

Optics of nanostructured dielectrics

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Abstract

We discuss the optical transport properties of complex photonic structures ranging from ordered photonic crystals to disordered strongly-scattering materials, with particular focus on the intermediate regime between complete order and disorder. We start by giving an overview of the field and explain the important analogies between the transport of optical waves in complex photonic materials and the transport of electrons in solids. We then discuss amplifying disordered materials that exhibit random laser action and show how liquid crystal infiltration can be used to control the scattering strength of random structures. Also we discuss the occurrence of narrow emission modes in random lasers. Liquid crystals are discussed as an example of a partially ordered system and particular attention is dedicated to quasi-crystalline materials. One-dimensional quasi-crystals can be realized by controlled etching of multi-layer structures in silicon. Transmission spectra of Fibonacci type quasi-crystals are reported and the (self-similar) light distributions of the transmission modes at the Fibonacci band edge are calculated and discussed.

Keywords: nanophotonics, complex systems, random laser, quasi-crystal, liquid crystal, light diffusion, photonic crystal

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The propagation of electromagnetic waves in complex dielectric nanostructures is full of surprises. Complex dielectrics are dielectric structures with an index of refraction that has variations on a length scale that is comparable to the wavelength. Such structures strongly scatter light. Of interest are periodically ordered structures like photonic crystals on one side, and disordered materials like powders, suspensions of microspheres, or strongly scattering random laser materials on the other. Ordered mesoscopic dielectric systems, with a lattice constant comparable to the wavelength, behave like a crystal for light waves and, at high enough refractive index contrast, a photonic bandgap is expected to occur [1, 2]. Light waves

in disordered materials, on the other hand, undergo random multiple scattering that to first order can be described as a diffusive type of transport. What makes disordered systems interesting is that interference effects can survive the random multiple scattering. Examples of such interference effects are coherent backscattering or weak localization [3], and short and long range intensity correlations [4].

Complex dielectric structures can be constructed as one, two, or three-dimensional systems. In one dimension a complex dielectric can be realized in the form of a multilayer structure, for instance by controlled etching of a semiconductor material [5]. One-dimensional structures have the advantage that they can be realized with almost arbitrary degree of disorder, which allows one to construct even complex deterministic non-periodic sequences like quasi-crystals. In two dimensions, complex dielectrics can be formed

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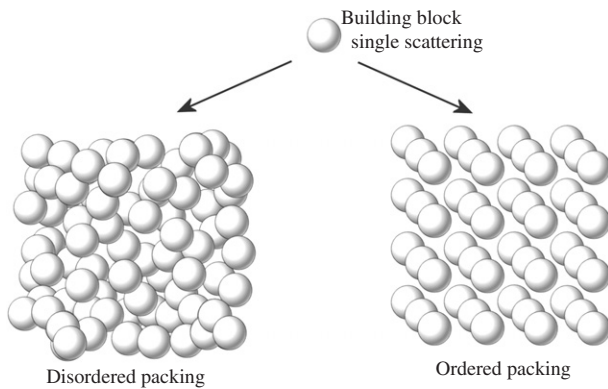


Figure 1. Microassembly of a complex photonic system. The building block (microsphere) can be packed in various ways, of which the two extremes are fully disordered assembly (left) and completely ordered assembly (right). Disordered assembly leads to random multiple light scattering and ordered assembly results in a photonic crystal or possibly a photonic band gap material.

by nano-patterning holes or pillars in a planar waveguide. Typically this involves expensive lithography techniques which are in some cases combined with controlled etching.

A possible building block for constructing a complex dielectric in three dimensions is a microsphere of diameter comparable to the wavelength. The single scattering from such a sphere has a rich structure due to internal resonances in the sphere, but its behaviour is well-understood and can be calculated using the formalism of Mie scattering [6]. A three-dimensional complex dielectric material can then be realized by microassembly of several microspheres. The spheres can be assembled in various ways ranging from completely disordered packing to a fully ordered assembly (see figure 1). In both cases the spheres have the same single scattering properties. Nevertheless the cumulative behaviour of the spheres after assembly will depend strongly on the way the spheres are packed together. This is due to the interference between the scattered waves and the way the waves are multiply scattered from one sphere to another. If the spheres are packed according to a crystal-like structure, the interference will be constructive only in certain well-defined directions giving rise to Bragg refraction and reflection. In the disordered case, the light waves will perform a random walk. The occurrence of interference effects is now less obvious to understand; however, in random systems interference effects also turn out to be very important.

Interference of light in random dielectric systems influences the transport of light in a way that is similar to the interference that occurs for electrons when they propagate in disordered conducting materials. As a result, light propagation in disordered systems shows many similarities with the propagation of electrons in (semi)conductors. Various phenomena that are common for electron transport have now also been found to exist for light waves [7]. Important examples are the photonic Hall effect [8], optical magneto-resistance [9], Anderson localization [10], and universal conductance fluctuations [11]. In the case of Anderson localization the interference effects are so strong that the transport comes to a halt and the light becomes localized in randomly distributed modes inside the system. A much simpler analogy between electron and photon transport is that of Ohm's

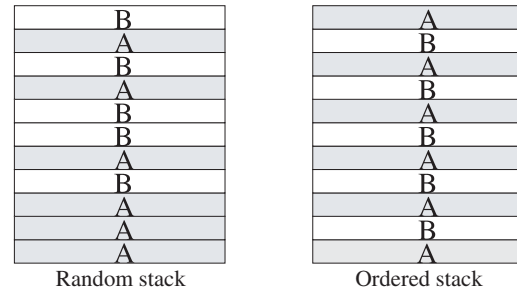


Figure 2. One-dimensional complex photonic systems. By stacking two types of layers (A and B), one can obtain random or ordered one-dimensional structures. In principle any desired stacking rule can be used which allows one to explore the regime in between complete order and disorder.

law of conductance. For electrons, Ohm's law tells us that the total resistance of a system increases linearly with its length. This is a direct consequence of the diffusive type of motion of the electrons. The same transport law holds for light waves in an opaque (but non-absorbing!) disordered dielectric, in which case the total diffuse transmitted light decreases linearly with the thickness of the system. Important applications of multiple light scattering include medical imaging [12] and diffusing wave spectroscopy, where interference in multiple scattering is used to study the dynamics of optically dense colloidal systems [13].

Also in ordered systems interference can give rise to dramatic effects. If the scattering of the spheres that constitute a photonic crystal is strong enough (that is the refractive index contrast between the spheres and their surrounding medium is large and their diameter is resonant with the wavelength), the interference can become destructive in all directions, for a certain range of frequencies. In analogy with the behaviour of electrons in semiconductors this range of optical frequencies is referred to as a photonic band gap [1, 2]. Inside a photonic band gap the density of light modes becomes zero, which means that even vacuum fluctuations are suppressed. A small impurity inside such a photonic band gap material will give rise to a naturally localized mode around this impurity.

Three-dimensional photonic materials can be assembled in practice by simple deposition of microspheres out of a suspension. Deposition can be accelerated by centrifuging the suspension. The amount of order/disorder in the system is controlled via the degree of polydispersity of the spheres. Below a polydispersity of about 5%, usually an ordered (crystalline) structure is formed. The behaviour of light in three-dimensional systems is often difficult to describe theoretically. The advantage of lower-dimensional structures is that an analytical theoretical description is often available, facilitating the interpretation of experimental results. Results on lower-dimensional structures can then be used to learn more about the complex behaviour of three-dimensional systems. In the case of one-dimensional (1D) structures one uses multi-layers of different refractive index and thickness that are stacked either periodically or randomly, or via any other desired packing rule (see figure 2).

Whereas knowledge of the propagation of light waves in completely ordered and disordered structures is now rapidly improving, little is known about the behaviour of optical

waves in the huge intermediate regime between total order and disorder. An example of a partially ordered system that we will discuss in this paper is a photonic quasi-crystal, in which the scattering elements are assembled in a non-periodic but deterministic way. To construct a quasi-crystal we resort to 1D systems. Another example of a partially ordered system is a liquid crystal in the nematic phase. Here the single scattering element is anisotropic due to directional ordering of the liquid crystal molecules along a common axis. On the extreme of complete disorder we will discuss random laser action in amplifying random materials. We will go into the possibility of observing narrow-band emission modes due to the amplification along rare long light paths in random systems, without the need for coherent feedback. One can therefore obtain narrow-band emission in the diffusive regime.

2. Disorder

Light waves in disordered materials perform a random walk which leads to a multiple scattering process. Of particular interest for photonic applications are disordered materials that provide optical amplification via stimulated emission. Letokhov [14] predicted theoretically that one can obtain laser-like emission from such materials using non-resonant feedback via multiple scattering of light. This phenomenon has been observed in experiments on strongly scattering powders [16]. The emission from such systems was shown to become narrow-banded above a threshold and to exhibit laser spiking [17]. This phenomenon, also known as diffusive random lasing [18], has been observed in a variety of materials like powdered laser crystals [16, 17], microparticles in laser dye solution [19], ceramic materials [20], and liquid crystal—dye-infiltrated porous glass [21]. The phenomenon has been treated analytically [18, 22] and through Monte Carlo simulations [23] to understand the temporal and spectral properties. The photon statistics of the emission from random lasers were calculated to be Poissonian above threshold, even in the regime of diffusive scattering without interference feedback [24]. Interesting lasing effects based on resonant feedback have also been observed in related complex dielectric systems like dye-doped cholesteric liquid crystals [25] and amplifying organic films [26, 27].

Interference effects can survive random multiple scattering also in the presence of optical gain in random laser materials. Coherent backscattering has been observed to survive the introduction of optical gain in random systems [28]. Lasing is often associated with interference (coherent feedback), and recent experiments on zinc oxide powder aimed at combining random lasing and strong localization [29, 30]. Localized optical modes could serve as cavities for random laser action. The combination of strong localization and gain has also been discussed theoretically, mainly for one-dimensional (1D) [31] and two-dimensional (2D) [32] random systems. In 1D the concept of lasing of localized modes is relatively easy to understand. Localization in 1D systems leads to narrow peaks in their transmission spectrum that correspond to internal resonances with high quality factor. These internal resonances can form a cavity for laser action that is randomly distributed in space.

Strong localization in 3D random systems requires very strong scattering and is much more difficult to achieve than localization in lower-dimensional systems [10]. The scattering strength of a disordered optical material is determined by its transport mean free path ℓ , defined as the length scale over which a propagating wave loses its memory of propagation direction. A material is in the multiple scattering regime when ℓ is considerably smaller than the system size L . In that case the transport of light can be described by a random walk with step length ℓ . If the scattering strength of a material is increased (and hence ℓ decreased), eventually a phase transition into an Anderson localized state is expected to occur at $k\ell \leq 1$, with k the wavevector of the light [10]. For $k\ell > 1$ the transport is diffusive, which is the case in most of the available disordered dielectric materials, whereas around $k\ell \sim 1$ a fascinating scaling regime exists in which the transport parameters depend on the system size [33].

Laser action in complex dielectric structures is not limited to random systems. In periodic systems like photonic crystals with gain, laser action can occur for frequencies just outside the photonic bandgap where the density of optical modes is high. These modes at the edge of the photonic bandgap have a high quality factor and long residence time due to coherent feedback from the periodic structure. This principle is used with great success in distributed feedback lasers [34], which are one-dimensional periodic structures with optical gain throughout the structure.

2.1. Tunable multiple scattering

In many experimental studies one would like to be able to vary the scattering strength of a sample without modifying its other properties. We have found a simple way to obtain external control over the diffusion constant of a random sample. Liquid crystals have the beautiful property that they go through various partially ordered phases when heated. The index of refraction is different in every liquid crystal phase. Of special interest is the nematic phase, due to its birefringence in the index of refraction (the index of refraction depends on the propagation direction and polarization of a light wave), which disappears when the liquid crystal is heated into the isotropic phase [35]. By infiltrating a random sample with a liquid crystal, one can obtain a system of which the diffusion constant strongly depends on temperature [36]. This is similar to, for instance, the concept of smart screens based on polymer dispersed liquid crystals that change their opacity with temperature [37]. For a random laser, having control over the diffusion constant has important consequences. The lasing threshold depends on the diffusion constant of the random material. This means that if we have a temperature-dependent diffusion constant, we are able to bring the random laser above and below threshold by changing its temperature.

One can realize such a temperature-tunable random laser in the following way. A carefully chosen glass (like SK11) is ground into a fine powder and sintered under high pressure. This results in strongly scattering discs of randomly assembled glass grains. These discs can be infiltrated by a laser dye like DCM (Lambdachrome 6500) dissolved in a liquid crystal like 4-cyano-4'-n-heptylbiphenyl (7CB) (see figure 3). The phase sequence of 7CB is crystalline' (15.0) crystalline (30.0)

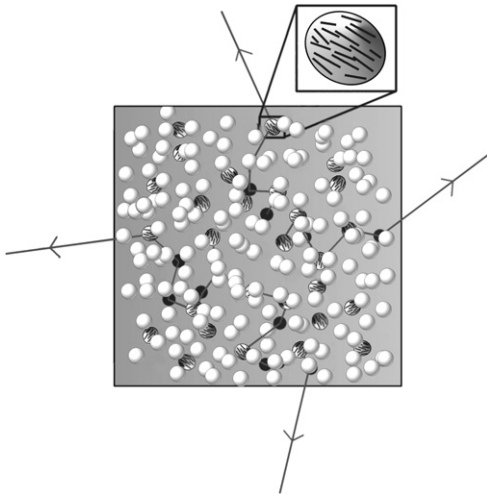


Figure 3. An amplifying disordered material with tunable diffusion coefficient. The laser dye is dissolved in liquid crystal and is excited by an external laser beam to provide optical gain. The refractive index contrast between glass powder and liquid crystal depends on temperature, and hence the scattering strength can be temperature controlled.

nematic (42.8) isotropic, where the numbers between brackets denote the phase transition temperatures in degrees Celsius. The refractive index of SK11 in the emission range of DCM is $n = 1.56$, whereas the refractive indices of 7CB in the nematic phase are $n_o = 1.52$ and $n_e = 1.68$ at 36.0°C . This means that in the nematic phase this liquid crystal infiltrated sintered glass is strongly scattering due to the refractive index mismatch between glass and liquid crystal: the value of $k\ell$ is small as is the value of the photon diffusion constant D . Above the phase transition temperature at 42.8°C this index mismatch nearly disappears, and the material there scatters weakly: the diffusion constant dramatically increases [36]. This control over the diffusion constant allows one to bring the random laser above and below threshold via temperature [21], and to study random laser action in a broad range of values of the disorder parameter $k\ell$. Note that the phase behaviour of a liquid crystal inside sintered glass is different from the typical phase behaviour of bulk liquid crystal, and that the nematic–isotropic phase transition is smeared out. This behaviour is common for liquid crystals in confined geometries like porous glasses, and is due to interaction between the liquid crystal molecules and the surface of the porous host [38]. By choosing different liquid crystal/glass combinations, one can obtain different tuning curves for the diffusion constant.

2.2. Narrow-band emission modes

Certain random laser materials have been reported to exhibit narrow emission spikes in the emission spectrum. The first observation of this phenomenon was on strongly scattering zinc oxide powders [29]. The initial interpretation of these observations was that Anderson localized modes provided feedback for random lasing. The scattering strength of these materials was about $k\ell = 3\text{--}5$, which indeed is close to the Anderson localization transition at $k\ell = 1$. Later on, however, the same narrow emission peaks were observed from random laser materials in almost the complete

diffuse scattering regime, even far away from the Anderson localization transition and up to at least $k\ell = 6 \times 10^3$ [39].

Indeed narrow emission spikes from random laser systems do not require the occurrence of interference effects or localized modes. Both in experiments and in Monte Carlo simulations we observed the narrow emission spikes and could identify their origin: each individual sharp peak originated from a *single* spontaneous emission event that followed an extremely long light path through the sample and was consequently strongly amplified [39]. Such long light paths have a negligible contribution to the net transport in passive systems, but we found that they can dominate the emission spectrum from an active system and lead to narrow emission peaks. The wavelength of these peaks is determined by the wavelength of the spontaneous emission event that originates the corresponding multiple scattering modes. Therefore shot to shot spectra are intrinsically different and single shot experiments are essential to observe the effect. This makes it distinct from interference effects that rely on coherent feedback from the system boundaries or from localized modes. Another signature is that the phenomenon depends only weakly on the disorder parameter $k\ell$.

In the case of Anderson localization, light is confined in a small region of space of dimensions determined by the localization length; however, the extended modes that we observe probe a large volume of the sample. The phenomenon is a direct consequence of the amplification of noise in the path length distribution in finite-size random systems, and is a general property of any random walk model with coherent gain. We therefore expect it to play an important role in most amplifying disordered materials studied in the context of random lasing.

3. Partial ordering

Liquid crystals in the nematic phase are opaque and therefore also give rise to multiple light scattering. This allows coherent backscattering to be observed from large nematic systems [40, 41]. The partial ordering of the nematic phase leads to an anisotropic scattering function [42], which makes nematic liquid crystals fundamentally different from common random media. This anisotropy in the scattering cross section leads, for large enough samples, to an anisotropic multiple scattering process, and monodomain nematics are therefore ideal systems to study anisotropic multiple light scattering. Anisotropic light diffusion has recently been observed on large monodomain nematics in cw experiments by Kao *et al* [43] and later in time-resolved experiments [44]. A lot of inspiring theoretical work is available on light propagation in opaque liquid crystals [45].

Comparison between time-resolved transmission measurements and static transmission measurements on monodomain nematic liquid crystals reveals a difference in the observed anisotropy whereas similar measurements on strongly anisotropic disordered GaP networks provide the same value [46]. The diffusion constant in an isotropic system can be written as the square of the average step size of the underlying random walk process (after correction for forward scattering) divided by the average time Δt it takes the random walker to cover this step. For an anisotropic system

like a liquid crystal the step length and time, in principle, both become anisotropic, and their value will depend on the propagation direction of the random walker. However, for sufficiently large systems, one should assume that Δt takes some average value, which allows one to write the perpendicular and parallel values of the diffusion constant in terms of one average Δt . (This is actually a necessary requirement to make the diffusion approximation in the first place.) It is clear that under these assumptions the *anisotropy* in the diffusion constant D_{\parallel}/D_{\perp} can be calculated from the *anisotropy* in the transport mean free path, since then we can write $D_{\parallel}/D_{\perp} = \ell_{\parallel}^2/\ell_{\perp}^2$. In static experiments one measures the transport mean free path of the system, whereas the diffusion constant is a dynamical property and can be measured only in time-resolved experiments. However, under the assumptions above, one can derive the *anisotropy* in the diffusion constant from the measured *anisotropy* in the mean free path, as was done in the static experiments of [43, 44, 46]. The above assumptions might not be valid for multiple scattering processes with non-Gaussian statistics (like Levy flights), in which the diffusion approximation breaks down, or for systems with long range correlations. It is not clear if the diffusion approximation is correct for nematic liquid crystals. An unambiguous way to measure the transport mean free path is via coherent backscattering [3], which is technically very challenging for liquid crystals due to their long mean free paths [40]. Recently, mean free path anisotropy in coherent backscattering was indeed observed on monodomain nematics [41], using an experimental apparatus with unprecedented experimental resolution.

4. Quasi-crystals

Quasi-crystals form another class of fascinating systems in between fully ordered and completely disordered. Quasi-crystals are non-periodic structures that are constructed following a deterministic generation rule [47]. If made from dielectric material, the resulting structure has interesting optical properties. Quasi-crystals of the Fibonacci type, for instance, exhibit an energy spectrum that consists of a self-similar Cantor set with zero Lebesgue measure [48]. The transmission spectrum of a Fibonacci system also contains forbidden frequency regions called ‘pseudo band gaps’ similar to the band gaps of a photonic crystal [49]. In the frequency regime outside these Fibonacci band gaps, the light waves are critically localized. A critically localized state is a localized state that decays weaker than exponentially in space, usually by a power law. This in contrast to (Anderson) localized states in fully disordered systems, which decay exponentially. The critically localized states inside a Fibonacci system have a rich self-similar structure [50]. This makes these systems very interesting for localization studies with classical waves. Kohmoto *et al* proposed to use quasi-crystals for light localization studies [51], and the spatial structure of critically localized acoustic modes has been observed by Desideri *et al* for surface acoustic waves on corrugated solids [52]. The first Fibonacci sequence for electron transport studies was realized by Merlin *et al* [53], which was followed by several experiments and theoretical studies on electron propagation in these systems [54]. The experimental work on light transport in this fascinating class of structures is limited so far. Important

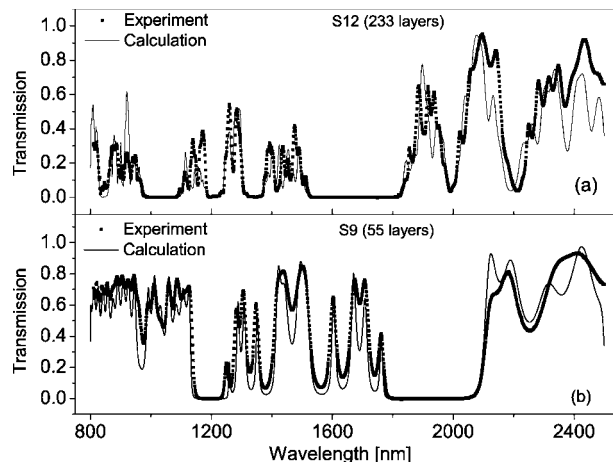


Figure 4. Transmission spectra of Fibonacci samples S_9 (a) and S_{12} (b). The solid lines are the results of a transfer-matrix calculation assuming optical path drifts of 1% (for S_9) and 4% (for S_{12}) and optical losses (absorption and scattering) of $\alpha \sim 120 \text{ cm}^{-1}$. The dots denote the measured spectra.

pioneering experiments were performed by Gellermann *et al* [55], who observed self-similarity in the transmission spectrum of Fibonacci dielectric multi-layers, and by Hattori *et al* [56], who measured the Fibonacci dispersion curves.

A Fibonacci quasi-crystal is a deterministic aperiodic structure that is formed by stacking two different compounds A and B according to the Fibonacci generation scheme: $S_{j+1} = \{S_{j-1}S_j\}$ for $j \geq 1$; with $S_0 = \{B\}$ and $S_1 = \{A\}$. The lower order Fibonacci sequences are therefore $S_2 = \{BA\}$, $S_3 = \{ABA\}$, $S_4 = \{BAABA\}$, etc. For a 1D dielectric Fibonacci sample, the elements A and B are dielectric layers with different refractive index and thickness. In our case layer A has a lower refractive index than layer B. The physical thickness of the layers can be chosen such that the optical thickness of both layers is equal to $\lambda_0/4$, where λ_0 is the central wavelength of the spectrum. This choice satisfies the maximum effective quasi-periodicity condition [55].

Fibonacci samples can be conveniently grown in porous silicon. Porous silicon has interesting optical properties [5] and, if grown in highly doped p-type Si substrate, behaves as an optically homogeneous dielectric material with an effective refractive index n determined by its porosity (volumetric fraction of air). As porous silicon is fabricated by a self-limiting electrochemical process, one can control the refractive index of each layer by varying the electrochemical current during the fabrication [5]. The thickness of the porous layer is determined by the etching duration.

Here we report on 9th and 12th order Fibonacci samples as realized in porous silicon. The transmission spectra of these samples are reported in figure 4. These samples were etched starting from (100)-oriented p⁺-type silicon (resistivity $0.01 \Omega \text{ cm}$). The electrolyte was prepared mixing a 30% volumetric fraction of aqueous HF (48 wt%) with ethanol. A current density of 50 mA cm^{-2} was applied for the high porosity layer A (refractive index $n_A = 1.45$) and 7 mA cm^{-2} for the low porosity layer B ($n_B = 2.13$). A high current step of 400 mA cm^{-2} was applied for 1 s to detach the multilayer structure from the substrate. This way free-standing Fibonacci samples of the 9th (55 layers) and 12th order

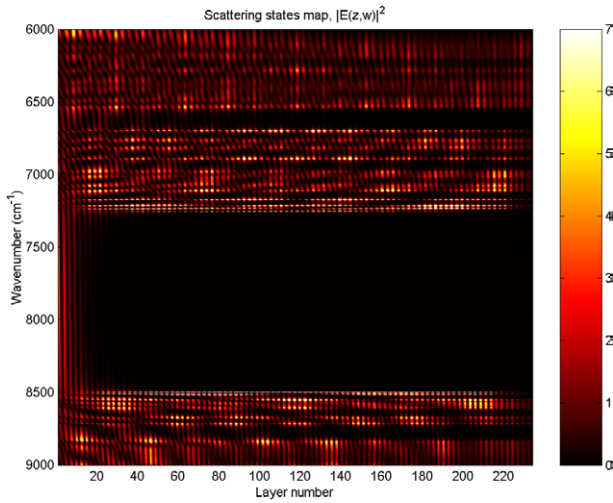


Figure 5. Scattering states map of an ideal 12th order Fibonacci system. The intensity of the light is denoted by a colour scale and plotted as a function of wavenumber and depth inside the sample.

(233 layers) centred in the near-infrared wavelength region (NIR) were grown. In this region, between 1 and 1.5 μm , the absorption of porous silicon is negligible. The etching process introduces a natural drift in the layer porosity and thereby in the optical thickness of the layers. This drift can be compensated quite efficiently by correcting the current during the etching process [57]. This is especially relevant in realizing thick structures, and limits the porosity drift to 4% even for the samples with 233 layers. Without compensation the natural drift in these thick samples is about 16% [58]. For the 9th order Fibonacci sample (55 layers) the compensation procedure limits the drift to 1%. A 5% drift value was reported in [55] for a 9th order Fibonacci quasi-crystal (55 layers) grown by electron-gun evaporation technique.

In the transmission spectra in figure 4, pseudo band gaps are clearly visible both for the 9th and 12th order Fibonacci samples. In order to interpret the transmission spectra and to check the parameters of the multilayers, we have calculated numerically their transmission spectra within a transfer-matrix approach [59]. The calculations for the S_9 and S_{12} sample assuming respectively a 1% and 4% drift and total optical losses (absorption and scattering) of $\sim 120 \text{ cm}^{-1}$ are also reported in figure 4 as a solid curve. The fine agreement with the experimental curve demonstrates the successful compensation of the porosity gradients.

The field distribution of the modes in one-dimensional structures like the above Fibonacci quasi-crystals can be calculated via a standard scattering-states method [59]. The magnitude of the electric (E) and magnetic (B) field at the left interface of a dielectric layer is simply related to E and B at the right interface of the same layer through the matrix relation:

$$\begin{pmatrix} E_l \\ B_l \end{pmatrix} = M \begin{pmatrix} E_r \\ B_r \end{pmatrix} = \begin{pmatrix} \cos \delta & i \frac{\sin \delta}{\gamma} \\ i\gamma \sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} E_r \\ B_r \end{pmatrix}, \quad (1)$$

where the indices l and r refer to the left and right interfaces of the layer, $\delta = k_0 n d$ is the phase change and γ is the inverse of the light velocity across the layer ($k_0 =$ wavevector in vacuum, $n =$ refractive index, $d =$ layer thickness). From the continuity requirements of the electric and magnetic fields,

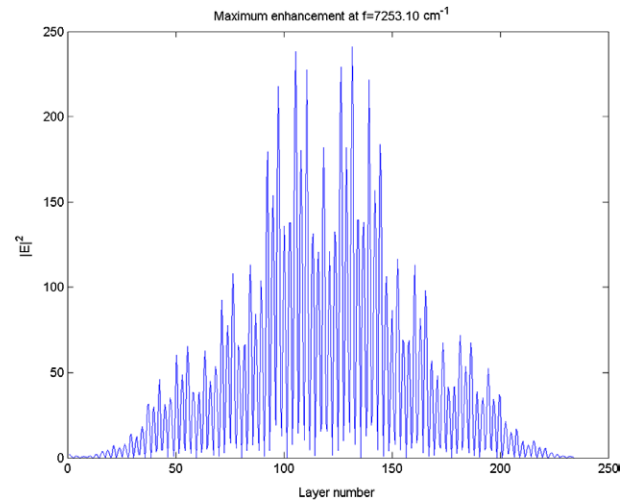


Figure 6. Intensity distribution of the light inside the sample at the wavenumber that corresponds to the first band edge resonance.

one can construct the transfer matrix M of the whole system by simple matrix multiplication.

For one-dimensional structures, it is also possible to compute the electric field distribution inside the sample. The reflectivity coefficient of the whole sample must be previously calculated, after which the incident and the total reflected light can be used as boundary conditions to compute the field amplitude of any layer inside the structure. The electric field on the k th interface can be expressed as

$$E(z^{(k)}, \omega) = \{1 + r(\omega)\} m_{22}^{(k)} - \gamma_0 \{1 - r(\omega)\} m_{12}^{(k)}, \quad (2)$$

where $m_{\alpha\beta}^{(k)}$ are the elements of the transmission matrix from the first to the k th interface. The intensity is then simply the square of the electric field, as plotted in figure 5 for an ideal 12th order Fibonacci sample. A Fibonacci pseudo-bandgap is clearly visible as a dark horizontal region. Just above and below the pseudo-bandgap the band edge modes are visible which correspond to peaks in the transmission spectrum.

The first two band edge modes are plotted in figures 6 and 7. These figures show the normalized field intensity distributions inside the sample and correspond to horizontal cuts through figure 5. In particular the first two band edge modes are the intensity distributions at wavenumber 7253.10 and 7233.61 cm^{-1} . The incoming field is normalized to 1. These distributions have a notable similarity to the band edge resonances occurring in photonic crystals [60] but are less regular. Band edge resonances in (finite-size) photonic crystals are due to a transient standing wave that is formed inside the sample and can temporarily store a substantial amount of energy. This is consistent with a large group velocity reduction and strong pulse stretching as observed in our experiments. Since this transient standing wave is formed from reflection by the sample boundaries, it has the characteristic intensity distribution of the various harmonics of a standing wave. Band edge resonances in photonic crystals are not localized since they extend over the full sample size, and the intensity does not decay exponentially to zero. In contrast, the Fibonacci band edge resonances decay via a power law due to their critically localized nature. Figures 6 and 7 also illustrate nicely

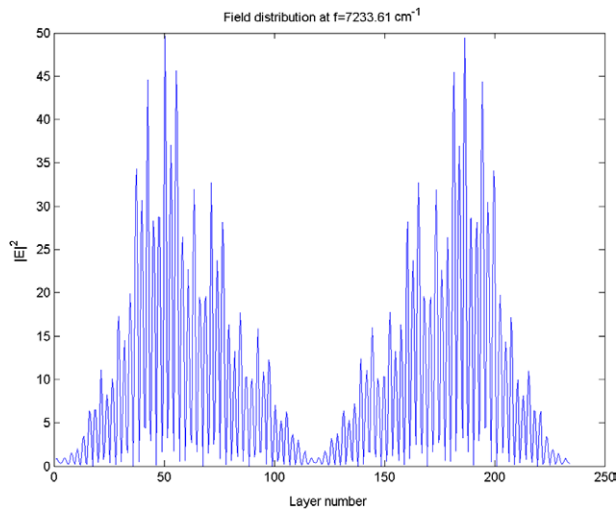


Figure 7. Intensity distribution of the light at the second banded resonance.

the field-enhancement effect: close to the band edge the field inside the sample becomes locally larger than one.

The deterministic generation rule of the Fibonacci structure leads to a self-similar transmission spectrum but also results in a characteristic intensity distribution inside the sample. The distributions plotted in figures 6 and 7 are clearly not common band edge resonances but exhibit a complex fine structure. Closer inspection via wavelet analysis shows that the intensity distribution is again self-similar. The transport of a short optical pulse through these resonances shows strong group velocity reduction and pulse stretching [58]. Fibonacci systems can provide an interesting alternative to regular photonic crystals for the realization of photonic devices like, for example, optical filters with a self-similar spectrum and a high wavelength selectivity in the band edge region. Another interesting future application of these materials could be realized in the field of random lasers, where the Fibonacci band edge resonances could serve as a new type of complex cavity that provides the feedback for laser action.

5. Conclusions

We have discussed several examples of light transport in complex photonic structures, in particular in partially ordered materials, quasi-crystals, and random laser systems. The vast regime between completely ordered and disordered structures is very rich and has yet to be fully explored. The transport of light in these various complex structures is very interesting from a fundamental point of view, and these new materials could find fascinating applications as photonic devices, speciality lightning, and in telecommunications.

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