

Fourier Analysis [6 CFU] (first semester)

Outline: The denomination *Fourier Analysis* refers to Fourier series, Fourier and Laplace transforms, and other classical integral transforms. These notions play a fundamental role in both classical and modern analysis, in particular in the study of ordinary and partial differential equations. This theory has also important applications to science and engineering: in particular it provides the language for quantum mechanics, signal theory, and other disciplines.

Prerequisites (in parenthesis the corresponding courses in Trento):

Differential and Integral Calculus, with Fourier series and ODEs (*Analisi I, II e III*).

Measure theory and Lebesgue integration (*Analisi III*).

Linear algebra (*Geometria I*).

General topology (*Geometria II*).

Banach and Hilbert spaces, functional spaces, linear and continuous operators (*Analisi Funzionale*).

Main contents: (details are provided by the lecture notes)

Review of Fourier series. Periodic functions. Trigonometric and exponential representation of Fourier series. Convergence of Fourier series. Fourier series in L^2 .

Schwartz distributions. Test functions. Distributions. Differentiation of distributions. Support and order of distributions. Compactly supported distributions. Tempered distributions. Convolution.

Fourier transform. The Fourier transform in L^1 , \mathcal{S} , \mathcal{S}' and L^2 . Fourier transform and ODEs.

Laplace transform. Laplace transform of functions and distributions. Laplace transform and ODEs.

Linear systems. Time-space and frequency-space. Time-invariance. Impulse response and transfer function. Filters and filter composition.

Fourier series and musical theory. Application of Fourier analysis to the construction of the musical scale. Equal temperament.

The formulation of examples and the solution of problems are an important part of the study of Fourier analysis. Several exercises will be provided and solved during the classes.

Texts: Teacher's lecture notes (available on the web).

Complementary texts (all available at the University Library in Povo):

C. Gasquet, P. Witomski: *Fourier analysis and applications*. Springer, New York 1999

G. Gilardi: *Analisi tre*. McGraw-Hill, Milano 1994

M. Marini: *Metodi matematici per lo studio delle reti elettriche*. C.E.D.A.M., Padova, 1999

Modality of exam: Written and oral examinations.