Arterial haemodynamical impact of aortic valvular and arterial stenoses. A mathematical-model study

Prof. Dr. Eleuterio F. Toro

Laboratory of Applied Mathematics
University of Trento, Italy
eleuterio.toro@unitn.it
http://www.ing.unitn.it/toro

Paper to read:
Background

- **The aortic valve**: is the valve located between the left ventricle and the aorta

![Aortic Valve Diagram](image)

Fig. 1. The heart and the aortic valve

- **Aortic valve stenosis**: it is a narrowing of heart’s aortic valve, which does not fully open, causing obstruction of blood flow from the heart to the aorta and to the rest of the body
- **Common valvular heart disease in industrialised countries**
- **Surgery**: is usually recommended for severe cases
Arterial stenoses

- Stenoses in large and medium-size arteries in various locations of the arterial tree are also common.
- Stenoses cause increased pressure drop and reduced flow rate, leading to ischemic events.
- Changes in geometry and material properties induce altered characteristics of arterial wave propagation and can also affect cardiac dynamics.
- This chosen topic for this course evaluation is concerned with the study of the global haemeodynamic impact of stenoses in various locations of the arterial tree, using a mathematical model.
- Mathematical model. The recommended paper (Liang et al. 2009) describes the construction of a global a mathematical model for the cardiovascular system and its use to study the impact of arterial stenoses.
Global multi-scale model

Fig. 2. Global multi-scale model
Major arteries: 1D model

- One-dimensional blood flow equations are used.

\[
\frac{\partial}{\partial t} \begin{pmatrix} A \\ U \end{pmatrix} + \frac{\partial}{\partial z} \begin{pmatrix} UA \\ \frac{U^2}{2} + \frac{P}{\rho} \end{pmatrix} = \begin{pmatrix} 0 \\ -K_R \frac{U}{A} \end{pmatrix}
\]

Fig. 3. 1D equations

- I note that they formulate the equations in terms of non-conservative variables, even though the equations can still be written in (mathematical) conservation-law form.

- For smooth solutions this formulation is still adequate.

- The equations are solved using a (very old) numerical method due to Lax and Wendroff (1960).

- To close the system they use a tube law (purely elastic) for flow in arteries (as distinct from veins).

\[
P - P_0 = \frac{E h_0}{r_0 (1 - \sigma^2)} \left( \sqrt{\frac{A}{A_0}} - 1 \right)
\]

Fig. 4. Tube law for arteries, with parameters \( p_0, h_0, r_0, A_0, \sigma, E \)
Heart, pulmonary and peripheral circulation: 0D models

- 0D models are also called ”lumped parameter models”. See lecture notes on background
- 0D models cannot resolve spatial variations, only time variations
- The governing equations are Ordinary Differential Equations (ODEs) with algebraic constraints
- They solve the ODEs using a 4th order Runge-Kutta method
Some results. Normal case

Fig. 5: Normal case. Blood pressure pulses in different regions of the cardiovascular system
Some results. Pathological cases

Fig. 6. Blood pressure pulses (a) in the left ventricle, (b) the ascending aorta, (c) the left brachial artery (c), and (d) the right anterior tibial artery

Fig. 6. Blood pressure pulses (a) in the left ventricle, (b) the ascending aorta, (c) the left brachial artery (c), and (d) the right anterior tibial artery
Ankle brachial pressure index

\[ ABI = \frac{P_{\text{ankle}}}{P_{\text{brachial}}} \]

A low ABI is taken as an indication of blocked arteries.
Conclusions

- Multi-scale model of the cardiovascular system constructed
- The model has been applied to study the global haemodynamic influences of an AV stenosis and arterial stenoses at various locations
- Five stenoses studied among those most frequently found in clinical practice
- Results show a strong location dependence of the global hemodynamic influences of an arterial stenosis
- Future model studies should focus on the influences of combined cardiac and arterial diseases that are often found in the elderly