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Varieties of Minimal Rational Tangents and Congruences of Lines

A classical counterexample to a modern conjecture

G. Occhetta

joint work with R. Muñoz and L.E. Solá Conde

MADRID, December 2012

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X smooth complex projective variety of dimension *n* Hom (\mathbb{P}^1, X) scheme parametrizing maps $f : \mathbb{P}^1 \to X$ Hom_{bir} $(\mathbb{P}^1, X) \subset$ Hom (\mathbb{P}^1, X) open subset

RatCurves^{*n*}(*X*) quotient of $\operatorname{Hom}^{n}_{bir}(\mathbb{P}^{1}, X)$ by $\operatorname{Aut}(\mathbb{P}^{1})$

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- RatCurves^{*n*}(*X*) quotient of $\operatorname{Hom}^{n}_{bir}(\mathbb{P}^{1}, X)$ by $\operatorname{Aut}(\mathbb{P}^{1})$
- **Family of rational curves**: $V \subset \text{RatCurves}^n(X)$ irreducible component

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$$\begin{array}{c} U \xrightarrow{i} X \\ \pi \\ V \\ V \end{array}$$

V is **dominating** if $\overline{i(U)} = X$;

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$$\begin{array}{c|c} U & \stackrel{i}{\longrightarrow} X \\ \pi \\ \downarrow \\ V \\ \end{array}$$

V is **dominating** if $\overline{i(U)} = X$; *V* is **minimal** if $\widetilde{V}_x := \pi(i^{-1}(x))$ is proper for a general *x* in Locus(*V*). For a general $x \in X$ the normalization V_x of \widetilde{V}_x is smooth.

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Given a minimal dominating family *V* of rational curves on *X* and a general point $x \in X$ the rational **tangent map** is

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Given a minimal dominating family V of rational curves on X and a general point $x \in X$ the rational **tangent map** is

$$\tau_x: V_x - \to \mathbb{P}(T_X|_x^{\vee})$$

$$\ell \to \mathbb{P}(T_\ell|_x^{\vee})$$

associating to a curve through x its tangent direction at x.

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We define the Variety of Minimal Rational Tangents to be

$$\mathcal{C}_x = \overline{\tau_x(V_x)}$$

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The subvariety

$$\mathcal{C} := ext{closure of} \bigcup_{ ext{general} x \in X} \mathcal{C}_x \subset \mathbb{P}(T_X^{\vee})$$

is called the total variety of minimal rational tangents of V.

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Theorem

• (*Kebekus*) $\tau_x : V_x \to C_x$ is a finite morphism.

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- (Hwang-Mok) $\tau_x : V_x \to C_x$ is birational.

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Trisecants to projected Severi

The subvariety

$$\mathcal{C} := ext{closure of} \bigcup_{ ext{general} x \in X} \mathcal{C}_x \subset \mathbb{P}(T_X^{\vee})$$

is called the **total variety of minimal rational tangents** of V.

Theorem

- (*Kebekus*) $\tau_x : V_x \to C_x$ is a finite morphism.
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- Thus $\tau_x : V_x \to C_x$ is the normalization.

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The VMRT is defined for any uniruled variety.

VMRT Prime Fanos

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X is a Fano manifold if $-K_X$ is ample.

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Theorem (Mori 1979)

Fano manifolds are uniruled.

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Theorem (Mori 1979)

Fano manifolds are uniruled.

A prime Fano manifold is a Fano manifold such that $\text{Pic}(X) \simeq \mathbb{Z}\langle H_X \rangle$. The index r_X of X is the largest integer such that

 $-K_X \sim r_X H_X$

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A rational curve $f : \mathbb{P}^1 \to C$ parametrized by V is called **standard** if

$$f^*T_X \simeq \mathcal{O}_{\mathbb{P}^1}(2) \oplus \mathcal{O}_{\mathbb{P}^1}(1)^{\oplus d} \oplus \mathcal{O}_{\mathbb{P}^1}^{\oplus n-d-1}.$$

where
$$d = -K_X \cdot C$$
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The tangent morphism is immersive at $p \in V_x$ if and only if $i(\pi^{-1}(p))$ is a standard rational curve.

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A rational curve $f : \mathbb{P}^1 \to C$ parametrized by V is called **standard** if

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where $d = -K_X \cdot C$.

The tangent morphism is immersive at $p \in V_x$ if and only if $i(\pi^{-1}(p))$ is a standard rational curve.

Proposition (Hwang)

Assume that X ⊂ P^N, and that V is a family of lines. Then, for a general x ∈ X, τ_x is an embedding and C_x is smooth.
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- 2 If X is a prime Fano of dimension n such that $r_X \ge (n+1)/2$ then the VMRT at a general point is non degenerate.

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- 2 If X is a prime Fano of dimension n such that $r_X \ge (n+1)/2$ then the VMRT at a general point is non degenerate.
- 3 The VMRT cannot be an irreducible linear subspace.

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meorem	cubic in \mathbb{P}^n	quadric \cap cubic in \mathbb{P}^{n-1}
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Fano bundles	Ш	Ш
Why Fano bundles?	\mathbb{Q}^n	\mathbb{Q}^{n-2}
A classification theorem	cubic in \mathbb{P}^n	quadric \cap cubic in \mathbb{P}^{n-1}
Congruences	$1 \qquad C \qquad U = D^{n} L \leq C$	
Generalities	hypersurface $X_d \subset P^n, d \leq n$	c.1 of hypersurfaces of deg $2, \ldots, d$
Special congruences		
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	cubic in \mathbb{P}^n	quadric \cap cubic in \mathbb{P}^{n-1}
ences	$1 \qquad C \qquad V = D^{\mu} I < C$	
ities	hypersurface $X_d \subset P^n$, $d \leq n$	c.1 of hypersurfaces of deg $2, \ldots, d$
congruences	$\mathbb{G}(k, n)$	$\mathbb{D}^k \searrow \mathbb{D}^{n-k-1}$
arieties	$\square(\kappa,n)$	1 ~ 1
ts to d Severi	$\mathbb{G}^{II}(k, 2m-1)$	$\mathbb{P}^k imes \mathbb{Q}^{2m-2k-3}$
	$\mathbb{G}^{II}(m-1,2m-1)$	$\mathbb{G}(1,m-1)$
	$\mathbb{G}^{III}(k, 2m-1)$	$\mathbb{P}_{\mathbb{P}^m}(\mathcal{O}(2)\oplus \mathcal{O}(1)^{2m-2k-2})$
	$\mathbb{G}^{III}(m-1,2m-1)$	$v_2(\mathbb{P}^{m-1})$

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Properties Examples	X	VMRT
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heorem	cubic in \mathbb{P}^n	quadric \cap cubic in \mathbb{P}^{n-1}
ongruences Jeneralities	hypersurface $X_d \subset P^n$, $d \leq n$	c.i of hypersurfaces of deg $2, \ldots, d$
ipecial congruences ieveri varieties	$\mathbb{G}(k,n)$	$\mathbb{P}^k imes \mathbb{P}^{n-k-1}$
Trisecants to projected Severi	$\mathbb{G}^{II}(k, 2m-1)$	$\mathbb{P}^k imes \mathbb{Q}^{2m-2k-3}$
	$\mathbb{G}^{II}(m-1,2m-1)$	$\mathbb{G}(1,m-1)$
	$\mathbb{G}^{III}(k,2m-1)$	$\mathbb{P}_{\mathbb{P}^m}(\mathcal{O}(2)\oplus\mathcal{O}(1)^{2m-2k-2})$
	$\mathbb{G}^{III}(m-1,2m-1)$	$v_2(\mathbb{P}^{m-1})$

VMRT and congruences of lines

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 $\mathbb{G}^{II}(m-1, 2m-1)$ orthogonal Grassmannian $\mathbb{G}^{III}(k, 2m-1)$ symplectic Grassmannian

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Is C_x irreducible if it has positive dimension? Is it at least true for prime Fano manifolds?

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The VRMT at every point of *X* is $\mathbb{P}^{s-1} \sqcup \mathbb{P}^{s-1}$.

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Theorem (Hwang-Mok 2004)

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X has the target rigidity property if, given an holomorphic surjective map $f: Y \to X$, all deformations of *f* come from automorphisms of the target.

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Let X be a prime Fano manifold, $X \neq \mathbb{P}^n$. Assume that at a general point $x \in X$ the VMRT $\mathcal{C}_x(X)$ is non-linear and of positive dimension.

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Let X be a prime Fano manifold. Then the VMRT of X at a general point is not a union of positive dimensional linear subspaces.

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A vector bundle \mathcal{E} over a smooth complex projective variety *X* is a **Fano bundle** if $\mathbb{P}_X(\mathcal{E})$ is a Fano manifold.

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A vector bundle \mathcal{E} over a smooth complex projective variety *X* is a **Fano bundle** if $\mathbb{P}_X(\mathcal{E})$ is a Fano manifold.

If \mathcal{E} is a Fano bundle on *X* then also *X* is a Fano manifold.

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Rank 2 Fano bundles over projective spaces and quadrics have been classified in the 90's (Ancona, Peternell, Sols, Szurek, Wiśniewski).

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Generalization: classify rank 2 Fano bundles over (Fano) manifolds with $b_2 = b_4 = 1$. Solved recently by Muñoz, _ , Solá Conde.

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As a side effect, a counterexample to the non linearity conjecture has been found.

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1 The assumption on b_4 provides the following:

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 - 2 Intersection theory combined with number theory provides n = 2, 3 or 5 if φ is of fiber type.

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Theorem

Let X be a Fano manifold satisfying $b_2 = b_4 = 1$, and let \mathcal{E} be an indecomposable rank two Fano bundle on X.

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• ψ is one of the embeddings given by

(P1)-(P5) ...,

- (D1) $\mathbb{P}^2 \subset \mathbb{G}^{II}(1,3)_{\mathbb{Q}^3} \subset \mathbb{G}(1,4)$ (lines in \mathbb{Q}^3 meeting a fixed line),
- **(D2)** $v_2(\mathbb{P}^2) \subset \mathbb{G}(1,3)$ ((bi)secant lines to $v_3(\mathbb{P}^1) \subset \mathbb{P}^3$),
- **(D3)** $V_5^3 \subset \mathbb{G}(1,4)$ (trisecant lines to the projection of $v_2(\mathbb{P}^2)$ into \mathbb{P}^4), **(C6)** ...

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- (D2) $V_2(\mathbb{P}^*) \subset \mathbb{G}(1, 3)$ ((b))secant lines to $V_3(\mathbb{P}^*) \subset \mathbb{P}^*$), (D3) $V_3^5 \subset \mathbb{G}(1, 4)$ (trisecant lines to the projection of $v_2(\mathbb{P}^2)$ into \mathbb{P}^4), (C6) ...
- ψ factorizes by a finite covering $\psi_1 : X \to X_1$ of one of the submanifolds of types (P1)-(P5) above and, either

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A congruence of lines \mathbb{P}^m is subvariety $X^{m-1} \subset \mathbb{G}(1,m)$.



order of X: number of lines parametrized by X passing through a general point of P^m, which is zero if π' is of fiber type or equal to the degree of π' otherwise.

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- fundamental locus: set of the fundamental points.

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• *X* smooth congruence of lines of **order** 1 in \mathbb{P}^m with $\operatorname{Pic}(X) \cong \mathbb{Z}$.

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- $Z^z \subset \mathbb{P}^m$ smooth fundamental locus.
- The images of the fibers of π' are linear spaces in $\mathbb{G}(1, m)$.

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- f fiber of $\pi: Y \to X \Longrightarrow \alpha := E \cdot f = (m-1)/(m-z-1)$.

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we get $\deg(Z) < \alpha^{m-z} \Longrightarrow Z$ cannot be a complete intersection.

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Remark

The case $\alpha = 2$ *does not happen.*

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X will be covered by linear spaces of dimension dim *X*/2, and have index dim *X*/2 + 1; there are no such Fanos with $Pic(X) \simeq \mathbb{Z}$.

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A **Severi** variety $X \subset \mathbb{P}^N$ is a smooth projective variety of dimension $\frac{2}{3}(N-2)$ which can be isomorphically projected to a projective space of smaller dimension.

Theorem (Zak)

- **2** The Veronese surface in \mathbb{P}^5 ;
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- **16** The variety $E^{16} \subset \mathbb{P}^{26}$.

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where σ and σ' denote, respectively, the blowing up of \mathbb{P}^{3k+2} along *S* and the blowing up of \mathbb{P}^{3k+2} along *S'* \cong *S*.

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Example

Let $X \subset \mathbb{G}(1, 3k + 1)$ be the closure of the family of trisecant lines to a general isomorphic projection $Z \subset \mathbb{P}^{3k+1}$ of a 2k-dimensional Severi variety $S \subset \mathbb{P}^{3k+2}$. Then X is a smooth congruence of order one with linear fibers.

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For every $P \in \mathbb{P}^{3k+1} \setminus Z$ there exists a unique trisecant line to *Z*.

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Trisecants through $P \in Z$.

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 $X \subset \mathbb{G}(1, 3k + 1)$ is a congruence of order one, and $Y = \mathbb{P}_X(\mathcal{Q}|_X)$ is the blow-up of \mathbb{P}^{3k+1} along *Z*.

Lines in *X* are the images of the fibers of $\pi' : Y \to \mathbb{P}^{3k+1}$.

Lines through a general $x \in X$ are the lines contained in three linear spaces passing through *x* and meeting only in *x*.

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The VMRT at *x* is the union of three disjoint linear spaces!

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