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Light propagation in one-dimensional porous silicon complex systems

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We discuss the optical properties of one-dimensional complex dielectric systems, in particular the time-resolved transmission through thick porous silicon quasiperiodic multi-layers. Both in numerical calculations and experiments we find dramatic distortion effects, *i.e.* pulse stretching and coherent beatings, when band-edge states are resonantly excited. Numerical simulations and experiments are in good agreement. We argue that porous silicon can be used conveniently for the fabrication of one-dimensional complex photonic structures.

1 Introduction In recent years, an increasing effort is dedicated to photonic structures that potentially can manipulate the emission, propagation, and detection of light waves. Photonic crystals, waveguides, microcavities, and random porous media are just a few examples of such photonic structures [1–4]. Porous silicon can be considered as a suitable material for the fabrication of photonic structures since its electrochemical growth process allows the realization of structures with arbitrary refractive index profiles [5, 6]. In addition, the fabrication of porous silicon is compatible with standard microelectronics processing.

Photonic crystals are periodic dielectric structures that at high enough refractive index contrast can exhibit a photonic band-gap (PBG) [7]. Photonic band-gap structures are characterized by the presence of a well-defined forbidden frequency band where mode propagation, spontaneous emission, and zero-point fluctuations are all absent. For finite-size samples, the density of optical modes (DOM) increases dramatically near the PBG edges, and the group velocity slows down significantly. This enhances spontaneous emission in luminescent materials and second harmonic generation in non linear systems. The appearance of gap solitons and bistability can be exploited in photonics devices such as band-edge laser [8].

Deterministic a-periodic structures, also called quasi-crystals, lack spatial periodicity. They can be considered as a class of complex dielectric structures in between ordered crystals and fully random structures. Whereas fully random structures give rise to Anderson localization effects (exponentially decaying localized modes) [9] the wave functions inside quasi-crystals are only weakly localized (less than exponentially localized). The weakly localized modes in quasi-crystals exhibit a rich structure including scaling [10]. More interestingly, for very large Fibonacci structures the transmission spectrum collapses to a

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Cantor set with self-similar structure [11]. On the other hand, deterministic a-periodic systems also exhibit band-gaps, like photonic crystals.

A quasi-crystal is constructed by applying an iterative recipe, called generating rule, to a set of "building blocks" (layers with defined thickness and refractive index). The Fibonacci structure is the most studied quasi-crystal because of its simple generating recipe. It requires only two building blocks A and B, stacked according to the generating rule: $S_{j+1} = \{S_{j-1}S_j\}$ for $j \geq 1$, where $S_0 = \{B\}$ and $S_1 = \{A\}$. The lower order Fibonacci crystals are therefore: $S_2 = \{BA\}$, $S_3 = \{ABA\}$, $S_4 = \{BAABA\}$, $S_5 = \{ABABAABA\}$, and so on.

In this work we will focus on the interesting properties of the modes near the band-edge of periodic and non-periodic systems. In the case of non-periodic systems, we also provide experimental results on time-resolved transmission of laser pulses at the band-edges.

2 Light propagation in complex systems Photon propagation in one-dimensional structures can be modeled conveniently by a simple transfer-matrix approach [12]. The electric (E) and magnetic (B) field at the left interface of a dielectric layer can be related to E and B at the right interface of the same layer through a simple matrix relation:

$$\begin{pmatrix} E_l \\ B_l \end{pmatrix} = M \begin{pmatrix} E_r \\ B_r \end{pmatrix} = \begin{pmatrix} \cos\delta & \dfrac{i \sin\delta}{\gamma} \\ i\gamma \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} E_r \\ B_r \end{pmatrix}, \tag{1}$$

where the indices l and r refer to the left and right interfaces of the layer, $\delta = k_0 nd$ is the phase change, k_0 is the wave number of the light in vacuum, n is the refractive index of the layer, d is the thickness of the sample, and γ is the inverse of the light velocity across the layer. From the continuity requirements of the parallel components of the electric and magnetic fields and by simple matrix multiplication, one can construct the transfer matrix M relating the input with the output amplitudes for the whole structure. The total complex transmission and reflection coefficients, $t(\omega)$ and $r(\omega)$, can be obtained by assuming a normalized incident radiation coming from the left and no incident radiation coming from the right.

The complex transmission coefficients contain information on both the amplitude and the phase of the transmitted light. The dispersion properties of the structure can be retrieved from the phase. Since quasi-crystals are non-periodic, we define k_{eff} via the wave number that connects the phases of the input and the output plane waves: $\varphi = k_{\text{eff}}\, d$, being φ the phase of the complex transmission coefficient. Given $k_{\text{eff}}(\omega)$, the mode density $\rho(\omega)$ and the group velocity $v_g(\omega)$ are easily calculated with: [13]

$$\rho(\omega) = \frac{\mathrm{d}k_{\text{eff}}}{\mathrm{d}\omega} \qquad v_g(\omega) = \frac{1}{\rho(\omega)}. \tag{2}$$

To compare periodic with a-periodic systems, we considered in our calculations two structures: a Distributed Bragg reflector (DBR) and a Fibonacci quasicrystal, both formed from porous silicon layers. The two building blocks are two layers of high and low porosity, respectively A and B, with refractive indices 1.59 and 2.22. For the DBR the thickness of layers A and B is 283 nm and 202 nm, respectively, so that its stop-band is centered at 1.8 μm. For the Fibonacci structure the two thicknesses are 148.5 nm and 99 nm, so that the band-edges are at approximately the same frequencies as for the DBR.

In Fig. 1, the transmission spectra, mode density, and group velocity of a 25-period DBR and of a Fibonacci of 12th order (233 layers) are compared. Even if the number of layers of the DBR is lower than that of the Fibonacci structure, the density of modes at the band-edges of the DBR is higher than in the Fibonacci case. These peaks correspond to band-edge modes, which are discrete states that appear due to the finite size of the sample (band-edge resonances). The maxima in the mode density correspond to minima in the group velocity. Indeed, inside the stop-band, where no optical modes are allowed, the

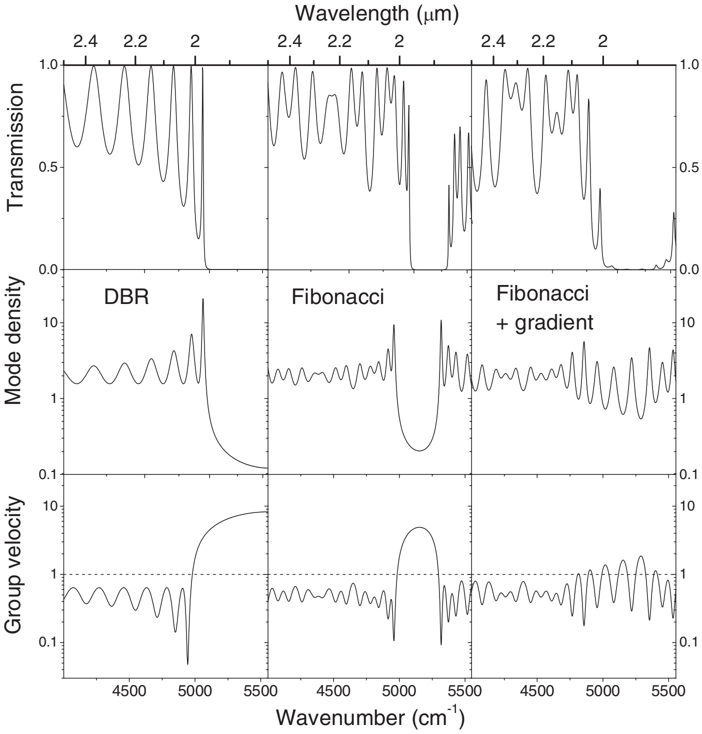

Fig. 1 Transmission spectrum (top), mode density (middle), and group velocity (bottom), calculated for three different samples. Left column: a 25-period DBR centered at a wavelength of 1.8 μm. Center column: a Fibonacci sample with 233 layers (12th order Fibonacci). Right column: the same Fibonacci with an added gradient on the layer parameters, corresponding to the experimentally studied samples. The units of the mode density are such that 1 corresponds to vacuum, and the group velocity is given in units of c_0.

group velocity becomes superluminal. Note that the group velocity is allowed to become superluminal, zero, or even negative without violating causality [14].

It is known that quasi-crystals can also exhibit stop-bands, although they lack periodicity. We see from Fig.1 that the Fibonacci structure indeed has at least one stop-band. By comparing the density of modes of the Fibonacci with the DBR around the band-edge, one observes a certain similarity. The decrease of the group velocity at the band-edge region is significant in both samples.

3 Experiment We have fabricated Fibonacci samples from porous silicon multilayers. A p^+-type wafer (ρ = 0.01 Ω cm) was electrochemically etched with an electrolyte containing a solution of aqueous 48%-wt HF and ethanol in a ratio 1:2. The resulting meso-porous structure allows to etch layers with a good range of porosities (40–85%) and a good depth-homogeneity [5]. The multilayer has been obtained by switching the current between two values corresponding to the two porosities of 69 and 47%. These two values of the porosity yield the refractive indices 1.6 and 2.2, respectively, of Fibonacci layer A and B. A Fibonacci structure of 12th order with 233 layers and 29 μm total thickness was grown in such a way.

In Fig. 2, the experimental as well as the calculated transmission spectra are reported. The numerical calculation takes into account the small drift in layer thickness (6%) and porosity (10%), and the lateral sample inhomogeneities of about 1% [5]. The drift causes the appearance of some optical modes inside the band-gap, but does not substantially change the properties of the band-edge states (Fig. 1 right). The correspondence between the measured and calculated transmission spectra allows us to calculate the time-dependent propagation of ultra-short light pulses through the Fibonacci structure.

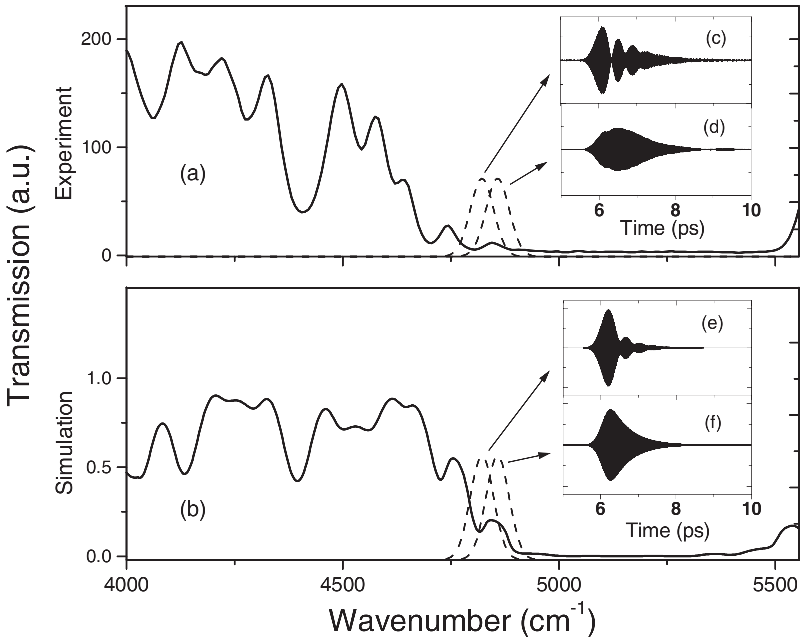

Fig. 2 Measured a) and calculated b) transmission spectra of the porous silicon Fibonacci sample. The dashed gaussian curves refer to the spectra of the two different input laser pulses. Panels c) and d): measured pulse profiles of the transmitted signal for the two different input pulses (central frequencies 4820 cm^{-1} and 4855 cm^{-1} respectively). Panels e) and f): calculated pulse profiles of the transmitted signal for the same two input pulses as in c) and d). The time on the *x*-axis of all panels is given with respect to the arrival of the pulse with no sample in the interferometer.

We have performed time-resolved transmission experiments in order to study the strong dispersion effects of the band-edge states experimentally. For this purpose we have used an interferometric technique based upon a Mach-Zehnder interferometer coupled to a Michelson interferometer [15]. The first interferometer is used to compose a double pulse by adding a reference pulse to the transmitted signal at a fixed time delay, such that the signal and reference pulses do NOT overlap in time. The second interferometer is then used to measure the auto-correlation of this double pulse, which effectively contains the cross-correlation function of signal and reference pulses. In this way it is possible to measure the time-resolved profile of the transmitted pulse [15]. The most interesting results are shown in Fig. 2. When the laser frequency is resonant with a band-edge mode (Fig. 2d), a strong stretching of the pulse is observed which demonstrates the slow group velocity of that optical mode in agreement with the calculation (Fig. 2f). When the laser frequency is resonant with two adjacent optical modes, we observe coherent beatings due to the mixing of the two modes with slightly different frequency (Fig. 2c). Indeed, the frequency of the beatings is equal to the frequency difference of the two modes. This effect is also observed in the calculation (Fig. 2e).

4 Conclusions

In this work we have studied light propagation through some photonic structures by calculating their complex transmission coefficient and their time-resolved response to a fast input pulse. We have also fabricated thick porous silicon Fibonacci quasi-crystals and measured their time-resolved transmission properties. Strong pulse distortions have been observed and compared to numerical calculations. The agreement between the numerical calculations and the experiments demonstrates that porous silicon is a suitable material for the fabrication of complex photonic structures.

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