Neuroimaging and mathematical modelling
Lesson 3: Diffusion Tensor Imaging

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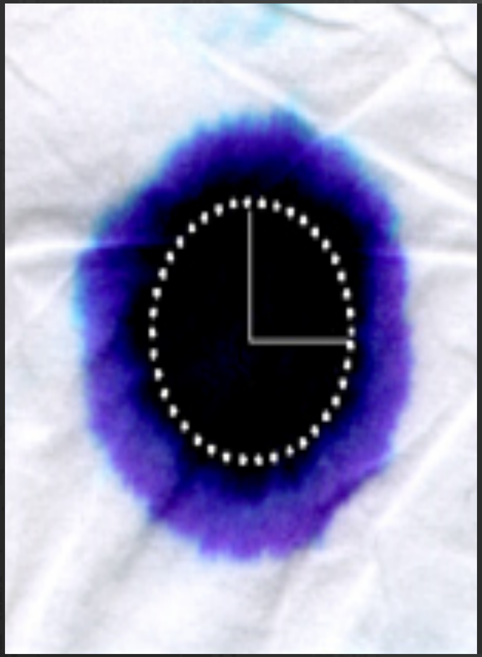
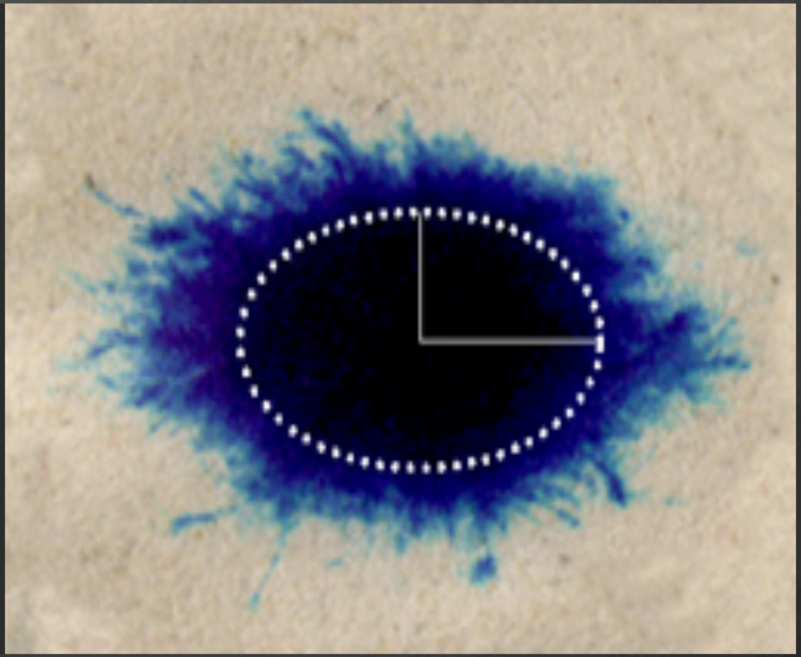
Diffusion



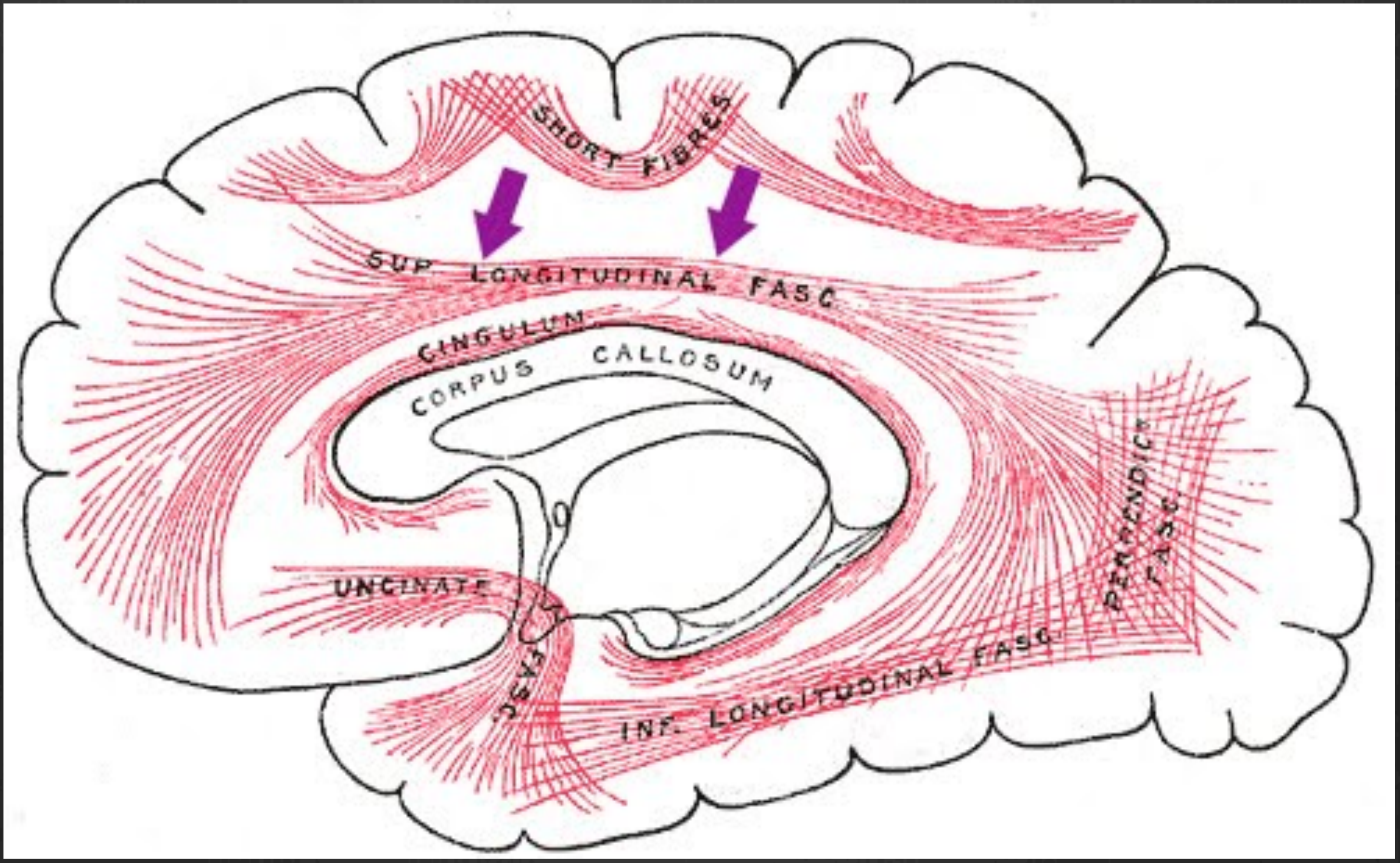
Kleenex



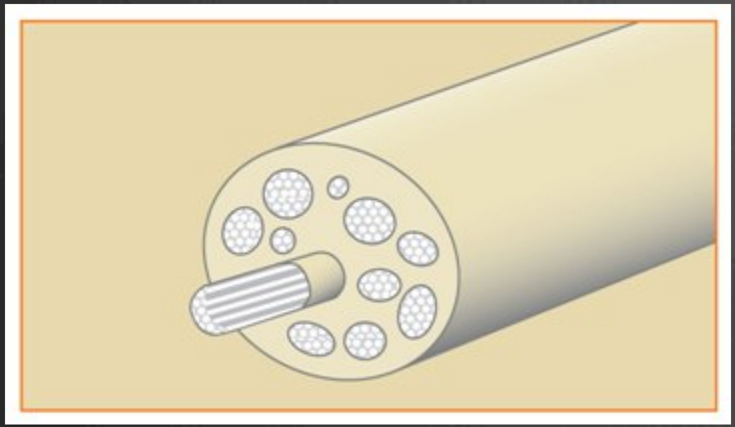
Newspaper

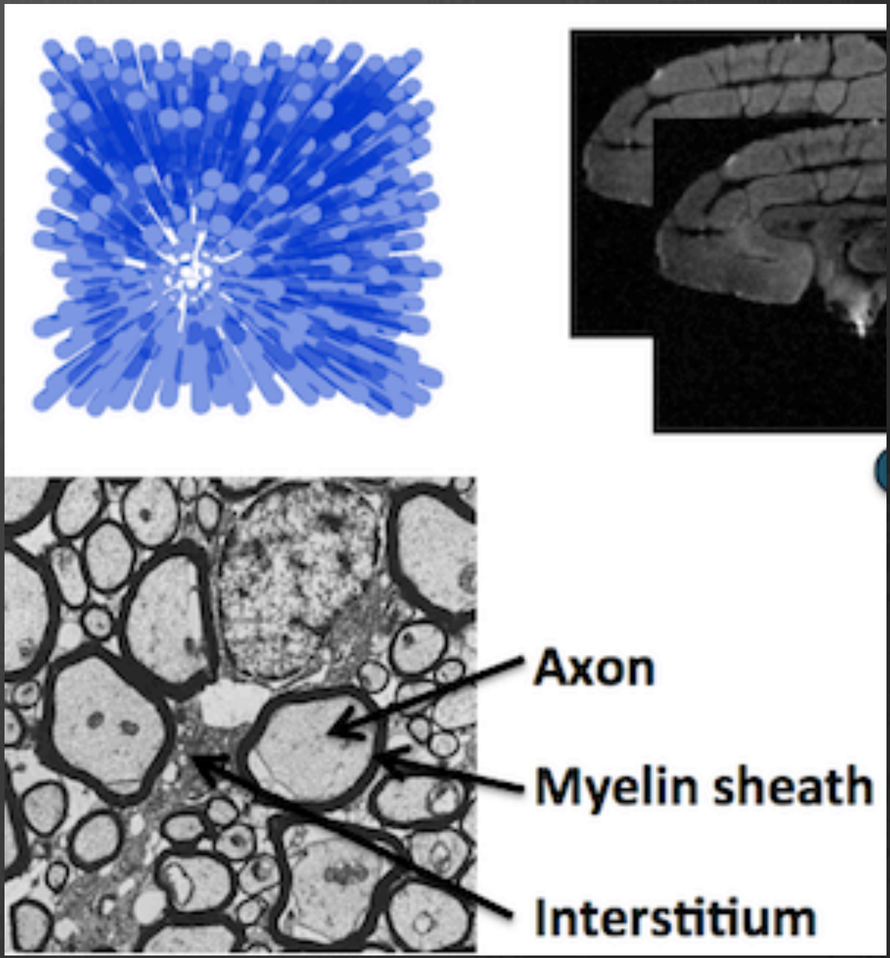
White matter: macroscopic

White matter: macroscopic

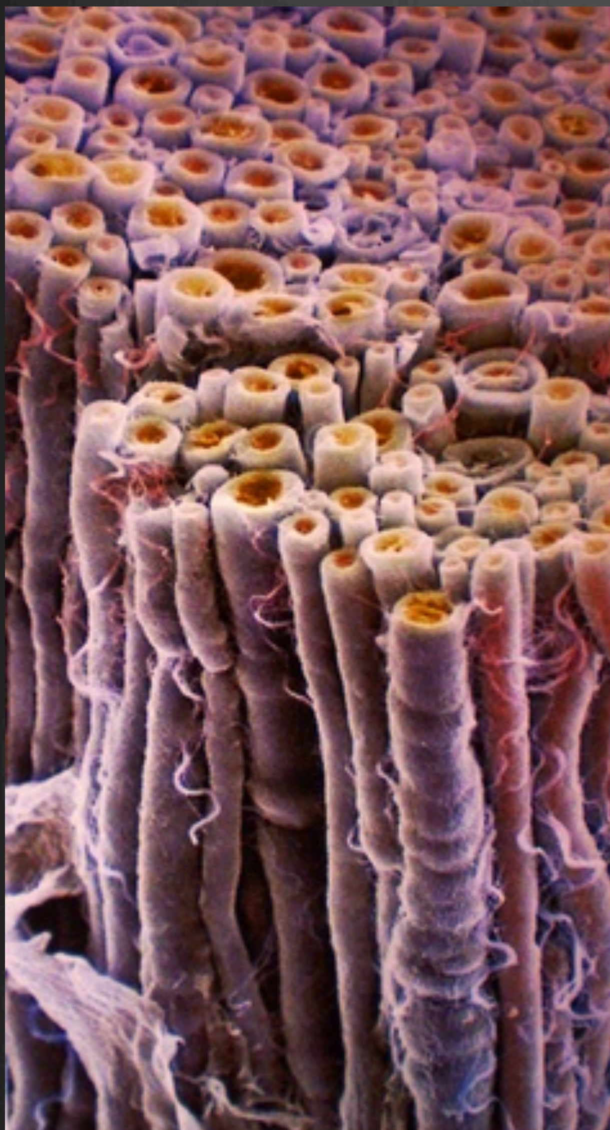


White matter: microscopy

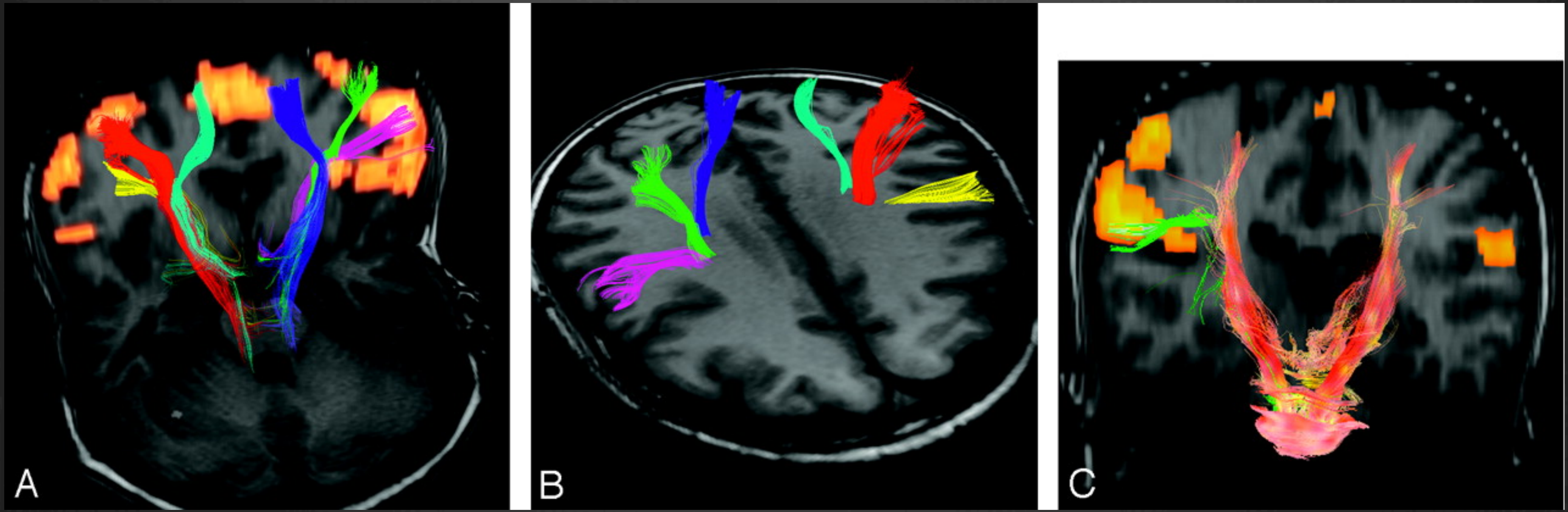




White matter: microscopy

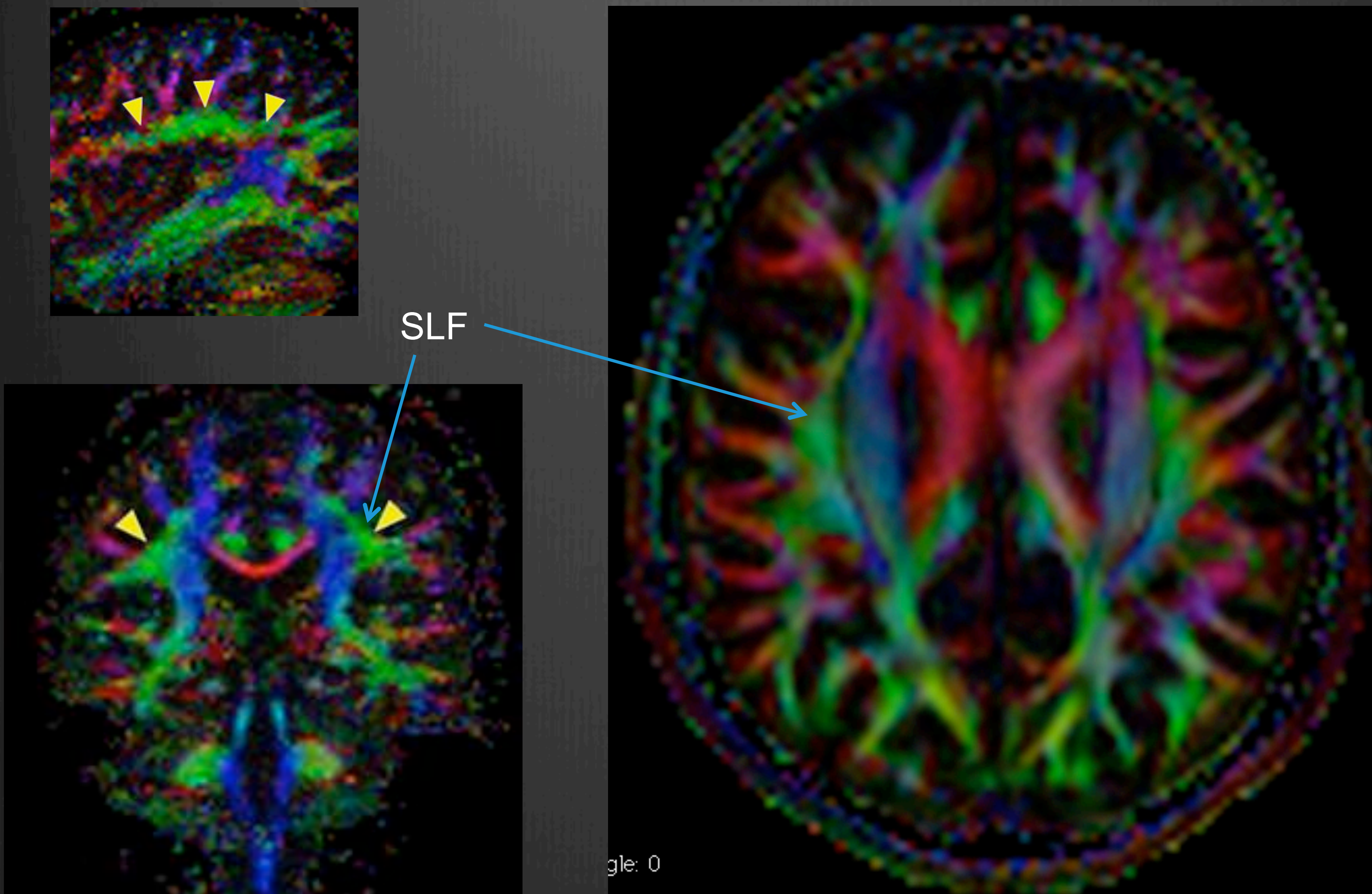


DTI: reconstruction

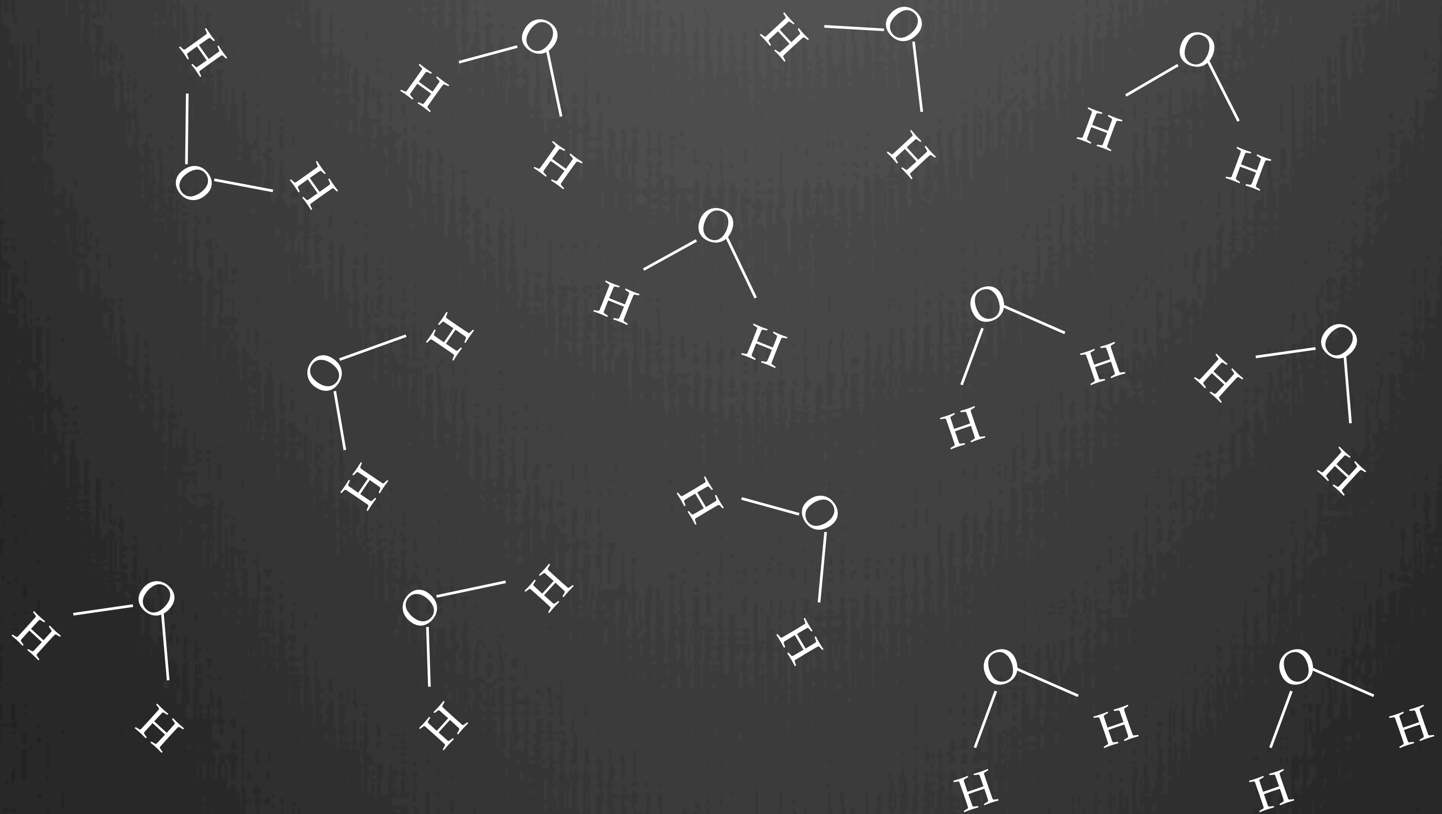


AJNR 2007

Diffusion tensor imaging (DTI)



Diffusion of water in water

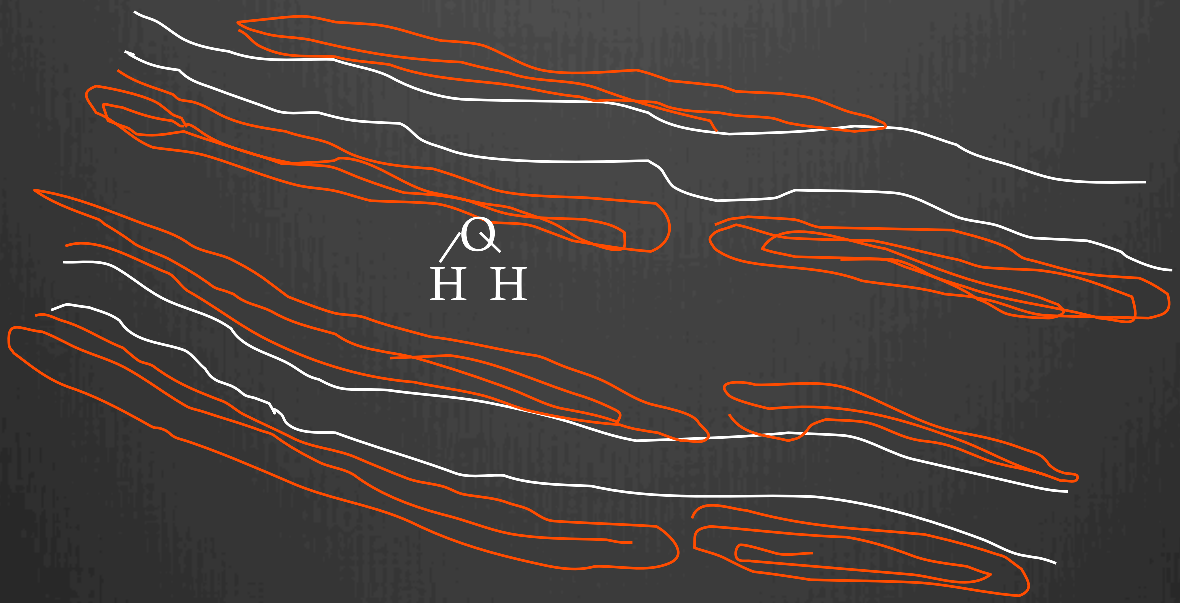


HBWL

Water diffusion in brain white matter

Anisotropic diffusion: Dx Dy DZ …: Diffusion tensor

Unequally restricted in space, preferred directions



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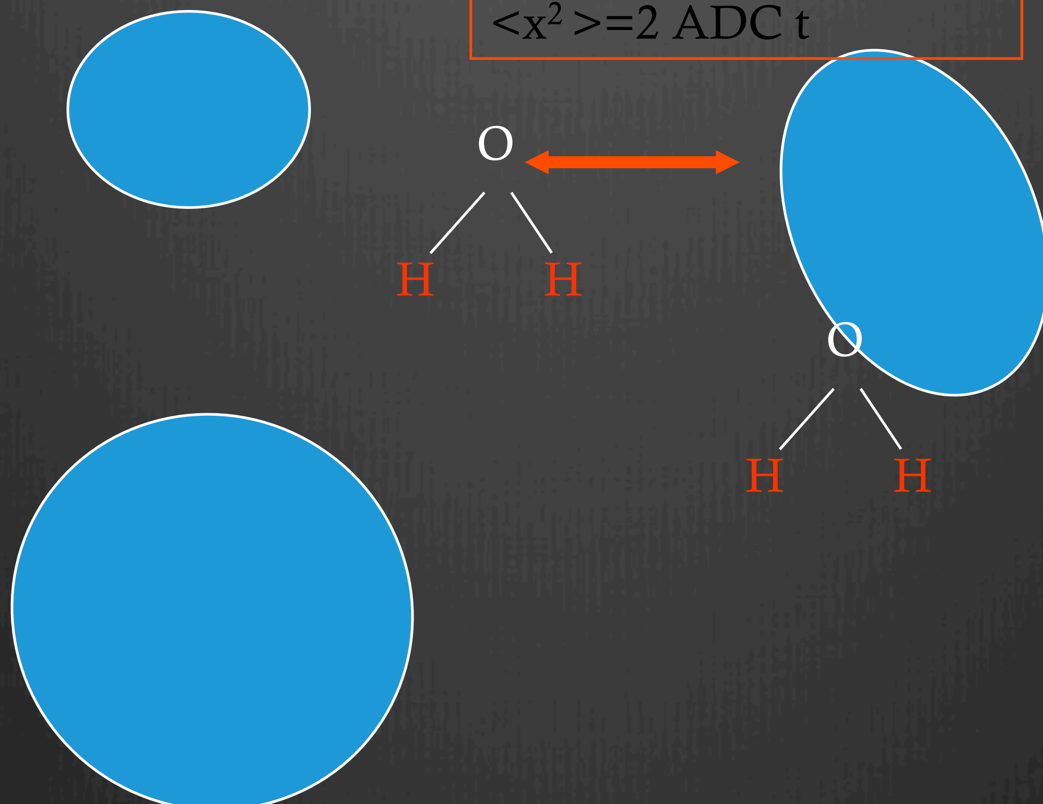
Water diffusion in biological tissue

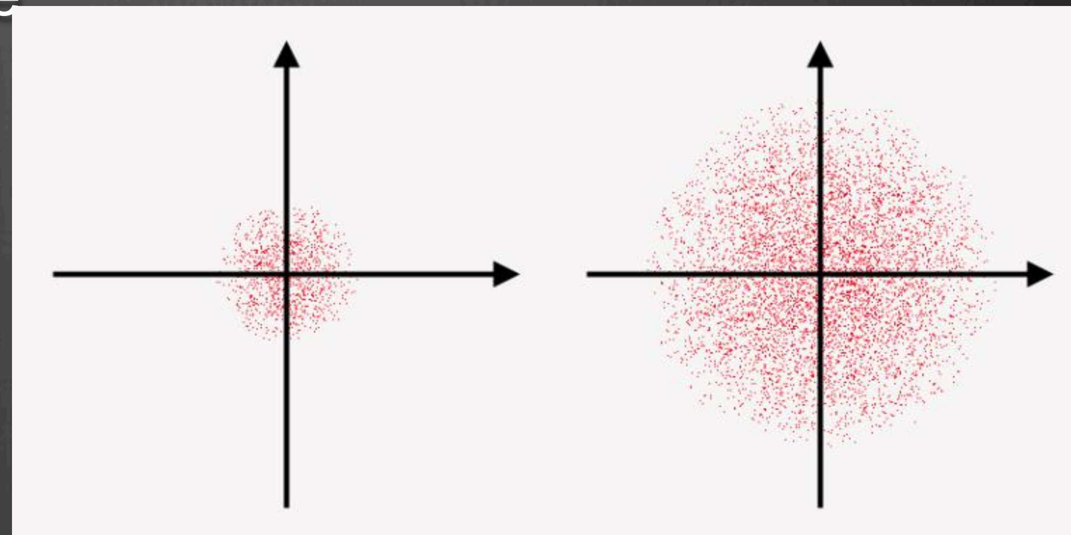
Still isotropic but restricted: apparent diffusion coefficient : ADC

$\langle x^2 \rangle = 2\ ADC\ t$



Stochastic Movements

- The type/size of molecule and surroundings affect ease of random movement
- Diffusion coefficient = ease of movement
- More time or higher Diff. coeff. → wider distribution
- This was described by A. Einstein in 1905
  Annalen der Physik 1905;17:549



Low D High D

$$D = \frac{Avg(x^2)}{2t}$$

Unit: mm^2/s

Example

$$D = \frac{Avg(x^2)}{2t}$$

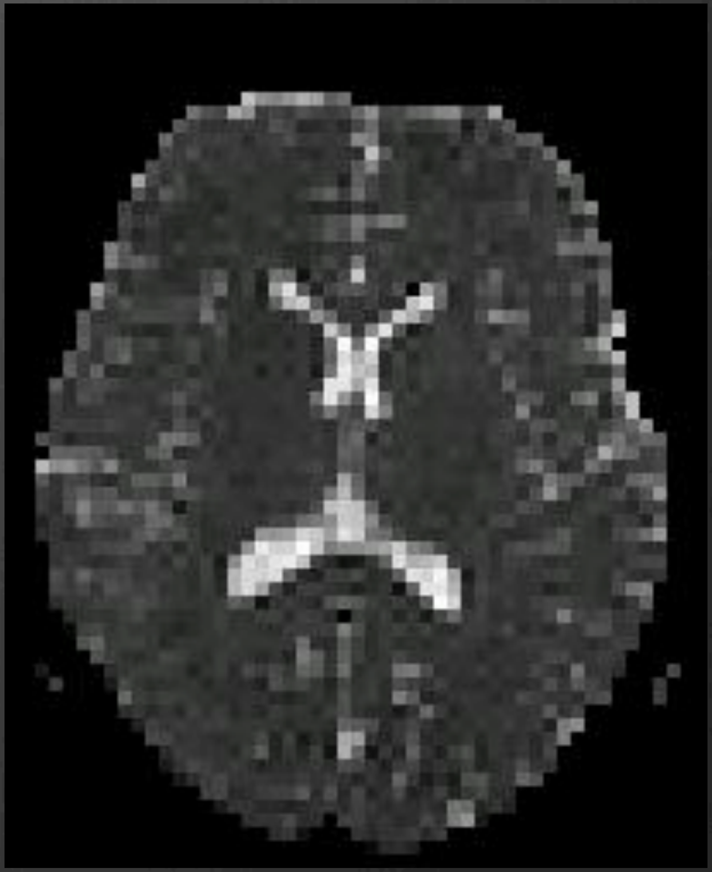
Unit: mm^2/s

D = 1.0x10^{-3} mm2/s diffusivity of water in the parenchyma
Se t = 35ms

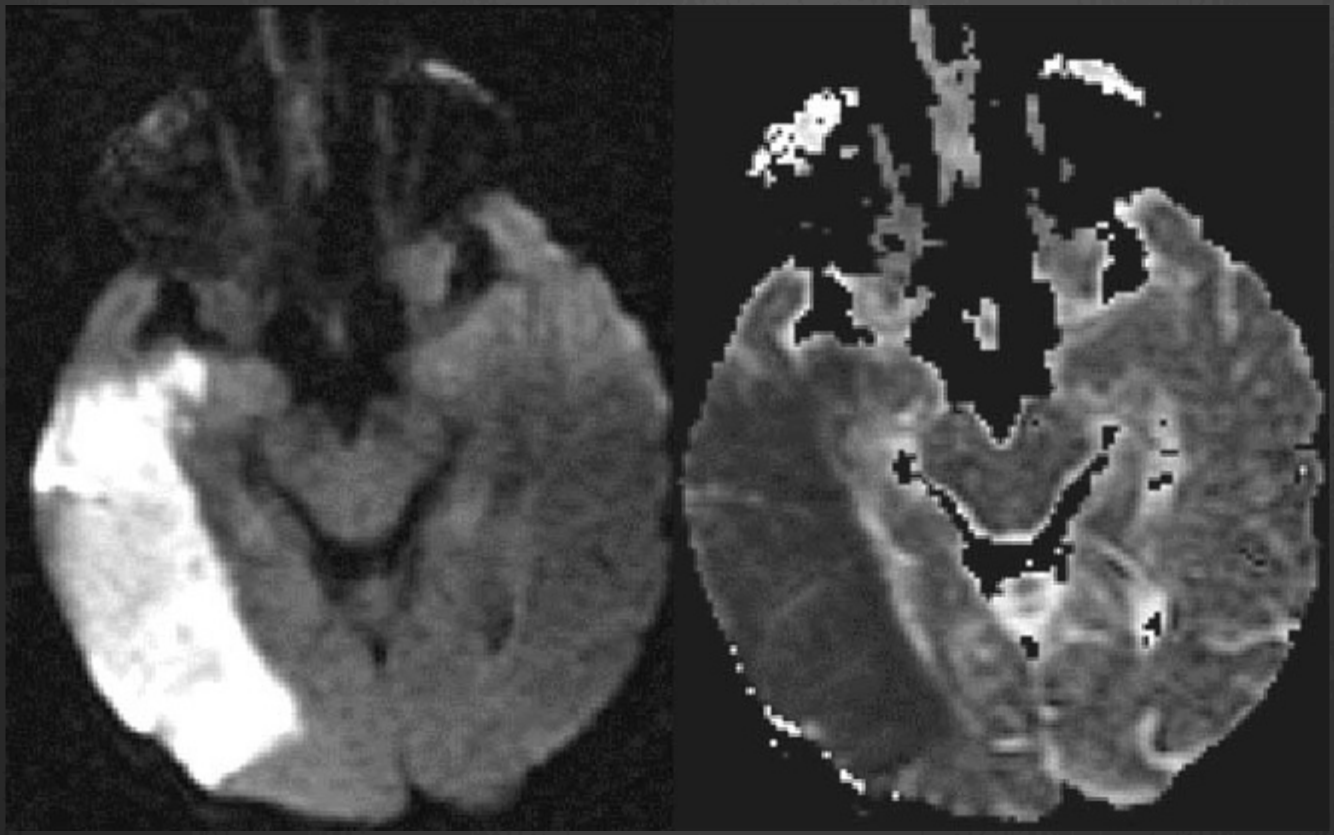
Then x = 8 micrometer which is the diameter of an axon.

Apparent Diffusion Coefficient (ADC)

- Areas with higher rate of diffusion are brighter
- Little contrast between gray and white matter
- DWI calculation of ADC, relative rate of diffusion, is useful clinically (e.g. stroke)
- Not of much use in research?



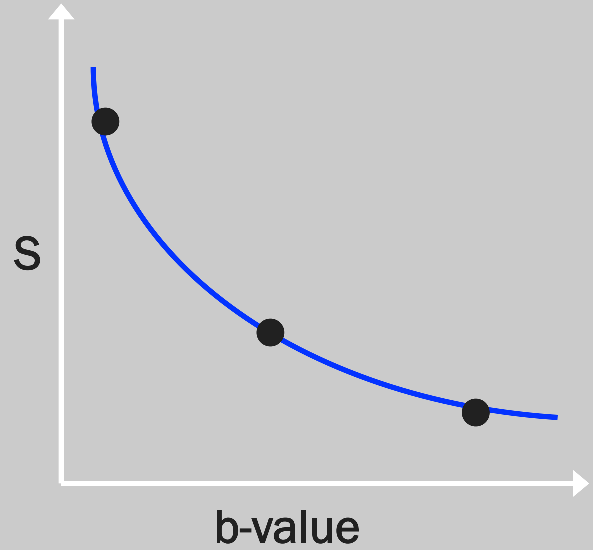
Brain Infarction (stroke)



Magnetic resonance imaging in acute stroke. Left: Diffusion-weighted MRI in acute ischemic stroke performed 35 minutes after symptom onset. Right: Apparent diffusion coefficient (ADC) map obtained from the same patient at the same time.

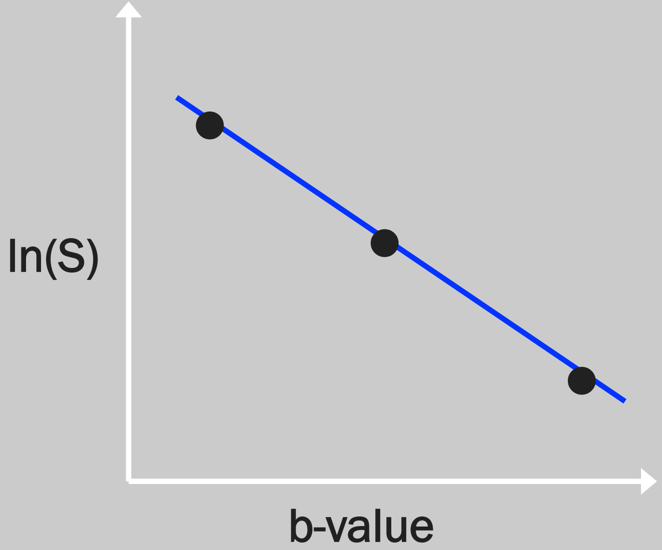
ADC (Apparent Diffusion Coefficient)

non-linear fitting
using image pixel values



$$S = S_0 \exp\left(-b \bullet ADC\right)$$

linear fitting using natural
log of image pixel values



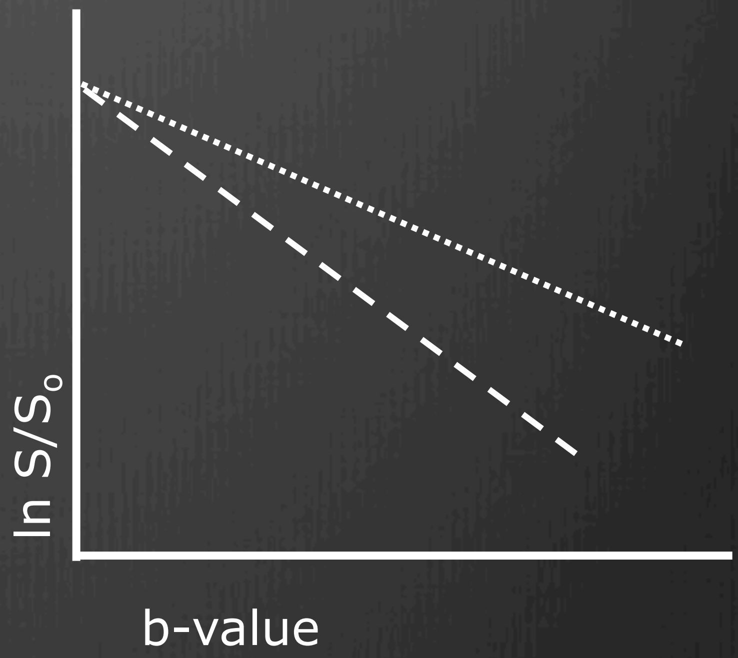
$$\ln\left(S\right) = \ln\left(S_0\right) - b \bullet ADC$$

Diffusion Weighted Imaging (DWI) Sequence

- Uses an EPI pulse sequence with bi-polar gradients applied during the sequence
  - First gradient disrupts the magnetic phases of all protons
  - Second gradient restores the phases of all stationary protons
- The restoration of signal is incomplete for protons that have moved (diffused) during the elapsed time

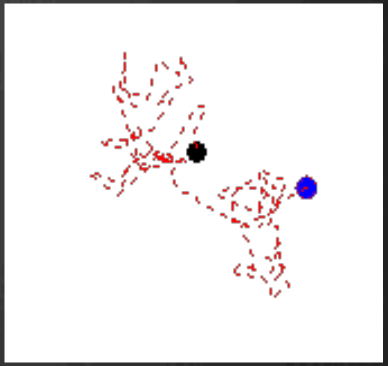
Basic DWI Calculation

- Additional parameter in DWI is the b-value which defines both how strong the bi-polar gradients are and their duration
- Areas where diffusion occurs most rapidly will exhibit a greater decrease in MR signal as the b-value increases
- Collect multiple images each with a different b-value
- Typically estimated with just 2 b-values



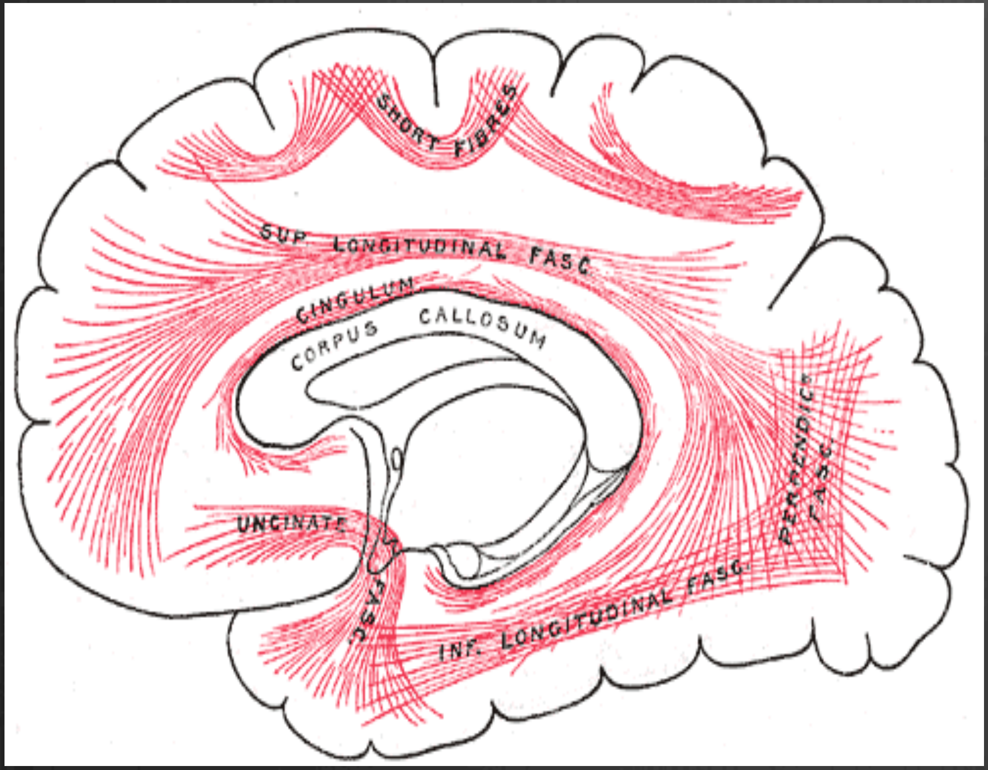
Diffusion Tensor Imaging

- DTI relates image intensities to the relative mobility of water molecules in tissue and the direction of the motion
- Motion of a water molecules is a random walk (Brownian motion)
- Areas with relatively high mean diffusion will appear dark on the Diffusion weighted MRI images

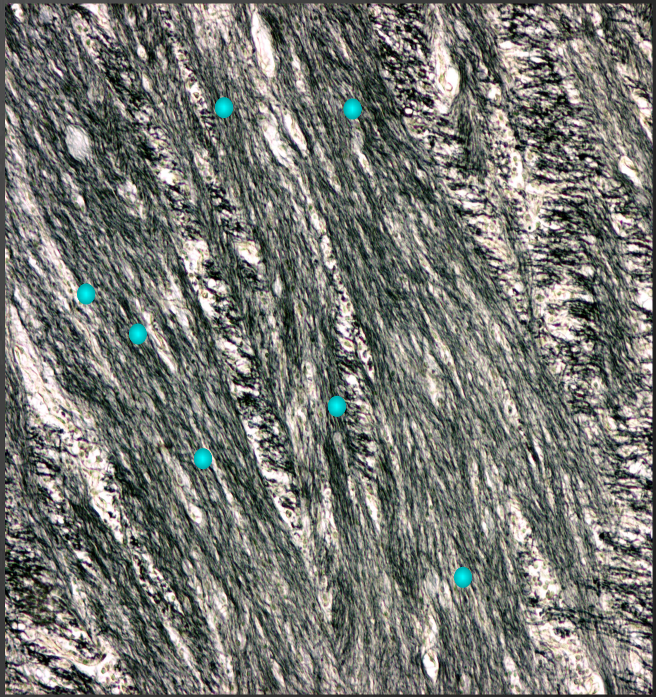


Currect terminology is important

Anatomy



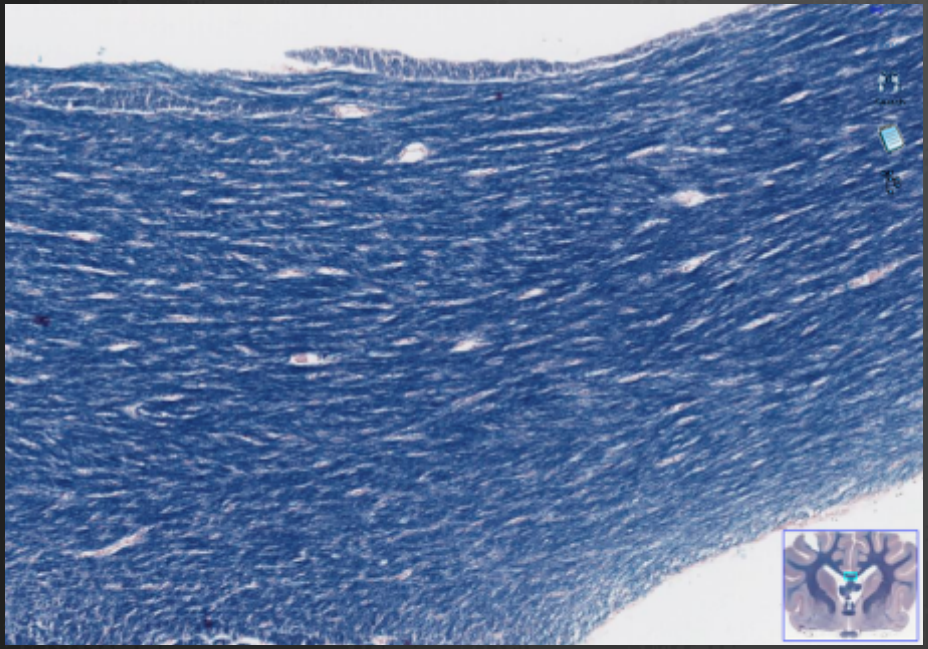
MR Tractography



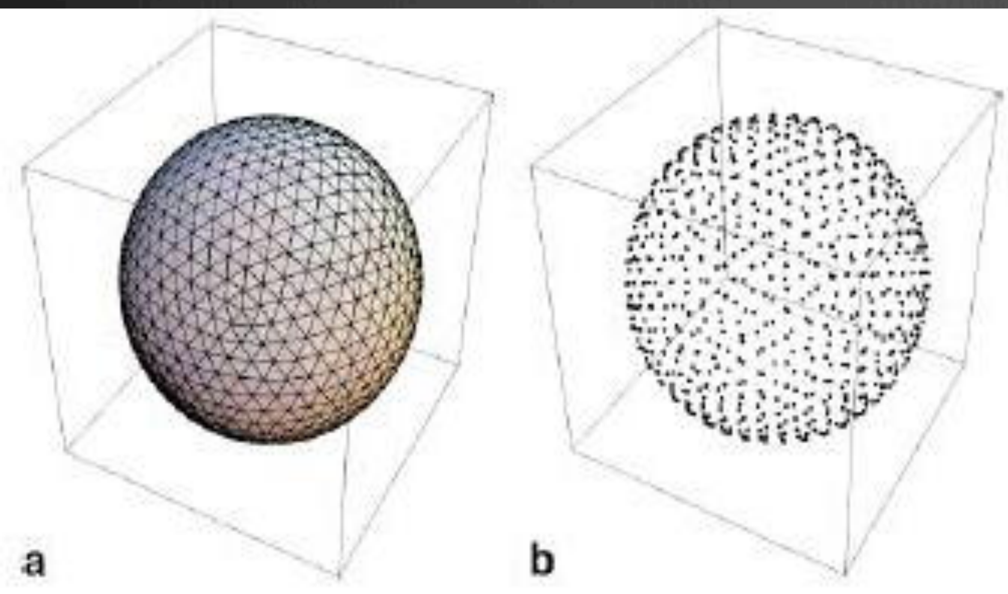
Streamlines

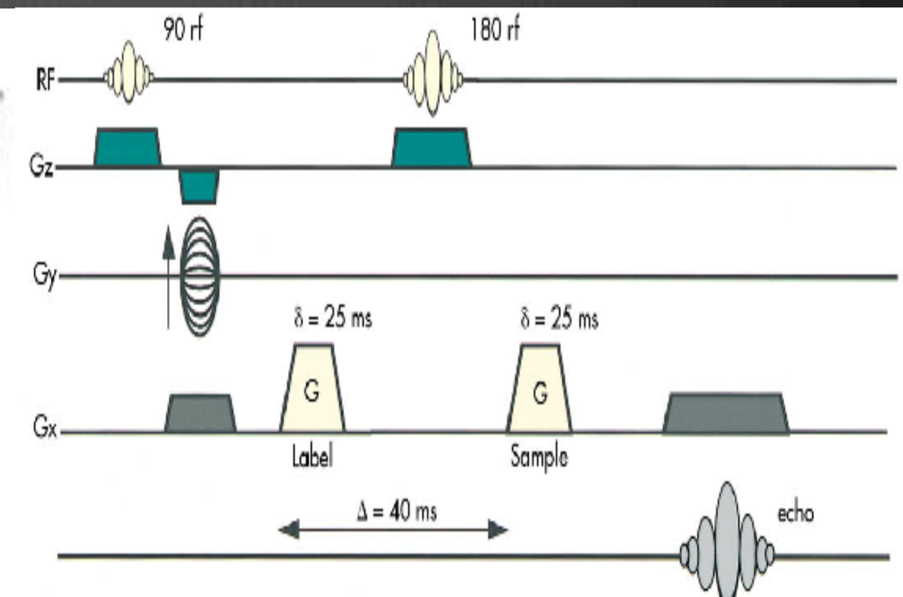
Trajectories

Diffusion TENSOR imaging (DTI)



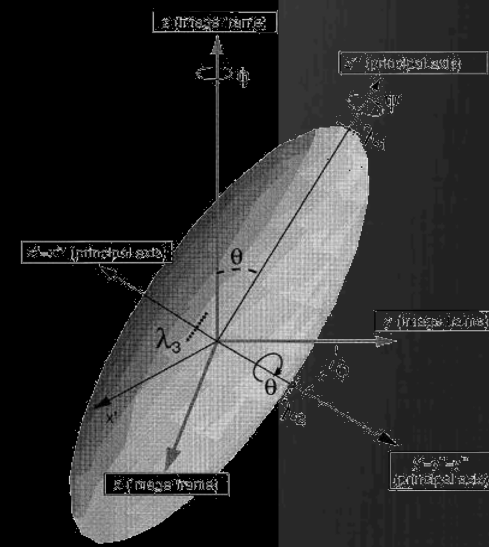
Diffusion Tensor Imaging (DTI)





Diffusion Tensor Imaging (DTI)

- Mobility in a given direction is described by ADC
- The tissue diffusivity is described by the tensor D
- The diffusion equation



$$Attenuation = \frac{A(b)}{A(0)} = exp\left(-b \sum_{i,j=1,2,3} x_{ij} \cdot D_{ij}\right) = exp(-b \cdot ADC)$$

- Diffusion is represented by a 3×3 tensor*

$$D = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{bmatrix} \rightarrow D_{diag} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

*P. Basser and D. Jone, NMR in Biomedicine, 2002.

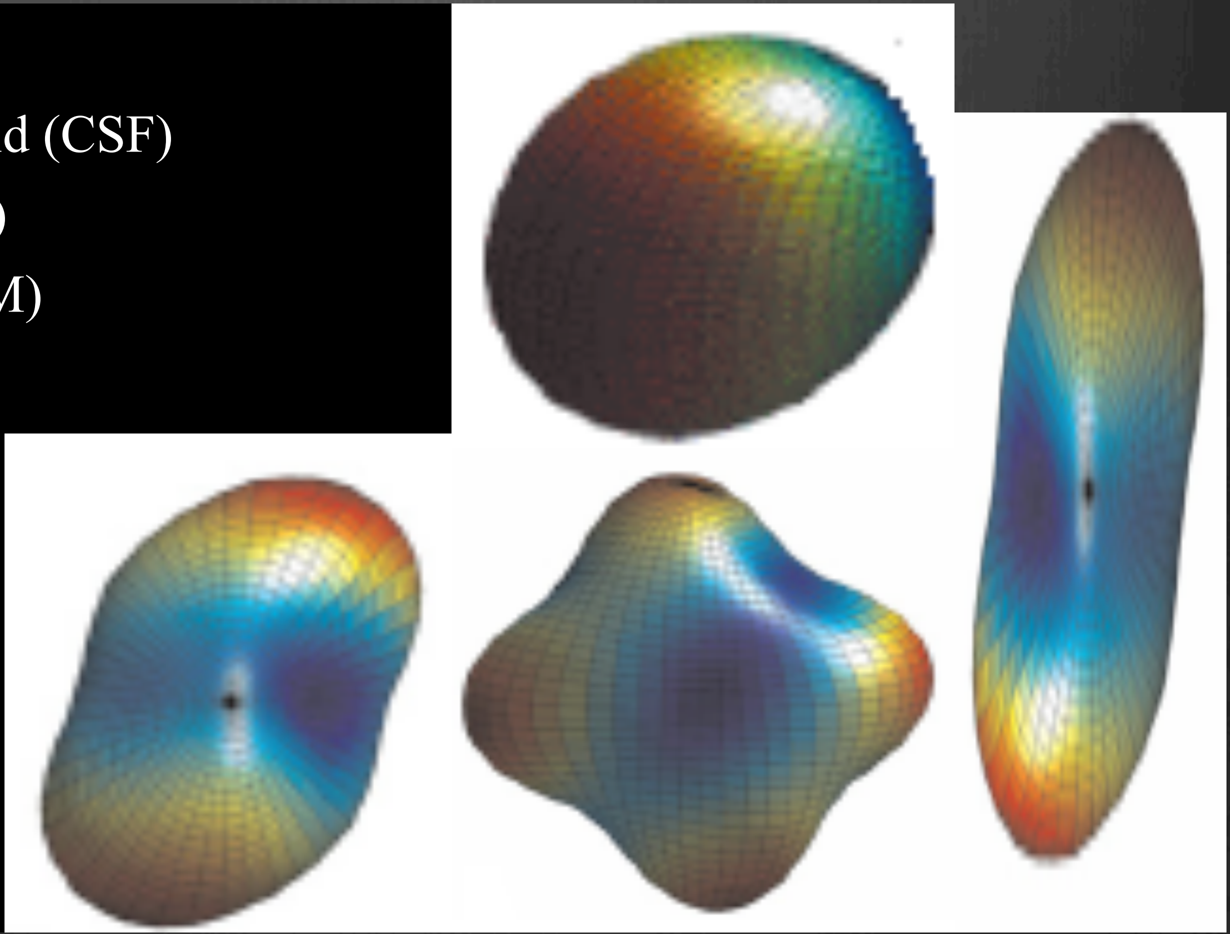
Diffusion Tensor

$$\mathbf{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{bmatrix}$$

- Diffusion properties described with a 3 X 3 symmetric tensor matrix
- Diagonal elements of **D** (*Dxx , Dyy , Dzz*) are the ADC values along *x*, *y* and *z* axes respectively
- Off-diagonal elements (*Dxy , Dxz , Dyz*) represent the correlation between molecular displacements in orthogonal directions

Diffusion Tensor Imaging (DTI)

- Brain Tissue types
  - Cerebrospinal fluid (CSF)
  - Gray matter (GM)
  - White matter (WM)
  - Mixing of tissues



Diffusion Tensor Imaging (DTI)

- Invariant Anisotropy Indices
  - Fractional Anisotropy (FA)*

$$FA = \frac{\sqrt{3[(\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_3 - \bar{\lambda})^2]}}{\sqrt{2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}}$$

  - Relative Anisotropy (RA)*

$$RA = \frac{\sqrt{[(\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_3 - \bar{\lambda})^2]}}{\sqrt{3}\bar{\lambda}}$$

  - Volume Ratio (VR)*

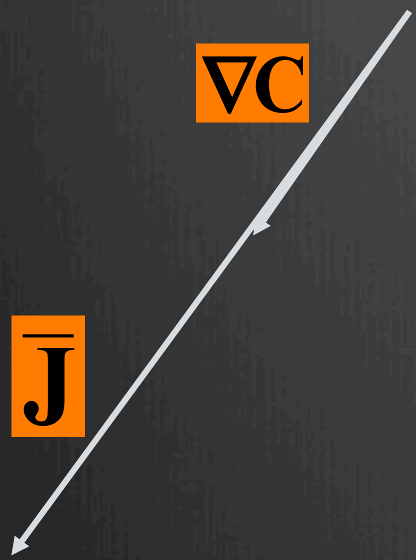
$$VR = \frac{\lambda_1 \lambda_2 \lambda_3}{\bar{\lambda}^3}$$

**D. LeBihan, NMR Biomedicine., 2002.*

DTI Calculation

- Eigenvalues of the diffusion tensor (λ_x, λ_y, and λ_z) provides length of the ellipsoid in the three principal directions of diffusivity
- Eigenvectors provide information about the direction of diffusion
- The eigenvector corresponding to the largest eigenvalue is used as the main direction of diffusion
- Maps are constructed of various measures of anisotropy from the eigenvalues and eigenvectors

Isotropic diffusion

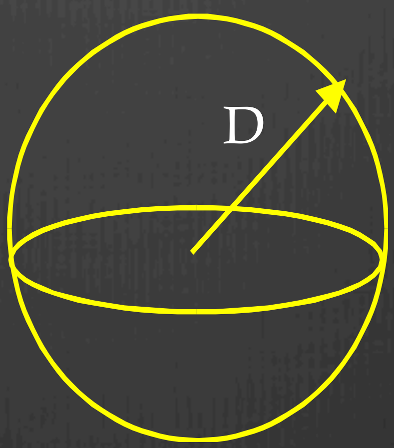


one - dimension

$$J = -D \frac{dC}{dX}$$

three - dimension

$$\bar{J} = -D \nabla C(x, y, z)$$



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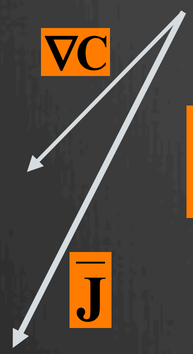
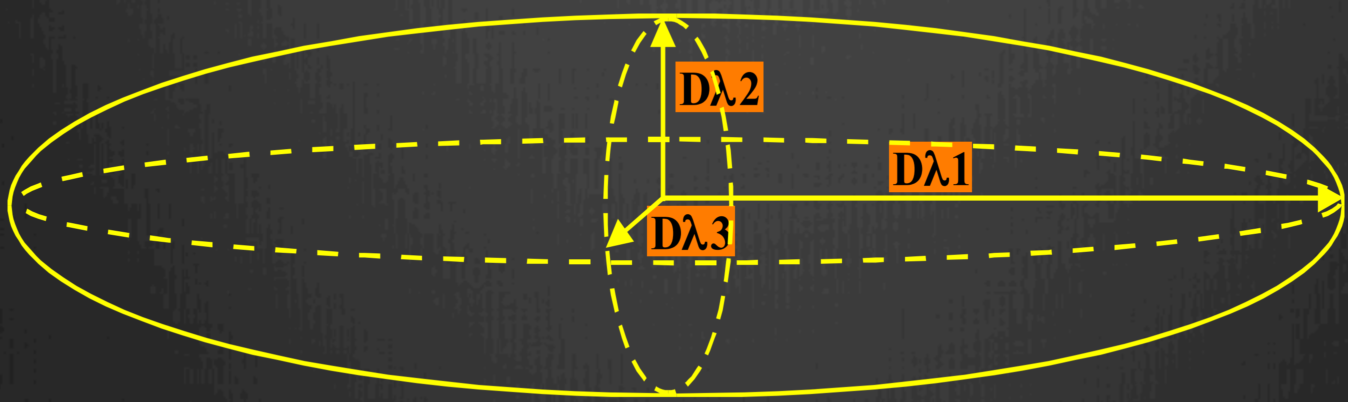
Anisotropic diffusion



$$\bar{J} = -\underline{D}\,\nabla C(x, y, z)$$

$$\underline{\mathbf{D}} = \begin{pmatrix} \mathbf{D_{xx}} & \mathbf{D_{xy}} & \mathbf{D_{xz}} \\ \mathbf{D_{yx}} & \mathbf{D_{yy}} & \mathbf{D_{yz}} \\ \mathbf{D_{zx}} & \mathbf{D_{zy}} & \mathbf{D_{zz}} \end{pmatrix}$$



ellipsoid

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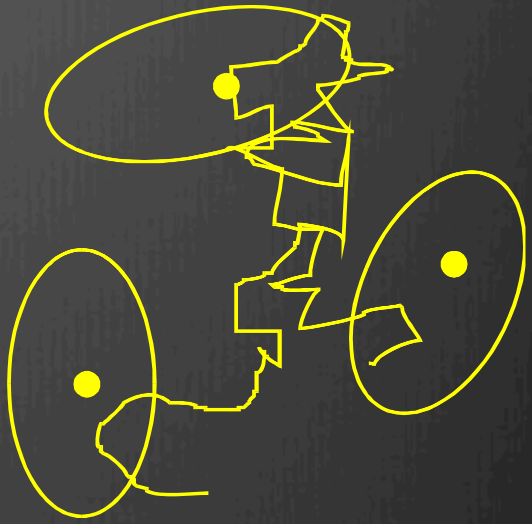
Apparent Difussion Coefficient (ADC)

Multicompartment system in vivo

- Intra cellular volume = 70%
- Extra cellular volume = 20%
- Vascular volume = 10%
- Membranes, mitochondria ect.



$$D_{ICV} < D_{ECV}$$

$$ADC = 0.7 - 1.2 \cdot 10^{-9}\ m^2/s$$

$$D_{H_2O} = 3 \cdot 10^{-9}\ m^2/s$$

$$T = 37^\circ C$$

Restricted diffusion

Dependence of diffusion time

Dependence of dirrections : Isotropic > < Anisotropic diffusion

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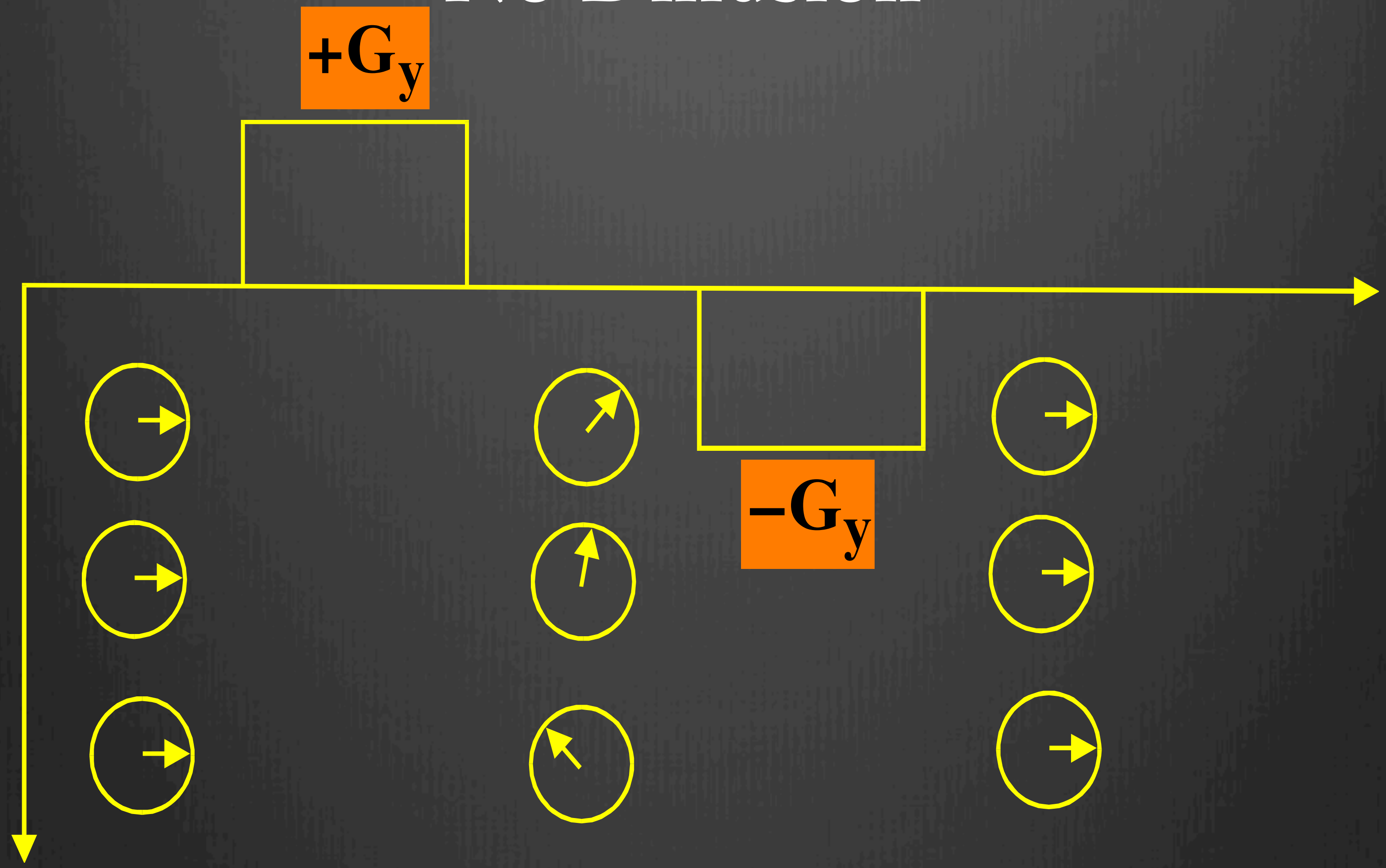
No Diffusion



y position

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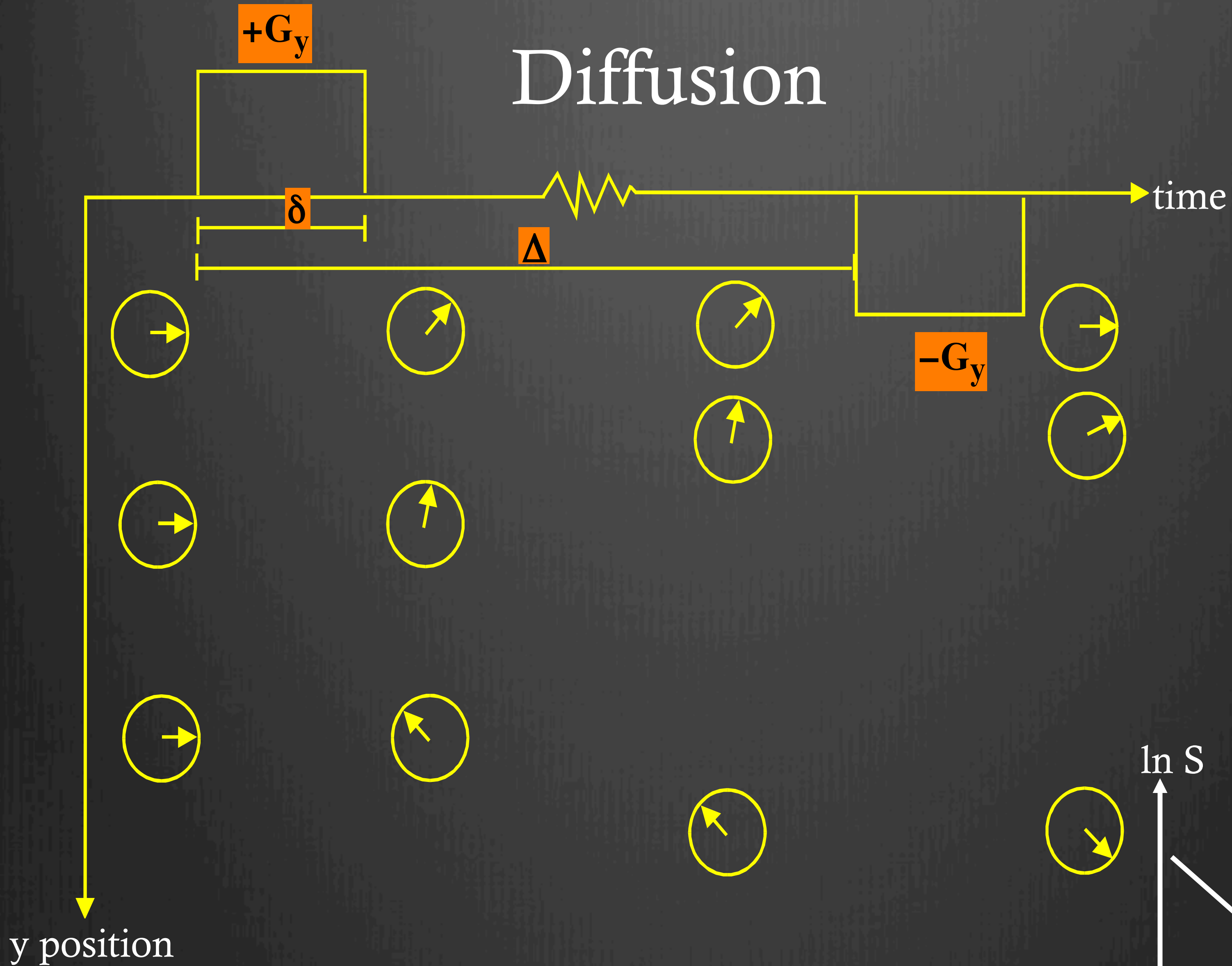
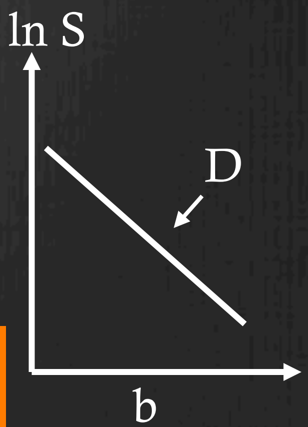
Diffusion



$$S(G_y) = S(0)\ \exp(-b \cdot D) \qquad b = \gamma^2 \cdot \delta^2 \cdot G^2\ (\Delta - \delta / 3)$$



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