Biomedical Application of Mathematics

Masters course in Mathematics

Specialism: *Modelling and Simulation for Biomedical Applications*

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Liang's model

Matlab 2015b

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Reference : Bonmassari's slides, Liang et al. [1], Lecture notes of Computational Haemodynamics [2] and Strocchi et al. [3]

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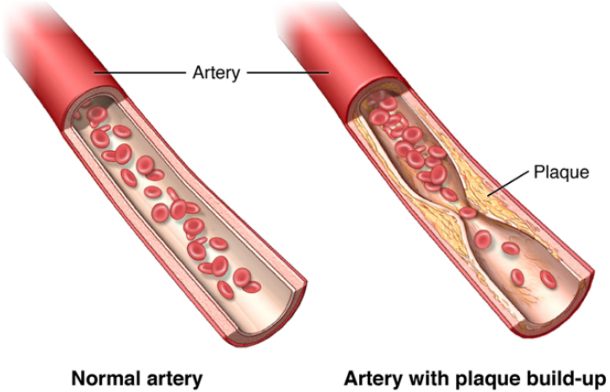
Stenosis

We are interested in studying the haemodynamic effects of an arterial stenosis in a global model of the human circulation.



A one-dimensional mathematical model for a compliant vessel

The one-dimensional equations for a compliant vessel (artery) are the following

$$\partial_t \begin{bmatrix} A \\ Au \end{bmatrix} + \partial_x \begin{bmatrix} Au \\ Au^2 + \gamma A^{\frac{3}{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ -Ru \end{bmatrix} \,, \tag{1}$$

where

- $A = A(x, t)$ is the cross-sectional area
- $u = u(x, t)$ is the velocity
- $q = A(x, t)u(x, t)$ is the flow
- Parameter γ contains the mechanical properties of the vessel
- R is the friction force per unit length

A lumped-parameter model

The microvasculature, heart chambers, lungs and veins can be model using lumped parameter models.
The equations are the following

$$\partial_t V = Q_{in} - Q_{out} \tag{2}$$

$$\partial_t Q = \frac{1}{L}\left(P_{up} - P_{do} - RQ\right) \tag{3}$$

where

- $V = V(t)$ is the volume of the chamber
- $Q = Q(t)$ is the inflow/outflow of the chamber
- $P = P(V)$ is the upstream/downstream pressure and is a function of $V(t)$
- R is the resistance to flow
- L is the inertia

Liang's model

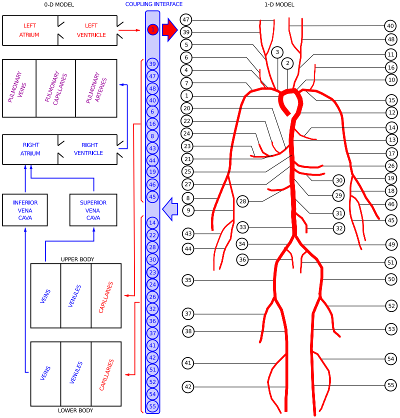


Figure : From Strocchi et al. [3]

The 1D model is composed by 55 arteries.

Liang's model

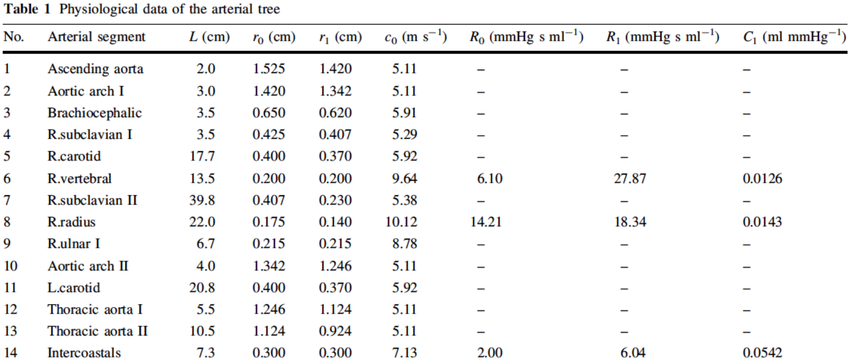


Table 1 Physiological data of the arterial tree

No.	Arterial segment	L (cm)	r_0 (cm)	r_1 (cm)	c_0 (m s^{-1})	R_0 (mmHg s ml^{-1})	R_1 (mmHg s ml^{-1})	C_1 (ml mmHg^{-1})
1	Ascending aorta	2.0	1.525	1.420	5.11	–	–	–
2	Aortic arch I	3.0	1.420	1.342	5.11	–	–	–
3	Brachiocephalic	3.5	0.650	0.620	5.91	–	–	–
4	R.subclavian I	3.5	0.425	0.407	5.29	–	–	–
5	R.carotid	17.7	0.400	0.370	5.92	–	–	–
6	R.vertebral	13.5	0.200	0.200	9.64	6.10	27.87	0.0126
7	R.subclavian II	39.8	0.407	0.230	5.38	–	–	–
8	R.radius	22.0	0.175	0.140	10.12	14.21	18.34	0.0143
9	R.ulnar I	6.7	0.215	0.215	8.78	–	–	–
10	Aortic arch II	4.0	1.342	1.246	5.11	–	–	–
11	L.carotid	20.8	0.400	0.370	5.92	–	–	–
12	Thoracic aorta I	5.5	1.246	1.124	5.11	–	–	–
13	Thoracic aorta II	10.5	1.124	0.924	5.11	–	–	–
14	Intercoastals	7.3	0.300	0.300	7.13	2.00	6.04	0.0542

Figure : Table n. 1 of [1]

We have modeled the healthy control (no stenosis), two stenoses: in the ascending aorta (no. 1) and in the thoracic aorta (n. 13).

Exam

You are asked to prepare an oral presentation with the following features:

- Characterize the cardiovascular system using the indicators introduced by Dr. Bonmassari: PV loop, mean arterial pressure, CI, etc.
- Discuss the differences between the healthy control and patients with the stenosis.
- Prepare an oral presentation (15-20 minutes) with slides.
- Create a zip file containing the slides and the scripts you have used to study the data, call it **yoursurname-BAM-2016-17.zip** and submit to christian.contarino@unitn.it and eleuterio.toro@unitn.it one week before the exam.

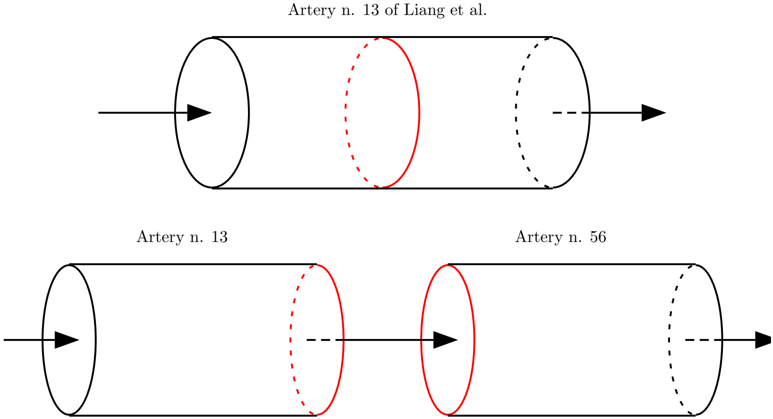
Data

- You have 3 folders : HealthyControl, StenosisAscAo and StenosisThorAo
    - HealthyControl : healthy control
    - StenosisAscAo : Stenosis in the ascending aorta (Artery No. 1).
    - StenosisThorAo : Stenosis in the thoracic aorta (Artery No. 13)
- Each folder contains dat-files : NameA.dat, NameP.dat, NameQ.dat and NameHeart.dat

Important

- Assume 77-kg person of 1.75 *m* in height.
- A cardiac cycle lasts 1 second.
- You have $55+1$ arteries

Stenosis in the thoracic aorta



Artery n. 13 of Liang et al.

Artery n. 13

Artery n. 56

Summary of the data : 1D model

- **NameP.dat**, **NameA.dat**, **NameQ.dat**
  - First column is time s
  - From column 2 to column 56, you find pressures, cross-sectional areas and flows in arteries from 1 to 55 of table 1 of Liang et al. [1]
  - Column 57 represents the second half of the vessel with the stenosis
    - For HealthyControl do not consider the 56th artery
    - Note that for the stenosis in the ascending aorta, artery no. 1 of [1], is composed of two vessels, whose column numbers are 2 (first proximal half) and 57 (second proximal half)
    - Note that for the stenosis in the thoracic aorta, artery no. 13 of [1] is composed of two vessels, whose column numbers are 14 (first proximal half) and 57 (second proximal half)
  - Pressure is measured in $mmHg$, cross-sectional area in m^2 and flow in mL/s
  - The data is given by considering sampling area and flow at the center of each artery

Summary of the data : 0D model

- **NameHeart.dat**
  - First column is the time s
  - Pressure is measured in $mmHg$, volume in mL and flow in mL/s

Column		
10	V_{RA}	Volume right atrium
11	Q_{TV}	Flow through the tricuspid valve
12	V_{RV}	Volume right ventricle
13	Q_{PV}	Flow through the pulmonary valve
20	V_{LA}	Volume left atrium
21	Q_{MV}	Flow through the mitral valve
22	V_{LV}	Volume left ventricle
23	Q_{AV}	Flow through the aortic valve
32	P_{LA}	Pressure left atrium
33	P_{LV}	Pressure left ventricle
34	P_{RA}	Pressure right atrium
35	P_{RV}	Pressure right ventricle

Useful commands in matlab

- Load the Healthy control's pressure

```
P=load('HealthyControl/SaveData/HealthyControlP.dat') %Load the .dat file
```

  - P(:,1) is the time. Time goes from 9 to 10 because we have simulated 10 cardiac cycles.
  - P(:,2) is the pressure of the first artery of the Liang's model, namely the ascending aorta.

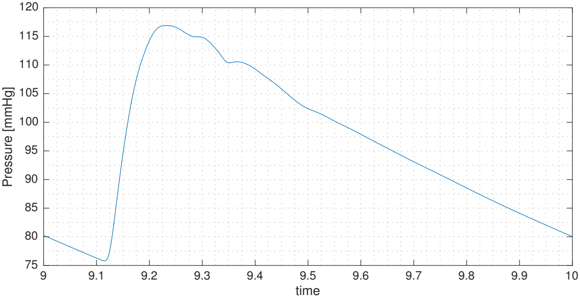
Useful commands in matlab

- Plot the pressure of the 13th artery

```
plot(P(:,1),P(:,14))  %P(:,1) is the time and P(:,14) is the pressure of the 13th artery
```

- Mean value of the 13th artery

```
mean(P(:,14))
ans =
  96.0276
```



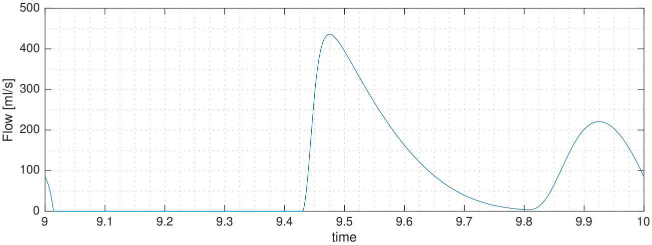
Useful commands in matlab

- Load the Heart information of the Healthy control

```
H=load('HealthyControl/SaveData/HealthyControlHeart.dat')
%Load the .dat file
```

- Plot the flow through the mitral valve

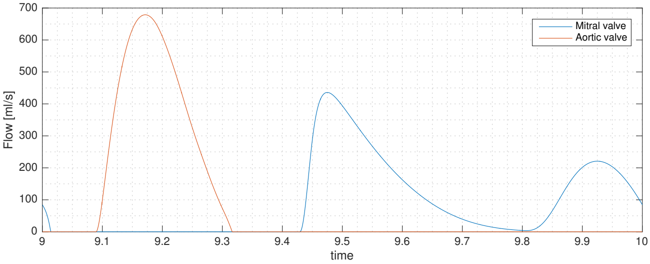
```
plot(H(:,1),H(:,21))  %H(:,1) is the time and H(:,21) is the flow through the mitral valve
```



Useful commands in matlab

- 2 plots in the same figure with a grid and a legend.

```
plot(H(:,1),H(:,21))
grid minor
xlabel('time')
ylabel('Flow [ml/s]')
hold on
plot(H(:,1),H(:,23))
legend('Mitral valve','Aortic valve')
```



Indicators

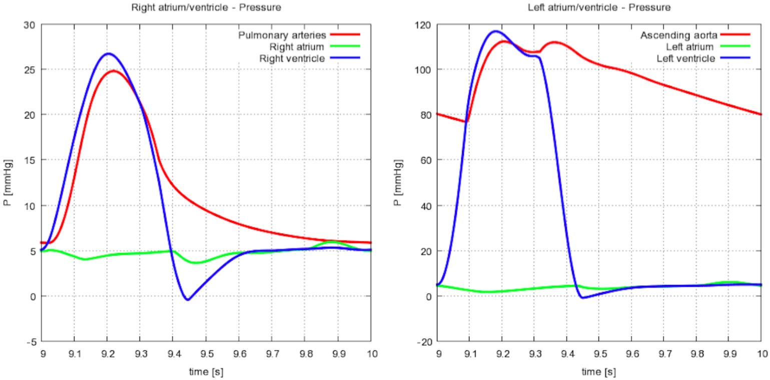
Here is an example of possible indicators you can use to characterize the cardiovascular system

- End-systolic pressure, volume
- End-diastolic pressure, volume
- Body Mass Index
- Frequency of contraction
- Pressure pulse
- Stroke volume and stroke volume index
- Cardiac output and cardiac index
- ABI
- Refer to the plots of [1] and [4]

What happens to the velocity near the stenosis?

Indicators

Try to replicate the following



References

- F. Liang, S. Takagi, R. Himeno, H. Liu, Multi-scale modeling of the human cardiovascular system with applications to aortic valvular and arterial stenoses, Medical & Biological Engineering & Computing 47 (7) (2009) 743–755.
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- E. F. Toro, Lecture Notes of Computational Haemodynamics (2015/2016).
- M. Strocchi, C. Contarino, E. F. Toro, R. Bonmassari, A global mathematical model for the simulation of stenoses and bypass placement in the human arterial system, submitted to Applied Mathematics and Computations.
- F. Liang, S. Takagi, R. Himeno, H. Liu, Biomechanical characterization of ventricular–arterial coupling during aging: A multi-scale model study, Journal of Biomechanics 42 (6) (2009) 692–704.
  `doi:10.1016/j.jbiomech.2009.01.010`.
  URL `http://dx.doi.org/10.1016/j.jbiomech.2009.01.010`