Arterial haemodynamical impact of aortic valvular and arterial stenoses. A mathematical-model study

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Paper to read:

Fuyou Liang, Shu Takagi, Ryutaro Himeno and Hao Liu.
Multi-scale modeling of the human cardiovascular system with applications to aortic valvular and arterial stenoses. Med Biol Eng Comput (2009) 47:743755 DOI 10.1007/s11517-009-0449-9

Background

- **The aortic valve:** is the valve located between the left ventricle and the aorta

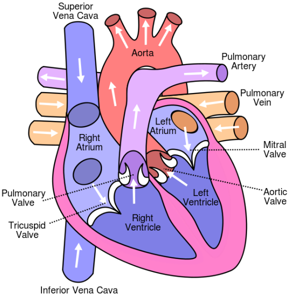

Fig. 1. The heart and the aortic valve

- **Aortic valve stenosis:** it is a narrowing of heart's aortic valve, which does not fully open, causing obstruction of blood flow from the heart to the aorta and to the rest of the body
- Common valvular heart disease in industrialised countries
- **Surgery:** is usually recommended for severe cases

Arterial stenoses

- Stenoses in large and medium-size arteries in various locations of the arterial tree are also common
- Stenoses cause increased pressure drop and reduced flow rate, leading to ischemic events
- Changes in geometry and material properties induce altered characteristics of arterial wave propagation and can also affect cardiac dynamics
- This chosen topic for this course evaluation is concerned with the **study of the global haemeodynamic impact of stenoses in various locations of the arterial tree, using a mathematical model**
- Mathematical model. The recommended paper (Liang et al. 2009) describes the construction of a global a mathematical model for the cardiovascular system and its use to study the impact of arterial stenoses

Global multi-scale model

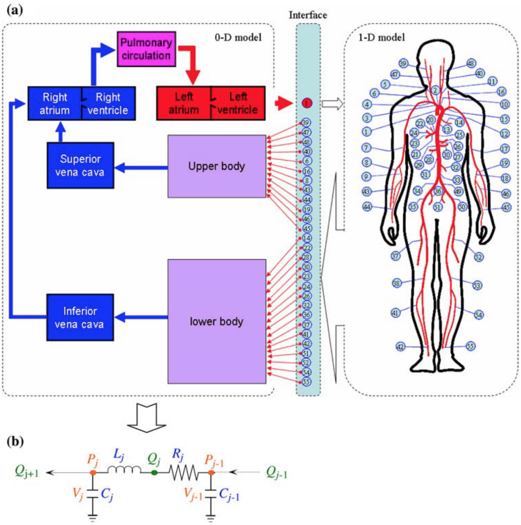

Fig. 2. Global multi-scale model

Major arteries: 1D model

- One-dimensional blood flow equations are used.

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ U \end{pmatrix} + \frac{\partial}{\partial z}\begin{pmatrix} UA \\ \frac{U^2}{2} + \frac{P}{\rho} \end{pmatrix} = \begin{pmatrix} 0 \\ -K_R \frac{U}{A} \end{pmatrix}$$

Fig. 3. 1D equations

- I note that they formulate the equations in terms of non-conservative variables, even though the equations can still be written in (mathematical) conservation-law form
- For smooth solutions this formulation is still adequate
- The equations are solved using a (very old) numerical method due to Lax and Wendroff (1960)
- To close the system they use a tube law (purely elastic) for flow in arteries (as distinct from veins)

$$P - P_0 = \frac{Eh_0}{r_0(1-\sigma^2)}\left(\sqrt{\frac{A}{A_0}} - 1\right)$$

Fig. 4. Tube law for arteries, with parameters p_0, h_0, r_0, A_0, σ, E

Heart, pulmonary and peripheral circulation: 0D models

- 0D models are also called "lumped parameter models". See lecture notes on background
- 0D models cannot resolve spatial variations, only time variations
- The governing equations are Ordinary Differential Equations (ODEs) with algebraic constraints
- They solve the ODEs using a 4th order Runge-Kutta method

Some results. Normal case

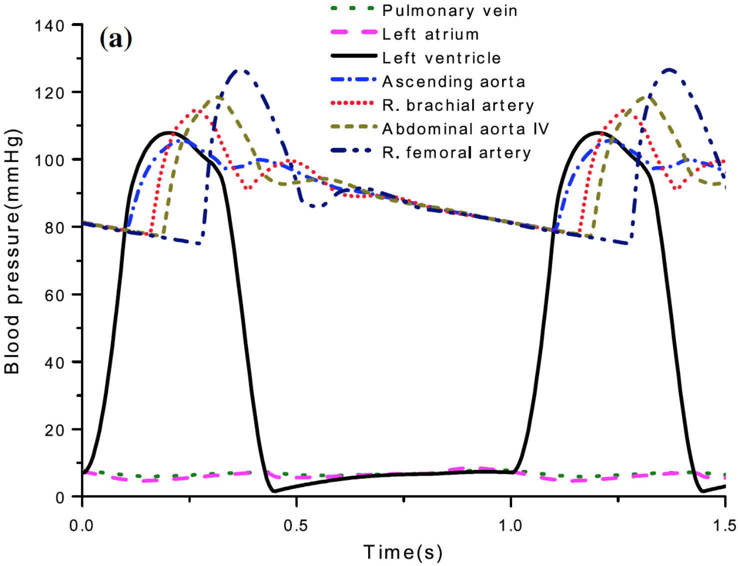

Fig. 5: Normal case. Blood pressure pulses in different regions of the cardiovascular system

Some results. Pathological cases

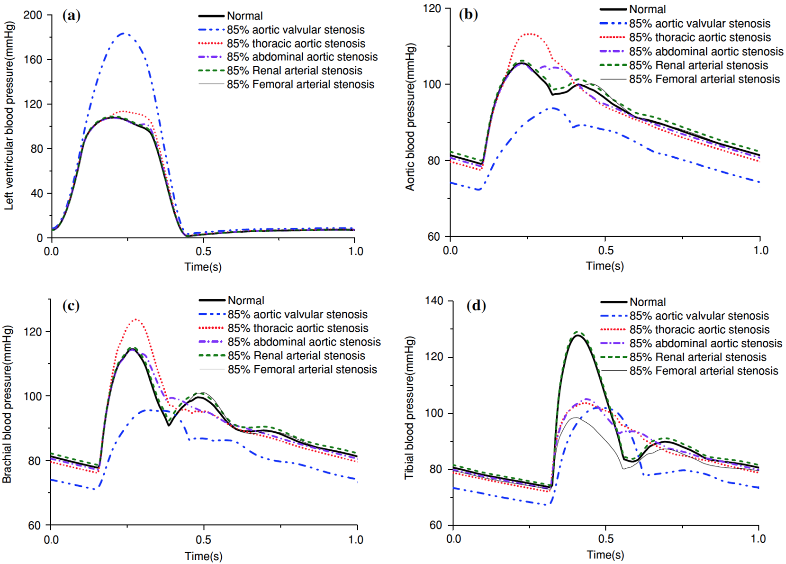

Fig. 6. Blood pressure pulses (a) in the left ventricle, (b) the ascending aorta, (c) the left brachial artery (c), and (d) the right anterior tibial artery

ABI

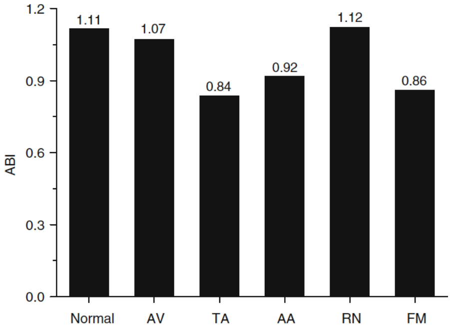

Fig. 7. ABI for the normal case and for the stenosis cases. AV, TA, AA, RN, and FM denote the locations of the stenoses, with AV aortic valve, TA thoracic aorta, AA abdominal aorta, RN renal artery, and FM femoral artery

- Ankle brachial pressure index

$$ABI = \frac{P_{ankle}}{P_{brachial}}$$

- A low ABI is taken as an indication of blocked arteries.

Conclusions

- Multi-scale model of the cardiovascular system constructed
- The model has been applied to study the global haemodynamic influences of an AV stenosis and arterial stenoses at various locations
- Five stenoses studied among those most frequently found in clinical practice
- Results show a strong location dependence of the global hemodynamic influences of an arterial stenosis
- Future model studies should focus on the influences of combined cardiac and arterial diseases that are often found in the elderly