the following approach:

1 Prove the conjecture for CP-manifolds of Picard number one.

Show that, given a CP-manifold X and a contraction f: X → Y, from the homogeneity of Y and of the fibers of f one can reconstruct the homogeneity of X.

The Picard number one case turned out to be very hard.

A possible different approach is the following:

- Prove the conjecture for a CP-manifold with "maximal" Picard number.
- Show that, given a CP-manifold X then X is dominated by a CP-manifold with "maximal" Picard number.

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Given a CP-manifold X, we define:

$$\tau(X) := \sum_{\mathsf{R}} (\ell(\mathsf{R}) - 2)$$

where the sum is taken over the extremal rays of $\overline{NE}(X)$.

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where the sum is taken over the extremal rays of $\overline{NE}(X)$.

In particular $\tau(X) = 0$ if and only if X is a Flag Type manifold.

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where the sum is taken over the extremal rays of $\overline{NE}(X)$. In particular $\tau(X) = 0$ if and only if X is a Flag Type manifold.

CP conjecture will then follow from:

Conjecture

Given a CP-manifold satisfying $\tau(X) > 0$, there exists a contraction $f: X' \to X$ from a CP-manifold X' satisfying $\tau(X') < \tau(X)$.

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